NUMERICAL PHYSICS

VOL. I

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NUMERICAL PHYSICS

VOL. I

(FOR CLASS XI)

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PREFACE TO THE SECOND EDITION

It is my great pleasure to present the second revised edition of the book in a very short period of time. It has been possible due to the interest which dear students and worthy teachers have shown in the book.

The book has been revised thoroughly. The chapter entitled 'Introduction' has been included. The contents of chapter 'Molecule, kinetic theory of gases and elasticity' has been reorganised and divided into four chapters under headings Molecule, Properties of Solids, Properties of Liquids and Properties of Gases.

The salient features of this revised edition are listed below :

1. Numerical problems drawn from various examinations including C.B.S.E.'S Exam. 1989 and Competitive Exams. of I.I.T., C.P.M.T. etc. have been included.
2. Difficult problems for brilliant students have been included in each chapter.
3. Each chapter includes a large number of 'objective type questions' with their answers given at the end of the book.

Suggestions for the improvement of the book will be acknowledged with thanks.

V.K. AGRAWAL

EXTRACTS FROM THE PREFACE TO THE FIRST EDITION

This book has been written according to the syllabus in Physics prescribed for class XI of Delhi and All India Senior School Certificate Examinations by the Central Board of Secondary Education, New Delhi.

Each topic has been tried to be made problem-based. The material included in the book is such that it makes the study of Physics more comprehensive and illustrative. It is my sincere hope that the book will be helpful to all students of Physics at this level.

V.K. AGRAWAL

CONTENTS

Chapter — *Page No.*

1. Introduction 1
2. Physical World and Measurements 6
3. Motion in One, Two and Three Dimensions and Laws of Motion 27
4. Work, Energy and Power 82
5. Rotational Motion 101
6. Gravitation 124
7. Molecules 142
8. Properties of Solids 156
9. Properties of Liquids 163
10. Properties of Gases 189
11. Heat and Thermodynamics 198
12. Harmonic Oscillations 223
13. Wave Motion 243

Answers 280
Appendix 288
Tables 297

1

Introduction

IMPORTANT DEFINITIONS

1. Science. The word 'science' in Latin means "to know". The well organized attempt of man to know about the nature and natural phenomena occurring around him is Science.

2. Physics. The word 'Physics' in Greek means 'nature'. It is that branch of Science which deals with nature, natural phenomenon, properties of materials and about different type of energies in nature.

3. Theory. On the basis of results of experimentation, the natural phenomena are explained in terms of specific law(s) or relationship(s). It is called 'theory'.

4. Physics and Technology. The technological development of any society is very closely related with the applications of Science in general and Physics in particular.

SOLVED EXAMPLES

Example 1. *Match the scientist's name with the discovery they made.*

A. *Scientist*	B. *Discovery*
1. Faraday	Law of gravitation.
2. Rutherford	Quantum model of hydrogen atom.
3. Chadwick	Unification of light and electromagnetism.
4. Bohr	Theory of Relativity.
5. Newton	Inelastic scattering of light by molecules.
6. Maxwell	Unification of weak and electromagnetic interaction.
7. Salam	Law of electromagnetic induction.
8. Einstein	Expansion of universe.
9. Raman	Neutron.
10. Hubble	Nuclear model of the atom.

Solution :

1.	Faraday	Law of electromagnetic induction.
2.	Rutherford	Nuclear model of the atom.
3.	Chadwick	Neutron.
4.	Bohr	Quantum model of hydrogen atom.
5.	Newton	Law of gravitation.
6.	Maxwell	Unification of light and electromagnetism.
7.	Salam	Unification of weak and electromagnetic interaction.
8.	Einstein	Theory of Relativity.
9.	Raman	Inelastic scattering of light by molecules.
10.	Hubble	Expansion of universe.

Example 2. *Match the scientist in Column A with the country of origin in Column B.*

A. *Scientist*	B. *Country of origin*
1. Michelson	Denmark
2. Newton	U.S.A.
3. Landau	Italy
4. Bhabha	France
5. Bohr	India
6. Archimedes	Germany
7. Galileo	U.S.S.R.
8. Curie	Britain
9. Heisenberg	Japan
10. Yukawa	Greece

Solution :

1. Michelson	U.S.A.
2. Newton	Britain
3. Landau	Germany
4. Bhabha	India
5. Bohr	Denmark
6. Archimedes	Greece
7. Galileo	Italy
8. Curie	France
9. Heisenberg	Japan
10. Yukawa	U.S.S.R.

Example 3. *Science, like an information and knowledge, can be put to good or bad use, depending on the user.*

(Given below are some of the applications of science. Formulate your views on whether the particular application is good, bad or something that cannot be so clearly categorised.

(*i*) Mass vaccination against smallpox to curb and finally eradicate this disease from the population. (This has already been succesfully done in India).

(*ii*) Television for eradication of illiteracy and for mass communication of news and ideas.

(*iii*) Parental sex determination.

(*iv*) Computers for increase in work efficiency.

(*v*) Putting artificial satellites around the earth.

(*vi*) Development of nuclear weapons.

(*vii*) Development of new and powerful techniques of chemical and biological warfare.

(*viii*) Purification of water for drinking.

(*ix*) Plastic surgery.

(*x*) Cloning.)

Solution :

(*i*) The smallpox kills many and those servived they get permanently disfigured. Many children becomes blind and deaf. Therefore mass vaccination against smallpox is good.

(*ii*) Television for eradication of illiteracy and for mass communication of news and ideas is good. But so many serials and feature films are telecasted and children want to view all of them. So their studies are suffered and eyesight is affected. So it is bad also.

(*iii*) Parental sex determination is good. The parents may decide whether they wants the child of that sex or not.

(*iv*) Computers for increase in work efficiency are good. There is no chance of personal errors and the efficiency increases.

(*v*) Putting artificial satellites around the earth is good. It helps us in transmitting T.V. programme and also help us in forecasting weather.

(*vi*) Development of nuclear weapons is not good. Nuclear weapons used in a war can kill many innocent people. They spread radioactivity which is very harmful to mankind.

(*vii*) Development of new and powerful techniques of chemical and biological warfare is good depending upon use. They have been devised for our safety and security. Misuse of them can not be ruled out.

(*viii*) Purification of drinking water is good. Many diseases are caused by drinking impure water taken directly from the earth.

(*ix*) Plastic surgery is good. It helps to make us beautiful and remove deformation.

(*x*) Cloning is good.

EXERCISE 1

1. Match the scientists with their inventions :

Scientists	*Inventions*
(*i*) Loi Galvani	(*a*) Transistor
(*ii*) Volta	(*b*) Telephone
(*iii*) Morse	(*c*) Electric current
(*iv*) Grahm Bell	(*d*) Television
(*v*) Baird	(*e*) Electric cell
(*vi*) William Shonkhle	(*f*) Telegraph.

2. Match the inventions with their inventors :

Inventions	*Inventors*
(*i*) Dynamo	(*a*) Roentgen
(*ii*) X-rays	(*b*) Faraday
(*iii*) Electric bulb	(*c*) James Watt
(*iv*) Radar	(*d*) Edison
(*v*) Steam engine	(*e*) Robert Watt

3. Match the following laws with their discoverer :

Laws	*Discoverers*
(*i*) Laws of motion	(*a*) Charles
(*ii*) Laws of gravitation	(*b*) Boyles
(*iii*) Law of floatation	(*c*) Robert Hooke
(*iv*) Law of gas pressure	(*d*) Pascal
(*v*) Law of elasticity	(*e*) Archimides
(*vi*) Law of gas volume	(*f*) Newton
(*vii*) Law of transmission of pressure in liquids.	

4. Match the following Scientists with their discovery :

Scientists	*Discovery*
(*i*) Autohon	(*a*) Thermos flask
(*ii*) Brush Bell	(*b*) Telescope
(*iil*) Dewar	(*c*) Atom Bomb
(*iv*) Galelio	(*d*) Aeroplane
(*v*) Wright Brothers	(*e*) Submarine

5. The discovery of the neutron was made by
   (*a*) Fermi (*b*) Chadwick
   (*c*) Rutherford (*d*) Niel Bohr

6. Prof. Albert Einstein was awarded the Nobel Prize in Physics for his theory of
   (*a*) Relativity (*b*) Specific heats
   (*c*) Scattering of α-particles (*d*) Distribution of charge in atom

7. The specific charge $\left(\frac{e}{m}\right)$ of an electron was determined by
   (*a*) Millikan (*b*) J.P. Thomson
   (*c*) J.J. Thomson (*d*) Albert Einstein

8. The charge on an electron was determined by
   (*a*) Albert Einstein (*b*) Michelson and Morley
   (*c*) Geiger and Marden (*d*) Millikan.

□□

2

Physical World and Measurements

IMPORTANT FORMULAE

(1) Dimensions :

	Quantity	Formula	Dimensions	Units
	Mechanics			
1.	Length		L	m
2.	Mass		M	kg
3.	Time		T	s
4.	Area	Length × Breadth	L^2	m^2
5.	Volume	Length × Breadth × Height	L^3	m^3
6.	Density	Mass/Volume	ML^{-3}	$kg\ m^{-3}$
7.	Velocity	Distance/Time	LT^{-1}	ms^{-1}
8.	Acceleration	Change in Velocity/ Time	LT^{-2}	ms^{-2}
9.	Force	Mass × Acceleration	MLT^{-2}	N
10.	Pressure & Stress	Force/Area	$ML^{-1}T^{-2}$	N/m^2
11.	Impulse	Force × Time	MLT^{-1}	$kg\ ms^{-1}$
12.	Work & Energy	Force × Displacement	ML^2T^{-2}	J
13.	Power	Work/Time	ML^2T^{-3}	w (watt)
14.	Surface Tension	Force/Length	MT^{-2}	Nm^{-1}
15.	Modulus of Elasticity	Stress/Strain	$ML^{-1}T^{-2}$	Nm^{-2}
16.	Angle	Arc/Radius	L^0	rad.
17.	Angular Velocity	Angle/Time	T^{-1}	$rad\ s^{-1}$

18.	Angular Acceleration	Change in Angular Velocity/Time	T^{-2}	rad s^{-2}
19.	Moment of Inertia	Mass × (Distance)^2	ML^2	kg m^2
20.	Torque (couple)	Force × Distance	ML^2T^{-2}	Nm
21.	Angular Momentum	Moment of Inertia × Angular Velocity	ML^2T^{-1}	kg m^2s^{-1}
22.	Gravitational constant	$G=\frac{Fx^2}{m_1m_2}$	$M^{-1}L^3T^{-2}$	Nm^2/kg^2
23.	Viscosity	Force/Area/ velocity gradiant	$ML^{-1}T^{-1}$	Pa.S
24.	Frequency	1/Time period	T^{-1}	S^{-1}
	Heat			
25.	Temperature	—	K	K
26.	Specific heat	Energy/mass/ Temperature difference	$L^2T^{-2}K^{-1}$	J $kg^{-1}K^{-1}$
27.	Thermal Conductivity	$\frac{\text{Heat}\times\text{Distance}}{\text{Area}\times\text{Temperature difference}\times\text{Time}}$	$MLT^{-3}k^{-1}$	J $m^{-1}k^{-1}s^{-1}$
28.	Latent heat	$\frac{\text{Heat}}{\text{Mass}}$	L^2T^{-2}	J kg^{-1}
29.	Coefficient of thermal expansion	$\frac{\text{Charge in dimension}}{\text{Initial dimension}\times\text{Rise in temperature}}$	k^{-1}	K^{-1}
	Electricity			
30.	Current	—	I	A
31.	Current density	Current/Area	IL^{-2}	Am^{-2}
32.	Charge	Current × time	IT	C
33.	Electric field	Force/charge	$MLT^{-3}I^{-1}$	N/C
34.	Electrical Potential	Work/charge	$ML^2T^{-3}I^{-1}$	J C^{-1} or V
35.	Electrical Capacitance	Charge/potential	$M^{-1}L^{-2}T^4I^2$	*f* (farad)
36.	Electrical inductance	Potential/Rate of charge of current	$ML^2T^{-2}I^{-2}$	H (henry)

37.	Permeability	$F=\mu\dfrac{I\,dl\sin\theta}{r^2}$	$ML^2T^{-2}I^{-1}$	—
38.	Magnetic field or Magnetic induction	Force/Charge/ Velocity	$MT^{-2}I^{-1}$	T (tesla)

(2) If a quantity '*a*' is supposed to depend upon quantities '*b*', '*c*', '*d*' etc., the relation between them is given by

$$a=k\,b^x\,c^y\,d^z \text{ etc.}$$

Where k is a dimensionless constant. The values of x, y, z, etc., can be determined by method of dimensions.

(3) Maximum permissible error :

(I) When $a=b.c.d.$

Max. permissible error in measurement of '*a*'

$$=\frac{\Delta b}{b}+\frac{\Delta c}{c}+\frac{\Delta d}{d}$$

(II) When $a=b^x\,c^y\,d^{-z}$

Max. permissible error in measurement of '*a*'

$$=x\frac{\Delta b}{b}+y\frac{\Delta c}{c}+z\frac{\Delta d}{d}$$

(4) Percentage error :

$$\%\text{ error}=\frac{\text{Difference between actual value \& calculated value}}{\text{Actual value}}\times 100$$

(5) 1 Fermi$=10^{-15}$ m, 1 Angstrom (Å)$=10^{-10}$ m, 1 light year$=9{\cdot}46\times10^{15}$ m.

SOLVED EXAMPLES

Example 1. *Find out the dimensions of viscosity and modulus of elasticity.*

Solution. The coefficient of viscosity or simply viscosity of a fluid is defined as the tangential force required per unit area to maintain a unit velocity gradient.

$$\therefore\ \text{Viscosity}=\frac{\text{Force}}{\text{Area}\times\text{Velocity gradient}}$$

$$=\frac{\text{Force}\times\text{Distance}}{\text{Area}\times\text{Change in velocity}}$$

$\therefore$ Dimension of viscosity

$$=\frac{MLT^{-2}\times L}{L^2\times LT^{-1}}$$

$$=ML^{-1}T^{-1}$$

Modulus of elasticity of a material is defined as the ratio of stress to the strain.

∴ Modulus of elasticity

$$= \frac{\text{Stress}}{\text{Strain}}$$

$$= \frac{\text{F/A}}{\text{Strain}}$$

$$= \frac{\text{F}}{\text{A} \times \text{Strain}}$$

∴ Dimension of elasticity

$$= \frac{\text{MLT}^{-2}}{\text{L}^2} = \text{ML}^{-1}\,\text{T}^{-2}$$

Example 2. *Find the dimensions of Electrical potential and electrical capacitance.*

Solution. Electrical potential

$$= \frac{\text{Work}}{\text{Charge}}$$

∴ Dimensions of potential $= \frac{\text{ML}^2\,\text{T}^{-2}}{\text{IT}}$

$$= \text{ML}^2\text{T}^{-3}\text{I}^{-1}$$

Electrical capacitance $= \frac{\text{Charge}}{\text{Potential}}$

∴ Dimensions of capacitance $= \frac{\text{IT}}{\text{ML}^2\text{T}^{-3}\text{I}^{-1}}$

$$= \text{M}^{-1}\text{L}^{-2}\text{T}^4\text{I}^2$$

Example 3. *Find out the dimensions of gravitational constant.*

Solution. We know $\text{F} = \text{G}\,\frac{m_1 m_2}{x^2}$

∴ Gravitational constant,

$$\text{G} = \frac{\text{F}.\,x^2}{m_1 m_2}$$

∴ Dimensions of gravitational constant

$$= \frac{\text{MLT}^{-2} \times \text{L}^2}{\text{M} \times \text{M}}$$

$$= \text{M}^{-1}\text{L}^3\text{T}^{-2}$$

Example 4. *What are the dimensions of power ?*

[*D.S.S.E. 1981*]

Solution.

$$\text{Power}=\frac{\text{Work}}{\text{Time}}$$

$$=\frac{\text{Force}\times\text{Displacement}}{\text{Time}}$$

$$=\frac{\text{MLT}^{-2}\times\text{L}}{\text{T}}$$

$$=\text{ML}^2\text{T}^{-3}$$

Example 5. *Check the dimensional consistency in the case of the following equations :* *[A.I.S.S E. 1981, 1985]*

(*a*) $S=ut+\frac{1}{2}at^2$

(*b*) $v=u+at$

(*c*) $\text{F.S.}=\frac{1}{2}mv^2-\frac{1}{2}mv_0^2$

Solution. (*a*) Dimension of $S=L$

Dimension of $ut=LT^{-1}\times T=L$

Dimension of $\frac{1}{2}at^2=LT^{-2}\times T^2=L$

Since dimension of each term in the given equation is same, the equation is correct dimensionally.

(*b*) $v=u+at$

Dimension of $v=LT^{-1}$

Dimension of $u=LT^{-1}$

Dimension of $at=LT^{-2}\times T=LT^{-1}$

Hence the given equation is dimensionally correct.

(*c*) $\text{F.S.}=\frac{1}{2}mv^2-\frac{1}{2}mv_0^2$

Dimension of $\text{F.S.}=MLT^{-2}\times L=ML^2T^{-2}$

Dimension of $\frac{1}{2}m_0v^2=M(LT^{-1})^2=ML^2T^{-2}$

Dimension of $\frac{1}{2}mv_0^2=M(LT^{-1})^2=ML^2T^{-2}$

Hence the given equation is dimensionally correct.

Example 6. *Check the consistency of the following equation :*

$$c=\sqrt{\frac{T}{m}}$$

Solution. $c=\sqrt{\frac{T}{m}}$ is the formula for the velocity of transverse waves on a string under tension (T), '*m*' is being mass per unit length of string.

$\therefore$ Dimension of $c=LT^{-1}$

Dimension of $\sqrt{\frac{T}{m}}=\sqrt{\frac{MLT^{-2}}{M/L}}$

$$=LT^{-1}$$

Hence the equation is dimensionally correct.

Example 7. *The velocity 'c' of sound waves through a medium may be assumed to depend on (i) the density 'd' of the medium (ii) its modulus of elasticity 'E'. Deduce by the method of dimensions the formula for velocity of sound.*

Solution. Let $C \propto d^x E^y$

or

$$c = kd^x E^y \text{ where } k \text{ is dimensionless constant.}$$

Putting the dimensions of each quantity, we have

$$LT^{-1} = (ML^{-3})^x \, (ML^{-1}\,T^{-2})^y$$

or

$$M^0 L^1 T^{-1} = M^{x+y} L^{-3x-y}\, T^{-2y}$$

$$\therefore \qquad -2y = -1 \text{ and } x+y=0$$

$$\therefore \qquad y=\tfrac{1}{2} \text{ and } x=-y=-\tfrac{1}{2}$$

Experimentally we know $k=1$

$$\therefore \qquad c = d^{-\frac{1}{2}}.\, E^{\frac{1}{2}}.$$

$$\therefore \qquad c = \sqrt{\frac{E}{d}}$$

Example 8. *A liquid is flowing steadily through a pipe. The volume of the liquid issuing out per second may be assumed to depend on (i) the coefficient of viscosity 'η', (ii) the radius 'r' of the pipe, and (iii) the pressure gradiant (p/l) along the pipe. Deduce by the method of dimensions formula for the volume of the liquid flowing out per second.*

Solution. Let volume issuing out per sec. be

$$V' \propto \eta^x r^y \left(\frac{p}{l}\right)^z$$

or

$$V' = k\eta^x r^y \left(\frac{p}{l}\right)^z$$

where k is a dimensionless constant.

Putting the dimensions of each quantity, we have

$$L^3T^{-1} = (ML^{-1}T^{-1})^x \, L^y \left(\frac{ML^{-1}T^{-2}}{L}\right)^z$$

or

$$M^0L^3T^{-1} = M^{x+z}\, L^{-x+y-2z}\, T^{-x-2z}$$

$$\therefore \qquad x+z=0$$

$$\therefore \qquad \begin{array}{r} -x-2z=-1 \\ \hline -z=-1 \end{array}$$

or

$$\boxed{z=1}$$

Now $x+z=0$

$$x+1=0$$

$$\boxed{x=-1}$$

$$-x+y-2z=3$$

$$-(-1)+y-2\,(1)=3$$

$$\boxed{y=4}$$

Experimentally we know $k=\frac{\pi}{8}$

$$\therefore \qquad V^1=\frac{\pi}{8}\,\eta^{-1}\,r^4\left(\frac{p}{l}\right)^1$$

$$\therefore \qquad \boxed{V'=\frac{\pi p r^4}{8\eta l}}$$

Example 9. *Assuming that time period of a simple pendulum depends upon length of pendulum and acceleration due to gravity, deduce the formula for time period of the simple pendulum.*

[*D.S.S.E. 1988*]

Solution : Let $t \propto l^x g^y$

or $t=kl^x g^y$ where k is a dimensionless constant.

Putting the dimensions of each quantity, we have

$$T=L^x(LT^{-2})^y$$

$$L^0T^1=L^{x+y}T^{-2y}$$

$$\therefore \qquad -2y=1$$

$$\text{or} \qquad \boxed{y=-\frac{1}{2}}$$

$$x+y=0 \quad \text{or} \quad x=-y$$

$$\text{or} \qquad \boxed{x=\frac{1}{2}}$$

Experimentally we know $k=2\pi$

$$t=2\pi\, l^{\frac{1}{2}}\; g^{-\frac{1}{2}}$$

or
$$\boxed{t=2\pi\sqrt{\frac{l}{g}}}$$

Example 10. *In the formula $x=3yz^2$, x and z have dimensions of capacitance and magnetic induction respectively. Find the dimensions of y in MKSQ system.* (*I.I.T.J.E.E. 1988*)

Solution. $y=\frac{x}{3z^2}$

$$\therefore \text{ Dimensions of } y=\frac{M^{-1}L^{-2}T^4I^2}{(MT^{-2}I^{-1})^2}$$
$$=M^{-3}L^{-2}T^8I^4$$
$$=M^{-3}L^{-2}T^4(T^4I^4)$$
$$=M^{-3}L^{-2}T^4Q^4$$

∴ Dimensions of y in MKSQ system
$$=K^{-3}M^{-2}S^4Q^4$$

[In MKSQ system M stands for metre (unit for L), K for kg (unit of M), S for sec. (unit of T) and Q for charge.]

Example 11. *What are the dimensions of (a) surface tension (b) modulus of elasticity and (c) power in terms of linear momentum (p), Area (A) and density (D).*

Solution. We know
$$p=MLT^{-1} \Rightarrow T=MLp^{-1}$$
$$A=L^2 \quad \Rightarrow L=A^{\frac{1}{2}}$$
$$D=ML^{-3} \Rightarrow M=DL^3$$

∴ We find,
$$L=A^{1/2}$$
$$M=DA^{3/2}$$
$$T=DA^2p^{-1}$$

(*a*) Dimensions of surface tension
$$=MT^{-2}=DA^{3/2}\,(DA^2p^{-1})^{-2}$$
$$=D^{-1}A^{-\frac{5}{2}}\ p^2$$

(*b*) Dimensions of modulus of elasticity
$$=ML^{-1}T^{-2}=DA^{3/2}\,(A^{1/2})^{-1}\,(DA^2p^{-1})^{-2}$$
$$=D^{-1}A^{-3}p^2$$

(*c*) Dimension of power
$$=ML^2T^{-3}=DA^{3/2}\,(A^{1/2})^2\,(DA^2\,p^{-1})^{-3}$$
$$=D^{-2}\,A^{-\frac{7}{2}}\,p^{-3}$$

Example 12. *In an experiment with simple pendulum, the time period measured was 40 seconds for 20 vibrations when the length of the pendulum was taken as 100 cm.*

(*a*) *If the least count of stop watch is 0·1 second and that of metre scale is 0·1 cm., calculate the maximum permissible error in measurement of value of 'g'.*

(*b*) *If actual value of 'g' at Delhi is 9·7915 ms*$^{-2}$, *calculate the percentage error.*

Solution. (*a*) We know that the time period of a simple pendulum is given by

$$T=2\pi\sqrt{\frac{l}{g}}$$

$$\therefore \qquad g=4\pi^2\frac{1}{T^2}=4\pi^2\frac{l}{(t/20)^2}$$

where t is the time of 20 vibrations.

Taking log of both sides,

$$\log g=\log 4+2\log\pi+\log l+2\log 20-2\log t$$

Differentiating, we have

$$\frac{\Delta g}{g}=0+0+\frac{\Delta l}{l}+0-2\frac{\Delta t}{t}$$

Converting the negative error into positive one for max. effect,

We have
$$\frac{\Delta g}{g}=\frac{\Delta l}{l}+\frac{2\Delta t}{t}$$

$$\therefore \qquad \frac{\Delta g}{g}=\frac{0{\cdot}1}{100}+\frac{2\times0{\cdot}1}{40}$$
$$=0{\cdot}001+0{\cdot}005$$
$$=0{\cdot}006.$$

$\therefore$ Max. permissible error$=0{\cdot}006\times100=0{\cdot}60\%$.

As maximum contribution to total error in the result is due to the individual error 0·5% in the measurement of time so 't' should be measured very carefully.

(*b*)
$$g=4\pi^2\frac{l}{T^2}$$

Now
$$l=100\text{ cm}=1\text{ m}$$

$$T=\frac{40}{20}=2\text{ sec.}$$

$$\therefore \quad g=4\pi^2 \frac{1}{2^2}$$

$$=\pi^2$$

$$=9{\cdot}8696 \text{ ms}^{-2}.$$

$$\therefore \quad \text{Percentage over}=\frac{(9{\cdot}8696-9{\cdot}7915)}{9{\cdot}7915}\times 100$$

$$=0{\cdot}7976\%.$$

Example 13. *In an experiment to determine the value of Young's modulus of elasticity of the steel, a wire of steel of 325 cm in length (measured by a metre scale with least count 0·1 cm) is loaded by 2 kg wt. and it is found that it stretches by 0·227 cm (measured by a micrometer having least count 0·001 cm). The diameter of the wire as measured by a Screw Gauge (Least count=0·001 cm) is found to be 0 043 cm. Calculate (a) maximum permissible error (b) percentage error if actual value of 'Y' for steel is* 20×10^{10} *N/m².*

Solution (*a*) $$Y=\frac{FL}{Al}=\frac{4Mg\,L}{\pi D^2 l}$$

Taking log of both sides,

$$\log Y=\log 4+\log M+\log g+\log L-\log \pi -2\log D-\log l$$

Differentiating, we have

$$\frac{\Delta Y}{Y}=0+0+0+\frac{\Delta L}{L}-0-2\frac{\Delta D}{D}-\frac{\Delta l}{l}$$

∴ Maximum permissible error

$$=\left(\frac{\Delta L}{L}+\frac{2\Delta D}{D}+\frac{\Delta l}{l}\right)\times 100\%$$

$$=\left(\frac{0{\cdot}1}{325}+\frac{2\times 0{\cdot}001}{0{\cdot}043}+\frac{0{\cdot}001}{0{\cdot}227}\right)\times 100\%$$

$$=(0{\cdot}00031+0{\cdot}0465+0{\cdot}0044)\times 100\%$$

$$={\cdot}05121\times 100\%$$

$$=5{\cdot}121\%.$$

As the maximum contribution to the total error in the result is due to individual error 4·65% in the measurement of D. Hence diameter should be measured most carefully.

(*b*) $$Y=\frac{4Mg\,L}{\pi D^2\,l}$$

$$=\frac{4\times 2\times 9{\cdot}8\times 3{\cdot}25\times 7}{22\times(43\times 10^{-5})^2\,(227\times 10^{-5})}\text{ N/m}^2$$

$$=19{\cdot}31\times 10^{10}\text{ N/m}^2$$

$$\therefore \text{ Percentage error} = \frac{(20\times10^{10}-19{\cdot}31\times10^{10})}{20\times10^{10}}\times100$$
$$=3{\cdot}45\%.$$

Example 14. *A calorie is a unit of heat energy and it equals about 4·2 J, where 1 J=1 kg m² S⁻². Suppose we employ a system of units in which the unit of mass equals* α *kg, the unit of length equals* β *m and the unit of time in* γ *S. Show that a calorie has a magnitude of* $4{\cdot}2\,\alpha^{-1}\,\beta^{-2}\,\gamma^{-2}$ *in terms of the new units.*

Solution. 1 calorie=4·2 Joule

The dimensions of energy $=ML^2T^{-2}$

$\therefore$ New unit of energy, $1=\alpha \text{ kg } (\beta \text{ m})^2 (\gamma \text{ S})^{-2}$

or $$1=\alpha\,\beta^2\,\gamma^{-2}\text{ kg m}^2\text{ S}^{-2}$$

$\therefore$ $$\alpha^{-1}\,\beta^{-2}\,\gamma^2=1\text{ kg m}^2\text{ S}^{-2}$$

or $$\alpha^{-1}\,\beta^{-2}\,\gamma^2=1\text{ J}$$

or $$1\text{ J}=\alpha^{-1}\,\beta^{-2}\,\gamma^2$$

or $$\frac{1}{4{\cdot}2}\text{ calorie}=\alpha^{-1}\,\beta^{-2}\,\gamma^2$$

or $$1\text{ calorie}=4{\cdot}2\,\alpha^{-1}\,\beta^{-2}\,\gamma^2.$$

Example 15. *A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of new unit, if light takes 8 min and 20 s to cover this distance.*

Solution. 8 min 20 s= 500 s

Since in each 1 *s* light travels 1 unit distance (in new unit of length).

$\therefore$ Distance between earth and sun=500 units.

Example 16. *The photograph of a house occupies an area of 1·75 cm² on a 35 mm slide. The slide is projected on to a screen and the area of the house on the screen is 1·55 m². What is linear magnification of the projector screen arrangement.*

Solution.

$$\text{Linear magnification}=\sqrt{\frac{1{\cdot}55\times10^4\text{ cm}^2}{1{\cdot}7\ \ 5\text{ cm}^2}}$$
$$=\sqrt{8857{\cdot}1428}$$
$$=94{\cdot}11.$$

Example 17. *The length, breadth and thickness of a rectangular sheet of metal are 4·234 m, 1·005 m and 2·01 cm respectively. Give the area and volume of sheet to correct significant figures.*

Solution. Area of all the 6 faces of the sheet.

$$=2(l\times b+b\times h+h\times l)$$
$$=2(4{\cdot}234\times 1{\cdot}005+1{\cdot}005\times {\cdot}0201+{\cdot}0201\times 4{\cdot}234)$$
$$=2(4{\cdot}25517+{\cdot}0202005+0{\cdot}0851034)$$
$$=8{\cdot}72\ \text{m}^2$$

Volume $=l\times b\times h$

$$=4{\cdot}234\times 1{\cdot}005\times {\cdot}0201\ \text{m}^3$$
$$=0{\cdot}0855\ \text{m}^3.$$

Note. Since thickness is given only in 3 significant figures so area and volume are calculated in 3 significant figures.

Example 18. *A physical quantity P is related to four observations a, b, c and d as follows :*

$$P=a^3b^2\sqrt{cd}$$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error is the quantity P. If the value of P calculated using above relation turns out to be 3·763 to what value should you round off the result ?

Solution. Since $P=a^3b^2\,(cd)^{1/2}$

$$\%\text{ error in } P=3(\%\text{ error in } a)+2(\%\text{ error in } b)$$
$$+\tfrac{1}{2}(\%\text{ error in } c+\%\text{ error in } d)$$
$$=3\times 1\%+2\times 3\%+\tfrac{1}{2}(4\%+2\%)$$
$$=12\%.$$

Since percentage error in P has 2 significant figures, P must also be expressed in 2 significant figures.

$\therefore$ 3·763 should be round off to 3·8.

Example 19. *The unit of length convenient on the atomic scale is known as Angstorm and is denoted by Å : 1Å$=10^{-10}$ m. The size of the hydrogen atom is about 0·5 Å. What is the total atomic volume in m^3 of 1 mole of hydrogen atoms ?*

Solution. 1 mole of hydrogen atoms$=6{\cdot}023\times 10^{23}$ atoms

Volume of 1 H_2 atom $=\dfrac{4}{3}\pi r^3$

$$=\frac{4}{3}\times\frac{22}{7}\,(0{\cdot}5\times 10^{-10}\ \text{m})^3$$

$\therefore$ **Volume of a mole of hydrogen atom**

$$=6{\cdot}023\times 10^{23}\times\frac{4}{3}\times\frac{22}{7}\times 0{\cdot}125\times 10^{-30}\ \text{m}^3$$
$$=3{\cdot}155\times 10^{-7}\ \text{m}^3.$$

Example 20. *The mole of an ideal gas at STP occupies 22·4 l (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen atom ? (Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large ?*

Solution. Atomic volume of a mole of hydrogen atoms

$$=3{\cdot}155\times10^{-7}\ \text{m}^3$$
$$=6{\cdot}023\times10^{23}\times\frac{4}{3}\times\frac{22}{7}\times(1\times10^{-10})^3$$
$$=2{\cdot}524\times10^{-6}\ \text{m}^3$$

(See Solved Example 19)

Molar volume of 1 mole of Hydrogen gas

$$=22{\cdot}4\ \text{litre}$$
$$=22{\cdot}4\times10^{-3}\ \text{m}^3$$
$$\frac{\text{Molar Volume}}{\text{Atomic Volume}}=\frac{22{\cdot}4\times10^{-3}}{2{\cdot}524\times10^{-6}}$$
$$=8874{\cdot}8.$$

Intermolecular spacing in a gas is much larger than the size of a molecule.

Example 21. *A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77·0 s. What is the distance of the enemy submarine ? (Speed of sound in water=1450 ms^{-1}).*

Solution. If x is the distance of the enemy submarine from ship, then

$$\text{Distance}=\text{Velocity}\times\text{Time}$$
$$2x=1450\times77\ \text{m}$$
$$x=725\times77\ \text{m}$$
$$=55{\cdot}825\ \text{km}.$$

EXERCISE 2

1. Deduce the dimensions of volocity, acceleration and force.
2. Deduce the dimensions of area, volume and density.
3. Deduce the dimensions of moment of a force and energy. [*H.P.S.S.E. 1988 ; A.I.S.S.E., 1985*]
4. Obtain the dimensions of angular momentum and viscosity. [*A.I.S.S.E. 1986*]
5. Deduce the dimensions of moment of inertia. [*A.I.S S.E. 1988*]
6. What are the dimensions of power ? [*A.I.S.S E. 1981*]
7. Deduce the dimensions of torque and angular velocity.
8. Find out the dimensions of surface tension and magnetic induction.

9. Find the dimensions of magnetic field. (*A.I.S.S.E. 1980*)

10. Check the dimensional consistency in the case of the following equations :

(*a*) Centripetal acceleration,

$$a = \frac{v^2}{R}$$ [*A.I.S.S.E. 1979*]

(*b*) $S = v_i t + \frac{1}{2} a t^2$ [*A.I.S.S.E. 1985*]

(*c*) $v_f^2 = v_i^2 + 2aS$.

(*d*) $v = \frac{1}{2l}\sqrt{\frac{T}{m}}$

(*e*) $t = 2\pi\sqrt{\frac{l}{g}}$

(*f*) $\nu = \frac{1}{2\pi}\sqrt{\frac{c}{I}}$

(*g*) $c = \sqrt{\frac{E}{d}}$

(*h*) $c = \sqrt{\frac{T}{m}}$

(*i*) $S = \frac{1}{2} g t^2$

(*j*) $S = \frac{v^2}{2g}$

(*k*) $T = \frac{rh\rho g}{2\cos\theta}$

(*l*) $F = 6\pi\eta r v$.

(*m*) Volume of liquid issuing out per sec.

$$V' = \frac{\pi p r^4}{8\eta l}$$

(*n*) F.S. $= mgh$

(*o*) $E = mc^2$

(*p*) $\rho = \frac{3g}{4\pi r G}$

(*q*) $P = \frac{1}{3}\frac{mNc^2}{V}$.

11. Find out the dimensions of gravitational constant and permeability.

12. Find out the dimensions of charge, electric field and inductance.

13. The time period T of a simple pendulum may depend upon (*i*) the length of the pendulum '*l*', (*ii*) acceleration due to gravity '*g*' and (*iii*) mass of the bob '*m*'. Obtain dimensionally a formula for the time period 'T'. [*A.I.S.S.E. 1986*]

14. Reynold number N_R (a dimensionless quantity) determines the condition of laminar flow of a viscous liquid through a pipe. N_R is the function of the density of liquid 'ρ', its average speed 'v' and the coefficient of viscosity of liquid 'η'. Given that N_R is also directly proportional to 'd' the diameter of the pipe. Show from dimensional consideration that

$$N_R \propto \frac{d\rho v}{\eta}$$

[The unit of η in S.I. system is kg $m^{-1}s^{-1}$] [*A.I.S.S.E. 1980*]

15. A gas bubble from an explosion under water oscillates with a period 't' proportional to $P^a d^b E^c$ where P is the pressure, d is the density and E is the energy of the explosion. Find the values of a, b and c. [*I.I.T. J.E.E. 1981*]

16. Derive an expression for excess of pressure inside a soap bubble by method of dimensions, assuming that excess pressure may depend upon (*i*) radius of the bubble 'r' and (*ii*) the surface tension of liquid 'T' (dimensionless constant $k=4$).

17. The capillary rise (h) in a tube may be assumed to depend upon (*i*) density of liquid 'ρ', (*ii*) surface tension of liquid 'T'. (*iii*) radius of tube 'r'. Given that 'h' is inversely proportional to acceleration due to gravity 'g'. Deduce an expression for capillary rise by method of dimensions.
[Dimensionless constant $k=2\cos\theta$]

18. The terminal velocity (v) of a spherical body moving in a liquid may assume to depend upon (*i*) viscosity of liquid 'η', (*ii*) radius of sphere 'r' and (*iii*) viscous force 'R'. Prove by method of dimensions that

$$v=\frac{R}{6\pi\eta r}$$

19. The largest mas 'm' of a stone that can be moved by flowing water in a river depends upon (*i*) the density of water 'ρ', (*ii*) velocity of flow of water 'v' and (*iii*) acceleration due to gravity 'g'. Deduce a relationship among m, ρ, v and g by method of dimensions.
[*Delhi University (1st year) 1974, 1978*]

20. Prove by the method of dimensions that period (t) of oscillation of a vibrating drop is given by

$$t=K\sqrt{\frac{r^3\rho}{T}}$$

where r=radius of drop, T=surface tension

and ρ=density of liquid.

21. The frequency 'v' of oscillation of sonometer wire may be assumed to depend on (*i*) the mass per unit length of the wire 'm' (*ii*) tension acting on the wire 'T' and (*iii*) length of the wire 'l'. Deduce by methods of dimension an expression for 'v'.

22. An artificial satellite of mass 'm' is revolving in a circular orbit around a planet of mass 'M' and radius 'R'. If the radius of the orbit of the satellite be r, justify by the method of dimensions that the time period of the satellite is given by

$$t=\frac{2\pi}{R}\sqrt{\frac{r^3}{g}}$$

[*D.U Pre. Med. 1977*]

23. If a planet moves round the Sun in a circular orbit and its time period 't' is found to depend upon the mean radius of the orbit 'r', the mass of the sun 'M' and the universal gravitational constant 'G'. Prove by the method of dimensions that

$$\frac{t^2}{\surd^3}=a \text{ constant} \qquad [A.I.S.S.E.\ 1981]$$

24. Prove by methods of dimensions that acceleration due to gravity 'g' inside the surface of the earth is given by

$$g=\frac{4}{3}\,G\pi R\rho$$

where G=gravitational constant,
R=radius of the earth,
and ρ=mean density of the earth.

25. An open U-tube of uniform area of cross-section contains mercury. Let the length of whole mercury column be L. The mercury in one limb is depressed a little and then allowed to oscillate freely. Prove by the methods of dimensions that time period of oscillation is given by

$$T=2\pi\sqrt{\frac{L}{2g}}$$

26. The frequency of oscillation of a tuning fork may be supposed to depend upon the length of the prong 'l', density of material 'ρ' and Young's modulus of elasticity 'E'. Deduce the expression for frequency of oscillation by method of dimensions.

27. The velocity of sound 'C' in air at constant temperature depends upon the pressure 'P' and density 'ρ' of air. Deduce by method of dimensions an expression for velocity of sound.

28. Obtain Stoke's law ($R=6\pi\eta rv$) by method of dimensions.

29. Find the dimensions of (*a*) Force, (*b*) momentum, (*c*) surface tension in terms of frequency, velocity and density as fundamental units.

30. Find the dimensions of (*a*) Linear momentum (*b*) angular momentum (*c*) Young's modulus in terms of velocity, acceleration and force as fundamental quantities.

31. Deduce by the method of dimensions an expression for the energy 'E' of a particle executing simple harmonic motion which may be assumed to depend upon (*i*) the mass 'm', (*ii*) the frequency 'v' and (*iii*) the amplitude of vibration 'a'.

32. The velocity v of water waves depends on wavelength 'λ', density of water 'ρ' and acceleration due to gravity 'g'. Deduce by the method of dimensions the relation between v, λ, ρ and g. [*A.I.S.S.E. 1980*]

15331

33. Find out the dimension of k in the following expression for the force acting between two charges q_1 and q_2 separated by a distance 'd'

$$F = k\ \frac{q_1 q_2}{d^2}$$

34. The hydrostatic pressure 'p' exerted by a liquid may be assumed to depend upon height 'h' density 'ρ' and acceleration due to gravity 'g'. Find how does p depend on h, ρ and g by method of dimensions.

35. Escape velocity 'v' for a planet may be assumed to depend upon the radius of the planet 'r' and acceleration due to gravity 'g' on the surface of the planet. Deduce relation between v, r, and g by method of dimensions.

36. The time period 'T' of a mass oscillating vertically on a spring may be assumed to depend upon force constant (*i.e.*, force per unit displacement) 'k' and (*ii*) mass of the body and (*iii*) acceleration due to gravity 'g'. Deduce by method of dimensions expression for T.

37. Obtain the dimensions of the coefficient of thermal conductivity of a solid.

38. Deduce by method of dimensions the expression for the
    (*a*) centripetal force in terms of mass, speed and radius of circular path.
    (*b*) energy in terms of mass of the body and its velocity.

39. A screw with threads of pitch 0·5 mm is available. How can you use it to make a screw gauge reading upon 0·01 mm. (*I.I.T. J.E.E. 1966*)

40. In a vernier callipers 10 vernier scale divisions coincide with 9 main scale division. Find the least count of the vernier calliper if main scale division is 1 mm.

41. A circular scale of a screw guage is divided into 100 divisions. It moves a linear distance of 5 mm when given 10 complete rotations. Find its pitch and least count.

42. The following observations were made in an experiment to find the radius of curvature of a concave mirror using a spherometer.

$$l = 4{\cdot}4 \text{ cm}, \qquad h = 0{\cdot}085 \text{ cm}$$

Find (*a*) the maximum permissible error and (*b*) percentage error if actual radius of curvature of concave mirror is 40 cm.

43. In an experiment to determine the value of Young's modulus of elasticity of brass, a wire of brass of length 312 cm (measured by a metre scale with least count 0·1 cm) is loaded by

1·5 kg wt and it is found that it stretches by 0·449 cm (measured by a spherometer having least count 0·001 cm). The diameter of the wire (as measured by a screw gauge of least count=0·001 cm) is found to be 0·37 mm. Calculate (*a*) maximum permissible error (*b*) percentage error if actual value of Y for brass is 9×10^{10} N/m^2.

44. In an experiment with simple pendulum, 20 vibration take 44·4 sec. (as measured by a stop watch having least count 0·1 sec.) for length of the pendulum as 120·5 cm. (as measured by a metre scale having least count 0·1 cm). Calculate (*a*) the maximum permissible error and (*ii*) the percentage error if the actual value of '*g*' at Delhi is 9·7915 ms^{-2}.

45. If in the above Question No. 44, 25 vibrations take 44·9 sec. for the length of pendulum as 79·8 cm. Calculate the two said errors in the question.

46. A laser is a source of very intense, monochromatic and unidirectional beam of light. A laser beam aimed at the Moon takes 2·56 S to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the earth ?

47. The sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K and its outer surface at a temperatures no substance remains in a solid or liquid phase. In what range do you except the mass density of the sun to be ? In the range of densities of solids and liquids or gases ? Check if your guess is correct from the following data : mass of the sun $2\cdot0\times10^{30}$ kg, radius of the sun$=7\cdot0\times10^8$ m.

OBJECTIVE TYPE QUESTIONS

48. Dimensions of energy are

(*a*) MLT^{-2} (*b*) $M^2L^2T^{-2}$

(*c*) ML^2T^{-2} (*d*) ML^2T^{-2}. [*A.I.I.M.S. 1982*]

49. The dimensions of the quantities in one (or more) of the following pairs are same. Identify the pair(s)

(*a*) Torque and work

(*b*) Angular momentum and work

(*c*) Energy and Young's modulus

(*d*) Light year and wavelength. [*I.I.T. J.E.E. 1986*]

50. L, C and R represent the physical quantities inductance, capacitance and resistance respectively. The combination which have the dimension of frequency are

(*a*) 1/RC (*b*) R/L

(*c*) $1/\sqrt{LC}$ (*d*) C/L. [*I.I.T. J.E.E. 1984*]

51. Match the physical quantities given in Column I with dimensions expressed in terms of mass (M), length (L), time (T) and charge (Q) given in Column II.

I	II
(*a*) Angular momentum	1. ML^2T^{-2}
(*b*) Latent heat	2. ML^2Q^{-2}
(*c*) Torque	3. ML^2T^{-1}
(*d*) Capacitance	4. $ML^3T^{-1}Q^{-2}$
(*e*) Inductance	5. $M^{-1}L^{-2}L^2Q^2$
(*f*) Resistivity	6. L^2T^{-2}

[*I.I.T. J.E.E. 1983*]

52. The dimensions of Plank's constant are the same as that of

(*a*) Energy
(*b*) Mass
(*c*) Frequency
(*d*) Angular momentum.

[*A.I.I.M.S. 1983*]

53. Which one of the following pairs have same dimension

(*a*) Work and power
(*b*) Momentum and impulse
(*c*) Density and relative density
(*d*) Planck's constant and energy. [*C.P.M.T. 1988*]

54. Dimensional formula for torque is

(*a*) L^2MT^2
(*b*) $L^{-1}M^1T^{-2}$
(*c*) L^2MT^{-2}
(*d*) LMT^{-2}.

[*M.A.M.C. 1983*]

55. Dimensional formula for gravitational constant is

(*a*) L^2MT^{-3}
(*b*) L^3MT^{-2}
(*c*) $L^3M^{-1}T^{-2}$
(*d*) $L^0M^0T^0$.

[*M.A.M.C. 1982*; *C.P.M.T. 1988*]

56. Which of the following combinations have same dimensions :

(*a*) Work, couple and power
(*b*) Impulse, momentum
(*c*) Force, momentum, pressure
(*d*) Pressure, force, stress. [*C.P.M.T. 1988*]

57. Match the dimensions given in Column B with physical quantities in Column A :

Column 'A'	Column 'B'
(*a*) Couple	1. ML^2T^{-3}
(*b*) Surface tension	2. $ML^{-1}T^{-2}$
(*c*) Momentum	3. ML^2T^{-2}

(*d*) Power 4. MT^{-2}
(*e*) Modulus of Elasticity 5. MLT^{-1}

58. The period of a body under SHM is represented by $T = p^a \rho^b \sigma^c$ where p is pressure, ρ is density and σ is the surface tension, the values of a, b and c are

(*a*) $-\frac{3}{2}, \frac{1}{2}, 1$ (*b*) $\frac{1}{2}, -\frac{3}{2}, 1$

(*c*) $-1, -2, 3$ (*d*) $1, 2, \frac{1}{3}$

[*B.H.U. 1983*]

59. The dimension of velocity gradient are
(*a*) $ML^{-1}T^{-1}$ (*b*) $M^0L^0T^{-1}$
(*c*) ML^0T^{-1} (*d*) none of them.

60. The dimensions of pressure in terms of density (D), velocity (V) and time (T) are
(*a*) DV^2 (*b*) DVT
(*c*) $DV^{-2}T^2$ (*d*) $DV^{-1}T^{-2}$.

61. What is the dimensions of pressure gradient in terms of linear mementum (P), Area (A) and density (D) ?
(*a*) $DA^{-\frac{5}{2}}P^{-2}$ (*b*) $D^{-1}A^{\frac{3}{2}}P^2$
(*c*) $D^{-1}-A^{\frac{5}{2}}P^2$ (*c*) $D^{-1}A^{-\frac{7}{2}}P^2$.

62. In Vander Waal's equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT,$$

the dimensions of a are
(*a*) $ML^{-5}T^{-2}$ (*b*) ML^5T^{-2}
(*c*) $M^0L^{-6}T^0$ (*d*) $M^0L^6T^0$.

63. The distance travelled by a body in the *n*th second is given by

$$S_{nth} = V_i + \tfrac{1}{2}a\,(2n+1),$$

the dimensions of 'V_i' and 'a' are—
(*a*) L^0T, LT^{-2} (*b*) LT^0, L
(*c*) LT^0, LT^{-1} (*d*) LT^{-1}, LT^{-2}.

64. Fill in the blanks :
(*a*) The volume of cube of side 5 cm is equal to……m^3.
(*b*) The surface area of a sphere of radius 2 cm is equal to ……mm^2.

(*c*) Sataabdi Express even moves with a speed 144 km/h. It covers......m in 1 S.

(*d*) The relative density of mercury is 13·6. Its density is ......g cm^{-3} or......kg m^{-3}.

65. State the number of significant figures in the following :

(*a*) $5{\cdot}278 \times 10^{12}$ (*b*) 0·0005208 m
(*c*) 0·520 s (*d*) 4·30 kg
(*e*) $8{\cdot}002 \times 10^{-5}$ N (*f*) 16·00 cm
(*g*) 20·006 mm. (*h*) 0·017 m.

66. Fill in the blanks :

(*a*) I m=......fermi. (*b*) 1 m=......light year
(*c*) 1 Å=......cm (*d*) 45 km/hr=......m/s
(*e*) I g/c.c.=......kg/m^3

67. The % error in the measurement of effective length (l) is 2% and in time period (T) is 1%. The maximum permissible error in the measurement of g will be

(*a*) 3% (*b*) 2%
(*c*) 6% (*d*) 4% [*C.P.M.T. 1988*]

68. The percentage error in the measurement of length of a square is 3%. The error in area will be

(*a*) 3% (*b*) 9%
(*c*) 6% (*d*) 12% [*C P.M.T. 1988*]

69. The density of water in C.G.S. is 1 g/c.c. The density in M.K.S. unit is

(*a*) 10 kg/m^3 (*b*) 1 kg/litre
(*c*) 10^{-3} kg/m^3 (*d*) 10^3 kg/m^3. [*C.P.M.T. 1988*]

□□

3

Motion in One, Two and Three Dimensions and Laws of Motion

IMPORTANT FORMULAE

1. For a body moving with a variable velocity, we take average velocity (V) which is the ratio of total distance (S) travelled by the body to the total time (t) taken, *i.e.*

$$V=\frac{S}{t}$$

2. For a body moving with uniform velocity, its velocity at any time is given by

$$U=\frac{S}{t}$$

3. The instantaneous velocity at any instant is given by

$$V=\frac{dS}{dt}$$

4. For a body moving with uniform acceleration 'a',

$$\left.\begin{aligned} V_f&=V_i+at \\ S&=V_it+\tfrac{1}{2}at^2 \\ V_f^2&=V_i^2+2aS \\ S_{nth}&=V_i+\tfrac{1}{2}a\,(2n+1) \end{aligned}\right\}\quad \begin{aligned} &\text{where } V_f=\text{final velocity} \\ &V_i=\text{Initial Velocity} \\ &S=\text{Distance travelled} \\ &S_{nth}=\text{Distance in } n\text{th Second.} \end{aligned}$$

[**Note.** Always consider $a=+g$ for a bodying falling freely for its vertical downward motion and $a=-g$ if it is projected up for its vertical upward motion where g is the acceleration due to gravity.]

5. For the motion of a body in a plane projected with velocity 'V'$_0$ making an angle 'θ' with the horizontal *i.e.* for projectile motion.

$$V_x=(V_0 \cos\theta)\ t$$

$$V_y=(V_0 \sin\theta)\ t-\tfrac{1}{2}gt^2$$

$$V=\sqrt{V_x^2+V_y^2}$$

$$\text{Time of flight}=\frac{2V_0 \sin\theta}{g}$$

$$\text{Range of projectile} = \frac{V_0^2 \text{Sin } 2\theta}{g}$$

$$\text{Maximum range} = \frac{V_0^2}{g}$$

6. Resolution of a vector 'V' into two perpendicular component give one component as V sin θ and the other component as V cos θ where θ is the angle which vector 'V' makes with component V cos θ.

7. $A = \lambda a + \mu b$ where λ and μ are real numbers.

8. The maximum force of static friction, $f_{ms} = \mu_s N$
where N=normal relation.

9. If a body is just to slide down on an inclined plane,

$$f_{ms} = mg \sin \theta$$

and $$N = mg \cos \theta$$

∴ $$\mu_s = \tan \theta$$

10. Force required not to allow the body to move down the inclined plane, $f_1 = mg\ (\sin \theta - \mu_s \cos \theta)$.

11. Force required to move the body up an inclined plane,

$$f_2 = mg\ (\sin \theta + \mu_s \cos \theta)$$

12. Angular velocity, $\omega = \frac{\theta}{t} = \frac{V}{r}$

∴ Linear velocity, $V = 2\pi nr$ where n=no. of revolutions per sec.

13. $F = ma$

14. $F = m_1 a_1 = m_2 a_2$

15. $mv = MV$

16. $(m_1 u_1 + m_2 u_2) = (m_1 v_1 + m_2 v_2)$

17. $(\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_1 u_2^2) = (\frac{1}{2} m_1^2 v_1^2 + \frac{1}{2} m_2 v_2^2)$

18. If a body of mass 'm_1' moving with velocity V_{1i} collides with a body of mass 'm_2' at rest, their respective velocity after collision will be.

$$\left.\begin{aligned} V_{1f} &= \frac{(m_1 - m_2)}{(m_1 + m_2)} V_{1i} \\ V_{2f} &= \frac{2m_1}{(m_1 + m_2)} V_{1i} \end{aligned}\right\}$$

It is supposed that the two bodies after collision moves in the same direction.

19. Resultant of two vectors P and Q acting at an angle 'θ' with each other

$$R = \sqrt{(P^2 + Q^2 + 2PQ \cos \theta)}$$

20. Impulse$=F.t=m(v-u)$

21. Apparent weight of a body in the lift.

$R=m(g+a)$ when lift moves upwards with uniform acceleration 'a',

$R=m(g-a)$ when lift moves downwards with uniform acceleration 'a',

and $R=mg$, if the lift moves with uniform velocity (since $a=o$)

SOLVED EXAMPLES

Example 1. *A car moving along a straight highway with speed 126 km/h is brought to rest within a distance of 200 m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop ?*

Solution. $V_i=126\times\frac{5}{18}$ m/s$=35$ m/s

$V_f=0$, $S=200$ m

By $2aS=V_f^2-V_i^2$

$2\times a\times 200=0-35\times 35$

$a=-3{\cdot}0625$ ms^{-2}

$\therefore$ Retardation$=3{\cdot}06$ ms^{-2} (only 3 significant figures)

By $V_f=V_i+at$

$0=35-3{\cdot}0625\ t$.

$t=\frac{35}{3{\cdot}06}=11{\cdot}4$ S

Example 2. *Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km/h in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 ms^{-2}. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them ?*

Solution. For train B.

$V_i=72$ km/h$=72\times\frac{5}{18}=20$ ms^{-1}

$a=1$ms^{-2}, $t=50$ S

$\therefore$ By $S=V_it+\frac{1}{2}at^2$,

$S=20\times 50+\frac{1}{2}\times 1\times 50^2$

$=1000+1250$

$S=2250$ m.

For train A,

$$U = 20 \text{ ms}^{-1}, \quad t = 50 \text{ S}$$

By
$$\begin{aligned} S &= U \times t \\ &= 20 \times 50 \\ &= 1000 \text{ m} \end{aligned}$$

∴ Original distance between two trains

$$\begin{aligned} &= (2250 - 1000) \\ &= 1250 \text{ m}. \end{aligned}$$

Example 3. *An aeroplane is flying due north with a velocity of 80 km/h. If wind blows due to east at 20 km/h. Find the resultant velocity of aeroplane with reference to earth.*

Solution. $V_1 = 80$ km/h, $V_2 = 20$ km/h

∴ Resultant velocity,

$$\begin{aligned} V &= \sqrt{(V_1^2 + V_2^2)} \\ &= \sqrt{(80^2 + 20^2)} = 80{\cdot}46 \text{ km/h} \end{aligned}$$

$$\tan\theta = \frac{V_1}{V_2} = \frac{80}{20} = 4$$

∴ $\theta = 76°$ north of east.

Example 4. *Fuel is consumed at the rate of 100 kg per second in a rocket. The exhaust gases are enjected at a speed of $4{\cdot}5 \times 10^4$ ms^{-1}. What is the thrust experienced by the rocket ?*

Solution. Force of ejecting exhaust gases,

$$\begin{aligned} F &= m\,a \\ &= \frac{m\,(V_f - V_i)}{t} \end{aligned}$$

Now $m = 100$ kg, $t = 1$ s

$$V_i = 0 \text{ and } V_f = 4{\cdot}5 \times 10^4 \text{ ms}^{-1}$$

∴
$$\begin{aligned} F &= \frac{100\,(4{\cdot}5 \times 10^4 - 0)}{1} \\ &= 4{\cdot}5 \times 10^6 \text{ N}. \end{aligned}$$

There will be a reaction (thrust) equal and opposite to action force (F).

∴ The thrust experienced by the rocket $= 4{\cdot}5 \times 10^6$ N.

Example 5. *A particle has a displacement of 12 m towards east and 5 m towards the north and then 6 m vertically upwards. Find the magnitude of the sum of these displacements.* [*D B.S.S.E , 1978*]

Solution. Since displacements of 12 m due east and 5 m due north are at right angle to each other, their resultant,

$$\begin{aligned} R &= \sqrt{12^2 + 5^2} \\ &= \sqrt{144 + 25} \\ &= \sqrt{169} \\ &= 13 \text{ m}. \end{aligned}$$

Resultant displacement R=13 m is in horizontal direction and displacement 6 m is in vertical direction. So they are at right angle to each other, hence the resultant of given three vectors,

$$R' = \sqrt{13^2+6^2}$$
$$= \sqrt{205}$$
$$= 14{\cdot}32 \text{ m.}$$

Example 6. *An aeroplane takes off at an angle of 60° to the horizontal. If the component of its velocity along horizontal is 200 km h^{-1}. What is the actual velocity ? Find also the vertical component of the velocity.*

Solution. Let V be the actual velocity of aeroplane, then horizontal component=V cos θ

$$\therefore \quad 200 = N \cos 60$$
$$\therefore \quad V = \frac{200}{\cos 60} = 400 \text{ km/h}$$

Vertical component=V sin θ

$$= 400 \sin 60$$
$$= 400 \times \frac{\sqrt{3}}{2}$$
$$= 200 \times 1{\cdot}732$$
$$= 346{\cdot}4 \text{ km/h.}$$

Example 7. *A truck of mass 2500 kg is moving at the speed of 54 km/h and is acted upon by two forces of 500 N due to engine and a retarding force of 100 N due to friction. Calculate the acceleration of truck and distance moved by it in 10 minutes.*

Solution. $F = 500 - 100 = 400$ N

$$m = 2500 \text{ kg}$$
$$a = \frac{F}{m} = \frac{400}{2500} = 0{\cdot}16 \text{ ms}^{-2}$$

Now $V_i = 54$ km/h $= 54 \times 5/18 = 15$ ms^{-1},

$t = 10$ min $= 10 \times 60 = 600$ s

$\therefore$ By $\quad S = V_i t + \frac{1}{2}at^2.$

$$S = 15 \times 600 + \tfrac{1}{2} \times 0{\cdot}16 \times 600^2$$
$$= 9000 + 28800 = 37800 \text{ m}$$
$$= 37{\cdot}8 \text{ km.}$$

Example 8. *A boy whose mass is 50 kg is standing on a weighing machine inside a lift which is moving with an acceleration of 2ms^{-2}. What is the weight of the boy as recorded by weighing machine when* *(a) lift is going upward, (b) lift is going downward, (c) lift is moving with uniform velocity. (Take $g = 10\ ms^{-2}$).*

Solution. (*a*) $R_1 = m(g+a)$

$$= 50(10+2)$$

$$= 600\ N = \frac{600}{10}\ \text{kg wt.} = 60\ \text{kg wt.}$$

(*b*) $R_2 = m(g-a)$

$$= 50(10-2)$$

$$= 400\ N = 40\ \text{kg wt.}$$

(*c*) $R_3 = mg$ [Since $a = 0$]

$$= 50\ \text{kg wt.}$$

Example 9. *Two objects of mass 5 kg and 3 kg respectively are connected by a light cord passing over a horizontal frictionless pulley. What is the acceleration of the system and tension in the cord ?*

Solution. $F = 5\times 9{\cdot}8 - 3\times 9{\cdot}8$

$$= 2\times 9{\cdot}8 = 19{\cdot}6\ N$$

$$m = (5+3) = 8\ kg$$

∴ The acceleration of the system.

$$a = \frac{F}{m}$$

$$= \frac{19{\cdot}6}{8} = 2{\cdot}45\ ms^{-2}$$

Appiying Newton's 2nd law of motion

$$(T - 3\times 9{\cdot}8) = 3\times 2{\cdot}45 \qquad [F = ma]$$

$$T = 3(9{\cdot}8 + 2{\cdot}45)$$

$$= 3\times 12{\cdot}25$$

$$= 36{\cdot}75\ N$$

[**Note**. The students may also work out the value of T considering the forces on the mass 5 kg alone].

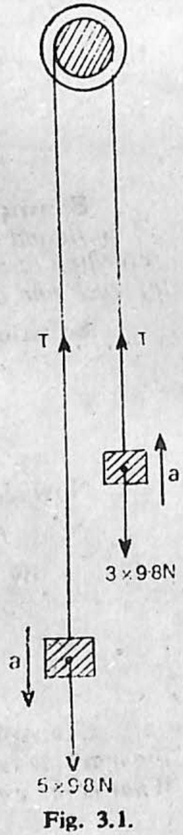

Fig. 3.1.

Example 10. *A roller of mass of 400 kg is attached by a light horizontal chain to a tractor of mass 1000 kg. The backward force of friction exerted by ground is 800 N. If the system has a forward acceleration of $2ms^{-2}$. Calculate the forward force of ground on tractor and the tension in the chain.*

Solution. If F is the forward force of the ground on the tractor, by Newton's 2nd law of motion,

$$(F-800)=2\times 1400$$

$$F=3600 \text{ N}$$

Considering the tractor alone, if T is the tension in chain,

$$(3600-T)=2\times 1000$$

$$T=1600 \text{ N}$$

(**Note.** The students may work out the value of T considering the roller alone).

Example 11. *A body of mass 2 kg makes an elastic collision with another body at rest and continues to move in the original direction with a speed equal to one-third of its original speed. Find the mass of the second body.*

Solution. $m_1=2$ kg, $m_2=?$

$$u_1=u, \qquad u_2=0$$

$$v_1=\tfrac{1}{3}u$$

$$\therefore \quad m_1v_1+m_2v_2=m_1u_1+m_2u_2$$

$$2\times\frac{u}{3}+m_2v_2=2\times u+0$$

$$\therefore \quad m_2v_2=\frac{4}{3}u \qquad \text{...(1)}$$

By $\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2+\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2$

$$2\times\left(\frac{u}{3}\right)^2+m_2v_2^2=2u^2+0$$

$$\therefore \quad m_2v_2^2=\frac{16}{9}u^2 \qquad \text{...(2)}$$

Dividing eqn. (2) by eqn. (1)

$$v_2=\frac{4}{3}u$$

$\therefore$ By eqn. (1),

$$m_2=1 \text{ kg.}$$

Example 12. *A bomb is dropped from an aeroplane when it is directly above a target at a height of 1960 m. The aeroplane is going horizontally with a speed of 540 km h^{-1}. By how much distance will the bomb miss the target ?*

Solution. For the vertical downward motion of the bomb,

$$u=0,\ s=1960 \text{ m},\ a=+g=9{\cdot}8 \text{ ms}^{-2}$$

∴ By $s=v_0t+\frac{1}{2}at^2$

$$1960=0+\frac{1}{2}\times 9{\cdot}8\times t^2$$

$$t^2=\frac{1960\times 2}{9{\cdot}8}=400$$

$$t=20\ s$$

During this 20 s the bomb will move in horizontal direction a distance given by

$$s=u\times t$$

$$=\left(540\times\frac{5}{18}\right)\times 20$$

$$=3000\ \text{m}$$

∴ The bomb will miss the target by 3 km.

Example 13. *Rain is falling vertically with a speed of 30 ms^{-1}. A man rides a bicycle with a speed of 10 ms^{-1} in the north to south direction. What is the direction in which he should hold his umbrella ?*

Solution. The relative velocity of air with respect to man (supporting him stationary) is 10 ms^{-1}.

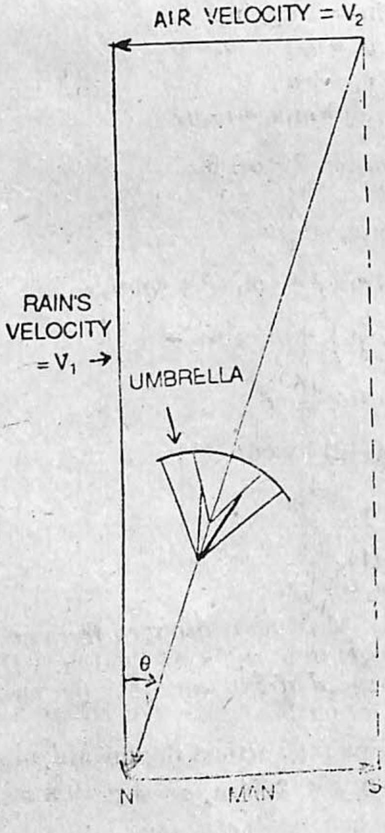

Fig. 3.2.

$\therefore \quad V_2 = 10\ \text{ms}^{-1}$ and $V_1 = 30\ \text{ms}^{-1}$

$$\therefore \quad \tan\theta = \frac{V_2}{V_1} = \frac{10}{30}$$

$$= \tfrac{1}{3} = 0{\cdot}3333$$

$$\therefore \quad \theta = 18^\circ\ 16'.$$

So the umbrella should be hold at 18° 16′ with vertical towards south.

Example 14. *In a harbour, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at the speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat ?*

Solution. In Fig. 3·3, $QE = PN = V_2$ (velocity of boat)

$$= 51\ \text{kmh}^{-1}$$

$$NE = V_1 \text{ (velocity of flag)}$$

$$= 72\ \text{kmh}^{-1}$$

$$\therefore \quad \sin\theta = \frac{QE}{NE} = \frac{51}{72}$$

$$\therefore \quad \sin\theta = 0{\cdot}7083$$

$\therefore$ From Natural sin table, $\theta = 5{\cdot}1^\circ$

$$\therefore \quad \angle QNS = (\theta + 45^\circ)$$

$$= (45{\cdot}1^\circ + 45^\circ)$$

$$= 90{\cdot}1^\circ$$

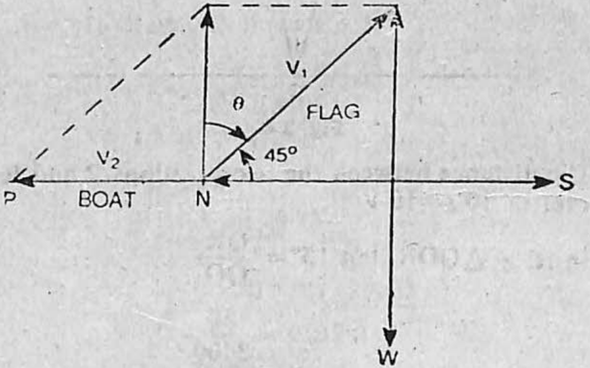

Fig. 3.3.

$\therefore$ The direction (NQ) of flag is at 90·1° from NS, *i.e.*, almost Eastward.

Example 15. *An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by aircraft positions 10 s apart is 30°, what is the speed of the aircraft ?*

Solution. Let velocity of aircraft be 'v' ms^{-1}.

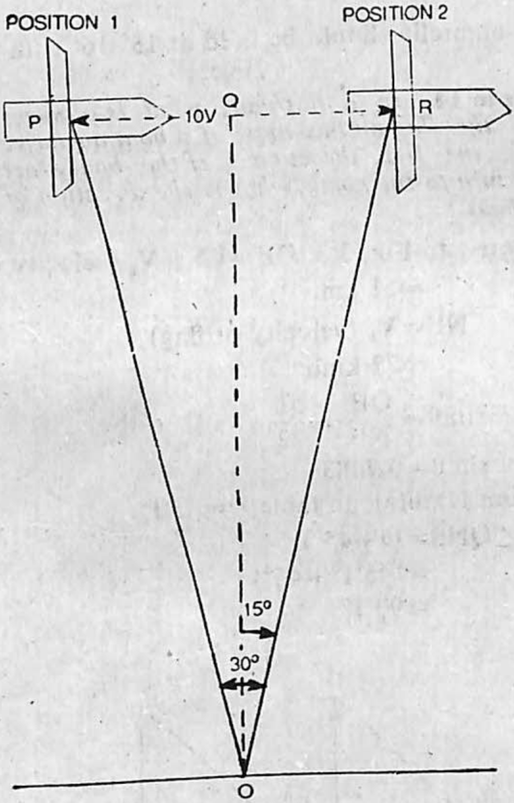

Fig. 3 4.

∴ The distance between the two positions P and R of aircraft at an interval of 10 $s = 10\ v$.

∴ In rt. ∠ΔOQR, tan $15° = \frac{QR}{QO}$

$$\therefore \quad 0{\cdot}2679 = \frac{5v}{3400}$$

$$\therefore \quad v = 182{\cdot}2 \text{ ms}^{-1}.$$

Example 16. *A man can swim with a speed of 4 km/h in still water. How long does he take to cross a river 1 km wide if the river* *flows steadily at 3 km/h and he makes his strokes normal to the river current ? How far down the river does he go when he reaches the other bank ? What is the velocity of swimming for man ?*

Solution. $S=1$ km, $U_1=4$ km/h

$$t=\frac{S}{U_1}$$

$$=\tfrac{1}{4}\text{ hr.}$$

$$t=15\text{ min.}$$

Due to flow of river at the speed of $U_2=3$ km/h, the man will go down the river by a distance,

$$S=U_2\times t$$

$$=3\times\tfrac{1}{4}\text{ km}$$

$$=750\text{ m}$$

If R is the resultant velocity of swimming for man,

$$R=\sqrt{U_1^2+U_2^2}$$

$$=\sqrt{4^2+3^2}$$

$$=5\text{ km/h.}$$

Example 17. *The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m/s can go without hitting the ceiling of the ball ?*

Solution. We know that for the vertical upward motion of the ball thrown with a velocity 40 m/s at an angle θ with the horizontal, $v_i=40\sin\theta$, $a=-g$ and $v_f=0$.

$\therefore$ By $2as=v_f^2-v_i^2$, we have

$$2\,(g)\times 25=0^2-(40\sin\theta)^2$$

$$\therefore\quad \sin\theta=\sqrt{\frac{2\times 9{\cdot}8\times 25}{40\times 40}}$$

$$=0{\cdot}5534,$$

$$\therefore\quad \theta=33{\cdot}6^\circ$$

$$\therefore\quad R=\frac{v_0^2\sin 2\theta}{g}$$

$$=\frac{40\times 40\times\sin(2\times 33{\cdot}6)}{9{\cdot}8}$$

$$=\frac{1600\times{\cdot}9219}{9{\cdot}8}$$

$$=150{\cdot}5\text{ m.}$$

Example 18. *A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball ?*

Solution. We know $R_{max} = \frac{v_0^2}{g}$

$$\therefore \quad v_0^2 = 100 \times g$$

For vertical upward motion of the ball projected vertically upward with the same velocity 'v_0',

$$s = ?, \; v_f = 0,$$

$$v_i = v_0 = \sqrt{100 \times g}$$

and $a = -g$

$\therefore$ By $2as = v_f^2 - v_i^2$

$$2(-g)\, s = o^2 - 100 \times g$$

$$s = \frac{100 \times g}{2g}$$

$$s = 50 \text{ m.}$$

Example 19. *A body of mass 5 kg is acted upon by two perpendicular forces 8 N and 6 N. Give the magnitude and direction of the acceleration of the body.*

Solution. Net force on the body,

$$F = \sqrt{(F_1^2 + F_2^2)}$$

$$= \sqrt{(8^2 + 6^2)}$$

$$F = 10 \text{ N}$$

and $m = 5$ kg

$\therefore$ By $F = ma$

$$a = \frac{10}{5} = 2 \text{ ms}^{-2}$$

If the direction of the acceleration is making an angle θ with F_1,

$$\tan \theta = \frac{F_2}{F_1} = \frac{6}{8} = 0{\cdot}75$$

$\therefore$ $\theta = 36° \, 52'$ with the direction of 8N force.

Example 20. *The driver of a three wheeler moving with a speed of 56 km/h sees a child standing in the middle of the road and brings his vehicle to rest in just 4 s time to save the child. What is the average retarding force on the vehicle ? The mass of the three wheeler is 400 kg and the mass of the driver is 65 kg.*

Solution. $v_i = 36 \text{ km/h} = 36 \times \frac{5}{18} = 10 \text{ ms}^{-1}$

$v_f = 0,\ t = 4s$

$\therefore$ By $v_f = v_i + at$

$0 = 10 + a \times 4$

$\therefore \quad a = -2{\cdot}5 \text{ ms}^{-2}$

$\therefore$ Average retardation $= 2{\cdot}5 \text{ ms}^{-2}$

$m = (400+65) = 465 \text{ kg}$

$\therefore$ Average retarding force,

$$\begin{aligned} \text{F} &= m \times \text{Average retardation} \\ &= 465 \times 2{\cdot}5 \\ &= 1162{\cdot}5 \text{ N.} \end{aligned}$$

Example 21. *A particle of mass 0·4 kg moving with a constant speed of 10 ms⁻¹ to the north is subject to a constant force of 8 N directed towards the south for 30 s. Take the instant the force is applied to be t=0, the position of the particle at that time to be x=0 and predict its position at t=5 s, 25 s and 100 s.*

Solution. $m = 0{\cdot}4 \text{ kg},\ \text{F} = 8 \text{ N}$

$$\therefore \quad a = \frac{\text{F}}{m} = \frac{-8}{0{\cdot}4} = -20 \text{ ms}^{-2}$$

Since the direction of force is opposite to that of velocity.

(*i*) For $t = -5 \text{ s},\ \text{U} = 10 \text{ ms}^{-1}$

(Since force is applied at $t=0$ that means the particle moved with uniform velocity up to $5s$).

$$\begin{aligned} \therefore \quad x &= \text{U} \times t \\ &= 10 \times -5 \\ &= -50 \text{ m} \end{aligned}$$

(*ii*) For $t = 25 \text{ s},\ v_i = 10 \text{ ms}^{-1},\ a = -20 \text{ ms}^{-2}$

$\therefore$ By $\text{S} = v_i t + \frac{1}{2} at^2$

$$\begin{aligned} x &= 10 \times 25 - \tfrac{1}{2} \times 20 \times 25^2 \\ &= (250 - 6250) \text{ m} \\ &= -6 \text{ km} \end{aligned}$$

(*iii*) For $t = 100$ s, we will calculate x in two parts.

In first part of motion the particle moved with $a = -20 \text{ ms}^{-2}$ for 30 s and in second part the particle moved for 70 s with a constant velocity it gained during 30 s.

$\therefore$ For $t = 30 \text{ s},\ v_i = 10 \text{ ms}^{-1},\ a = -20 \text{ ms}^{-2}$,

by $\text{S} = v_i t + \frac{1}{2} at^2$

$$\begin{aligned} x_1 &= 10 \times 30 - \tfrac{1}{2} \times 20 \times 30^2 \\ &= -8700 \text{ m} \end{aligned}$$

and by $v_f = v_i + at$,

$$v_f = 10 - 20 \times 30$$

$$v_f = -590 \text{ ms}^{-1}.$$

It will become uniform velocity for the further journey since now force ceases to act.

$\therefore$ For $t = 70$ s, $U = -590$ ms^{-1}

$\therefore$ By $S = U \times t$

$$x_2 = -590 \times 70$$

$$x_2 = -41300 \text{ m}$$

$$\therefore \quad x = (x_1 + x_2)$$

$$= (-8700 - 41300)$$

$$= -50 \text{ km}.$$

Example 22. *A car starts from rest and accelerates uniformly with 2ms^{-2}. At $t = 10$ s, a stone is dropped out of window (1 m high) of the car. What are the (a) velocity and (b) acceleration of the stone at $t = 10{\cdot}1$ s? (Neglect the air resistance).*

Solution. (*a*) For $t = 10$ s

$$v_i = 0, \; a = 2 \text{ ms}^{-2}$$

$\therefore$ By $v_f = v_i + at$

$$= 0 + 2 \times 10$$

$$v_f = 20 \text{ m/s}.$$

At $t = 10$ s, the stone is dropped in air so for further journey of stone it will move with constant velocity at 20 m/s.

$\therefore$ At $t = 10{\cdot}1$ s,

Velocity of stone in horizontal direction,

$$v_x = 20 \text{ m/s}$$

For the velocity of stone in vertical direction (V_y) after $0{\cdot}1$ s from dropping,

$$v_i = 0, \quad t = 0{\cdot}1 \text{ s}$$

$$a = +g \quad = 9{\cdot}8 \text{ ms}^{-2}$$

$\therefore$ By $v_f = v_i + at$

$$v_y = 9{\cdot}8 \times 1 = 0{\cdot}98 \text{ ms}^{-2}$$

$\therefore$ Resultant velocity of stone at $10{\cdot}1$ s,

$$V = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{20^2 + 0{\cdot}98^2}$$

$$= 20{\cdot}024 \text{ ms}^{-1}$$

If the resultant velocity is making an angle 'θ' with vertical,

$$\tan\theta = \frac{vx}{v_y}$$

$$= \frac{20}{0{\cdot}981}$$

$$\tan\theta = 20{\cdot}408$$

$$\therefore \qquad \theta = 87^\circ\ 12'$$

(*b*) Since acceleration in horizontal direction is zero, so net acceleration of stone is only in vertical downward direction which is, $g = 9{\cdot}8\ \text{ms}^{-2}$ at any time, so also at $t = 10{\cdot}1$ s, the acceleration of stone,

$$a = 9{\cdot}8\ \text{ms}^{-2}, \text{ vertically downward.}$$

Example 23. *A helicopter of mass 1000 kg rises with a vertical acceleration of 15 ms^{-2}. The crew and the passengers weigh 300 kg. Give the magnitude and direction of*

(*a*) *force on the floor by the crew and passengers.*

(*b*) *action of the rotor of the helicopter on the surrounding air.*

(*c*) *force on the helicopter due to surrounding air. (Take $g = 10$ ms^{-2}).*

Solution. (*a*) Force on the floor

$$= m(g+a)$$

$$= 300\ (10+15)$$

$$= 7500 \text{ N}$$

(*b*) Actual acceleration of helicopter in vertical upward direction,

$$a = (15+10) = 25\ \text{ms}^{-2}$$

Total mass which is rising in air,

$$m = (1000+300) = 1300 \text{ kg}$$

∴ Action on the surrounding air by the rotor of helicopter,

$$F = ma$$

$$= 1300 \times 25$$

$$= 32500 \text{ N}$$

(*c*) Since to every action there is an equal and opposite reaction, the force on the helicopter due to surrounding air = 32500 N.

Example 24. *A stream of water flowing horizontally with a speed of 15 ms^{-1} gushes out of a tube of cross-sectional area 10^{-2} m^2 and hits at a vertical wall nearby. What is the force exerted by the impact of water on the wall, assuming it does not rebound ?*

Solution. $F = ma$

$$= (V\rho)\left(\frac{v_f - v_i}{t}\right)$$

$$= \frac{V}{t}.\rho.(v_f - v_i)$$

where $\frac{V}{t}$ (volume of water gushes out per sec. out of the tube)

$$= \frac{A \times x}{t} = A \times v_i = 10^{-2} \times 15$$

ρ (density of water) $= 1000 \text{ kg m}^{-3}$

$v_f = 0$ and $v_i = 15 \text{ ms}^{-1}$

$\therefore$ $F = 10^{-2} \times 15 \times 1000\ (0 - 15)$

$= -2250 \text{ N}$

$\therefore$ Force of impact of water on the wall

$= 2250 \text{ N}$

Example 25. *A shell of mass 0·02 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 ms^{-1}. What is the recoil speed of the gun ?*

Solution. $m = 0{\cdot}02 \text{ kg}$; $v = 80 \text{ ms}^{-1}$

$M = 100 \text{ kg}$ and $V = ?$

We known $MV = mv$

$$100\ V = {\cdot}02 \times 80$$

$$V = \frac{{\cdot}02 \times 80}{100}$$

$$= {\cdot}016 \text{ ms}^{-1}$$

$$= 1{\cdot}6 \text{ cm s}^{-1}$$

Example 26. *A balloon is ascending at the uniform rate of 14 ms^{-1}. At a height of 98 m above the ground a packet is dropped from the balloon. After how much time and with what velocity does it reach the ground ?*

Solution. A body shares same velocity as the velocity of its carrier.

$\therefore$ For the vertical downward motion of the packet dropped from balloon,

$v_i = -14 \text{ ms}^{-1}$, $a = +g = 9{\cdot}8 \text{ ms}^{-2}$ and

$S = 28 \text{ m}$.

$\therefore$ By $v_f^2 = v_i^2 + 2aS$

$$= (-14)^2 + 2 \times 9{\cdot}8 \times 98$$

$$= 196 + 1920{\cdot}8$$

$$v_f = \sqrt{2116{\cdot}8}$$

$$= 46 \text{ ms}^{-1}$$

Now by

$$v_f = v_i + at$$

$$46 = -14 + 9{\cdot}8\ t$$

$$t = \frac{60}{9{\cdot}8} = 6{\cdot}12\ \text{s}$$

So packet reaches the ground with velocity 46 ms^{-1} and 6·12 s after dropping it from balloon.

[**Note.** The students may calculate the same velocity and time considering the motion of packet in two parts : (1) first it goes vertically upward till its velocity becomes zero and then, (2) falls vertically downward to reach the ground.]

Example 27. *A parachutist bails out from an aeroplane and after dropping through a distance of 40 m he opens the parachute and deaccelerates at 2 ms^{-2}. If he reaches the ground with a speed of 2 ms^{-1}, how long was he in air ? At what height did he bail out from the plane ?*

Solution. Before the parachut opens, the parachutist falls freely, so for his vertical downward motion (the aeroplane is flying horizontally),

$$v_i = 0,\ a = +g = 9{\cdot}8\ \text{ms}^{-2} \text{ and } S = 40\ \text{m}$$

∴ By

$$S = v_i t + \tfrac{1}{2} at^2$$

$$40 = 0 + \tfrac{1}{2} \times 9{\cdot}8 \times t^2$$

$$t^2 = \frac{40 \times 2}{9{\cdot}8} = \frac{400}{49}$$

$$t = \frac{20}{7}\ s = 2{\cdot}86\ s$$

and by

$$2\ aS = v_f^2 - v_i^2$$

$$2 \times 9{\cdot}8 \times 40 = v_f^2 - 0^2$$

∴

$$v_f^2 = 49 \times 16$$

$$v_f = 7 \times 4 = 28\ \text{ms}^{-1}$$

For further motion of the parachutist, 28 ms^{-1} becomes initial velocity.

∴

$$v_i = 28\ \text{ms}^{-1}, v_f = 2\ \text{ms}^{-1},\ a = -2\ \text{ms}^{-2}$$

∴ By

$$v_f = v_i + at$$

$$2 = 28 - 2\ t$$

$$t = \frac{-26}{-2} = 13\ \text{s}$$

and by

$$2\ as = v_f^2 - v_i^2$$

$$-2 \times 2\ s = 2^2 - 28^2$$

$$s = \frac{-780}{-4} = 195\ m$$

∴ The time for which parachutist was in air

$$=2{\cdot}86+13=15{\cdot}86 \text{ s}$$

and the height at which he bails out $=40+195=235$ m

Example 28. *A batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to 54 km/h. What is the impulse imparted to the ball ? (Mass of the ball is 0·15 kg).*

Solution. The component of initial and final velocity of 54 km/h may be considered along the bisector of angle 45° between initial and final directions.

$$\therefore \quad v_i=v_f=54 \cos 22\ 5 \text{ km/h}$$

$$=(54\times 0{\cdot}9239)\frac{5}{18}\text{ ms}^{-1}$$

$$=13{\cdot}86 \text{ ms}^{-1}$$

∴ Impulse imparted $=m\,[v_f-v_i]$

$$=0{\cdot}15\,[13{\cdot}86-(-13{\cdot}86)]$$

$=4{\cdot}16$ kg ms^{-1} directed along the bisector of the initial and final directions.

Example 29. *Two bodies A and B of masses 5 kg and 10 kg are in contact with each other. They rest on the horizontal table, against a vertical rigid partition, each in contact with table and body B is being in between body 'A' and partition. The coefficient of friction between the bodies and the table is 0·15. A horizontal force of 200 N is applied perpendicularly towards partition at A. What are (a) the reaction of partition, (b) the action and reaction forces between A and B ? (c) What happens when the partition is removed ? Does the answer to (b) change, when the bodies are in motion. Ignore difference between* μ_s *and* μ_k.

Solution. (*a*) Frictional force on two bodies,

$$f_{m_s}=\mu\,\text{N}$$

$$=\mu\,(m_1+m)_2\,g$$

$$=0{\cdot}15\,(5+10)\,9{\cdot}8$$

$=22$ N in opposite direction of the applied force

∴ Net force on the partition

$$=(200-22)=178 \text{ N}$$

∴ The reaction of the partition $=178$ N

(*b*) Frictional force on body '*a*',

$$f'_{m_s}=\mu\,m_1 g$$

$$=0{\cdot}15\times 5\times 9{\cdot}8$$

$=7$ N, in opposite direction of the applied force

∴ Net force on body 'B' by body 'A'

$$=(200-7)=193\ \text{N}$$

∴ The action and the reaction force between the bodies A and B

$$=193\ \text{N}.$$

(*c*) The net force on two bodies,

$$F=178\ \text{N}$$

So when partition is removed they will together move with an acceleration,

$$a=\frac{F}{m}$$

$$=\frac{178}{(5+10)}$$

$$=11{\cdot}9\ \text{ms}^{-2}$$

Example 30. *A bullet of mass 0·01 kg is fired horizontally into a wooden block at rest on a horizontal surface. The value of* μ_s *between the block and the surface is 0·25. The bullet gets embedded in the block and the combination moves 20 m before coming to rest. With what speed did the bullet strike the block ?* [*A.I.S.S.E. 1982*]

Solution. Kinetic energy of the bullet,

$$\text{K.E.}=\frac{1}{2}mv^2$$

$$=\frac{1}{2}\times0{\cdot}1\times v^2$$

The normal reaction on block,

$$N=mg=4\times9{\cdot}8\ \text{N}$$

∴ The frictional force on block,

$$f_{ms}=\mu N$$

$$=0{\cdot}25\times4\times9{\cdot}8\ \text{N}=9{\cdot}8\ \text{N}$$

∴ Force required to move the block,

$$F=9{\cdot}8\ \text{N}$$

∴ Work done on block,

$$W=F\times S$$

$$=9{\cdot}8\times20$$

$$=196\ \text{J}$$

Work on block is done by K.E. of bullet,

$$\therefore\quad \frac{1}{2}\times{\cdot}01\times v^2=196$$

$$v^2 = \frac{196 \times 2}{\cdot 01}$$

$$v = \frac{14\sqrt{2}}{0 \cdot 1} = 140 \times 1 \cdot 414$$

$$= 197 \cdot 96 \text{ ms}^{-1}$$

Example 31. *A box of mass 4 kg rests upon an inclined plane. The inclination of the plane to the horizontal is gradually increased. It is found that when the slope of the plane is 1 in 3, the box starts sliding down the plane (Given $g = 9 \cdot 8$ ms^{-2}).*

(*a*) *Find the coefficient of friction between the box and the plane.*

(*b*) *What force applied to the box parallel to the plane will just make it move up the plane ?*

Solution. (*a*) When the box is just to slide down on the inclined plane,

$$\mu = \tan \theta$$

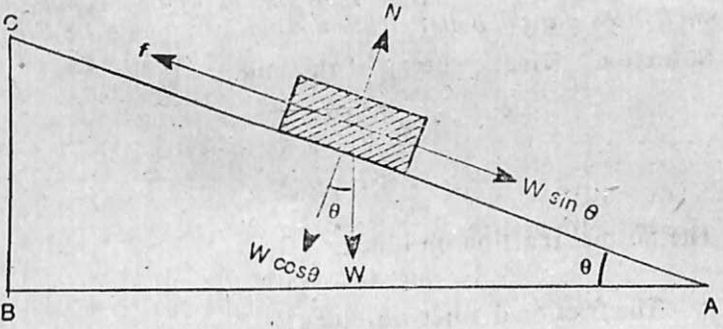

Fig. 3.5.

Given $\sin \theta = \frac{1}{3}$

$$\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - (\tfrac{1}{3})^2}$$

$$= \frac{2\sqrt{2}}{3}$$

$$\therefore \quad \mu = \frac{\sin \theta}{\cos \theta}$$

$$= \frac{1/3}{2\sqrt{2}/3} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$= \frac{1 \cdot 414}{4} = 0 \cdot 35$$

(*b*) If F is the minimum force required to move the box up the plane, then

$$\begin{aligned} F &= W \sin\theta + f \\ &= W\sin\theta + W\sin\theta \\ &= 2W\sin\theta \\ &= 2\times(4\times 9{\cdot}8)\tfrac{1}{3} = 26{\cdot}13 \text{ N} \end{aligned}$$

Example 32. *When an automobile moving with a speed of 36 km/h reaches the foot of an upward inclined road of angle 30°, its engine is switched off. The coefficient of friction involved is 0·1, how much distance will the automobile move before coming to rest. (Assume, $g=10\ ms^{-2}$)* [*A.I.S.S.E. 1980*]

Solution. Net downward force acting on automobile on inclined plane,

$$F = mg(\sin\theta + \mu\cos\theta)$$

∴ Retardation of automobile,

$$\begin{aligned} a &= \frac{F}{m} = g\,(\sin\theta + \mu\cos\theta) \\ &= 10\,(\sin 30 + 0{\cdot}1\,\cos 30) \\ &= 10\left(\frac{1}{2} + 0{\cdot}1\times\frac{\sqrt{3}}{2}\right) \\ &= 10(0{\cdot}5 + {\cdot}0866) \\ a &= 5{\cdot}866\ \text{ms}^{-2} \end{aligned}$$

Now $V_i = 36\ \text{km/h} = 36\times\frac{5}{18} = 10\ \text{m/s},\ V_f = 0$

and $a = -5{\cdot}866\ \text{ms}^{-2}$ (since it is retardation)

∴ By $2as = V_f^2 - V_i^2$

$$\begin{aligned} 2(-5{\cdot}866)s &= 0 - 10^2 \\ s &= \frac{100}{2\times 5{\cdot}866} \\ s &= 8{\cdot}5\ \text{m} \end{aligned}$$

Example 33. *From the top of a multi-storyed building 39·2 m high, a boy projects a stone vertically upwards with an initial velocity of $9{\cdot}8\ ms^{-1}$ such that it finally drops to the ground.*

(*a*) *When will the stone reach the ground ?*

(*b*) *When will it pass through the point of projection ?*

(*c*) *What will be its velocity just before striking the ground ?*

Solution. (*a*) For vertical downward motion of stone,

$$v_i = -9{\cdot}8\ \text{ms}^{-1}, \qquad S = 39{\cdot}2\ \text{m}, \qquad a = +g = 9{\cdot}8\ \text{ms}^{-2}$$

$\therefore$ By $\quad S=v_i t+\frac{1}{2}at^2,$

$$39{\cdot}2=-9{\cdot}8\,t+\tfrac{1}{2}\,9{\cdot}8t^2$$

or $\quad t^2-2t-8=0$

$$(t-4)(t+2)=0$$

$\therefore$ $\quad t=4$ s or -2 s

The $-$ive value is not admissible, so the stone reach the ground 4 seconds after it is projected.

(*b*) When stone passes back from point of projection in vertical downward direction, $S=0$ (since resultant displacement is zero)

$$v_i=-9{\cdot}8\ \text{ms}^{-1} \text{ and } a=+g=9{\cdot}8\ \text{ms}^{-1}$$

$\therefore$ By $\quad S=V_i t+\frac{1}{2}at^2,$

$$0=-9{\cdot}8t+\tfrac{1}{2}9{\cdot}8t^2$$

$\therefore$ $\quad t^2-2t=0$

or $\quad t(t-2)=0$

$\therefore$ $\quad t=0$ or 2 s

$t=0$ corresponds to the start of the motion. So the stone passes through the point of projection 2 seconds after it is projected.

For vertical downward motion of stone,

$$V_i=-9{\cdot}8\ \text{ms}^{-1},\quad t=4\ \text{s},\qquad a=+g=9{\cdot}8\ \text{ms}^{-2}$$

$\therefore$ By $\quad V_f=V_i+at$

$$=-9{\cdot}8+9{\cdot}8\times 4$$

$$=29{\cdot}4\ \text{ms}^{-1}$$

[**Note.** The students may try this question in two parts—(1) from the point of projection to the top point where its velocity becomes zero and then (2) from top point to ground. Answers will be the same.]

Example 34. *A body of mass 1 kg initially at rest explodes and breaks into three fragments of masses in the ratio 1 : 1 : 3. The two pieces of equal mass fly off perpendicular to each other with a speed of 30 ms⁻¹ each. What is the velocity of heavier fragment?* [*I.I.T. J.E.E. 1982*]

Solution. Let the masses of the fragments be m_1, m_2 and m_3. Then

$$m_1 : m_2 : m_3=1 : 1 : 3$$

and $\quad m_1+m_2+m_3=1$ kg

$\therefore$ $\quad m_1=\dfrac{1}{5}$ kg, $\quad m_3=\dfrac{1}{5}$ kg

and $\quad m_3=\dfrac{3}{5}$ kg

and $V_1 = V_2 = 30\ \text{ms}^{-1}$ (given) and $V_3 = ?$

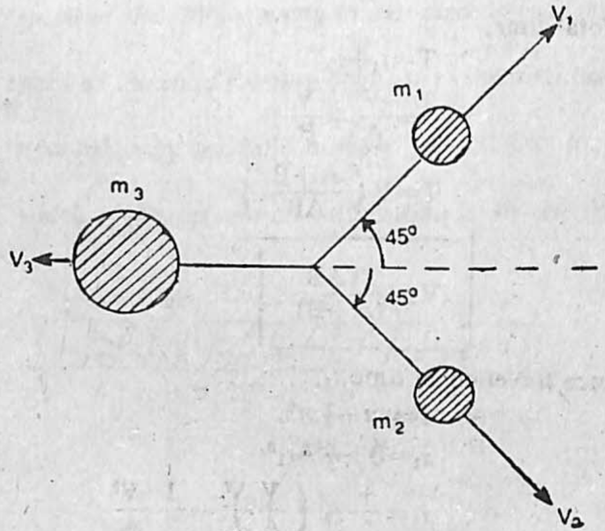

Fig. 3.6.

∴ By principle of conservation of linear momentum along V_3,

$$m_3V_3 = m_1V_1 \cos 45 + m_2V_2 \cos 45$$

$$\frac{3}{5} \times V_3 = \frac{1}{5} \times 30 \times \frac{1}{\sqrt{2}} + \frac{1}{5} \times 30 \times \frac{1}{\sqrt{2}}$$

∴ $V_3 = 14{\cdot}14\ \text{ms}^{-1}$, making an angle of $(180-45) = 135°$ with each of lighter fragments.

Example 35. *A car accelerates from rest at constant rate 'A' for sometime after which it retards at a constant rate 'B' to come to rest. If the total time lapses is 'T' seconds, evaluate the maximum velocity reached and the total distance travelled in terms of A, B and T.* *[A.I.S.S.E. 1983]*

Solution. Let the car accelerates for time t_1 and gain maximum velocity 'V' during this time and retards to come to rest for time t_2. Then for the motion of car with acceleration 'A'.

By $$v_f = v_i + at,$$
$$V = 0 + At_1$$

∴ $$t_1 = \frac{V}{A}$$

and then for the motion of car with retardation $= -B$ to come to rest,

By $$v_f = v_i + at$$
$$0 = V - Bt_2$$

$$\therefore \qquad t_2 = \frac{V}{B}$$

$\therefore$ Total time,

$$T = t_1 + t_2$$

$$= \frac{V}{A} + \frac{V}{B}$$

$$\therefore \qquad T = V\left(\frac{A+B}{AB}\right)$$

or

$$\boxed{V = \frac{TAB}{(A+B)}}$$

Distance travelled in time t_1,

By

$$s = v_i t + \tfrac{1}{2}at^2,$$

$$s_1 = 0 + \tfrac{1}{2}At_1^2$$

$$s_1 = \frac{1}{2}A\left(\frac{V}{A}\right)^2 = \frac{1}{2}\frac{V^2}{A}$$

Similarly distance travelled by the car in time 't_2',

By

$$2as = v_f^2 - v_i^2,$$

$$2(-B)s_2 = 0 - (V)^2$$

$$s_2 = \frac{1}{2}\ \frac{V^2}{B}$$

$\therefore$ Total distance travelled by car,

$$S = s_1 + s_2$$

$$= \frac{1}{2}\ \frac{V^2}{A} + \frac{1}{2}\ \frac{V^2}{B}$$

$$= \frac{1}{2}V^2\left(\frac{1}{A} + \frac{1}{B}\right)$$

$$= \frac{1}{2}V^2\left(\frac{A+B}{AB}\right)$$

$$= \frac{1}{2}\left(\frac{TAB}{A+B}\right)^2\left(\frac{A+B}{AB}\right)$$

$$= \frac{T^2AB}{2(A+B)}.$$

Example 36. *On a long horizontally moving belt, a child runs to and fro with a speed of 9 km/h (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt* *moves with speed of 4 km/h. For an observer on a stationary platform out, what is the*

(*a*) *speed of the child running in the direction of motion of the belt ;*

(*b*) *speed of the child running opposite to the direction of motion of the belt ;*

(*c*) *time taken by the child to move from father to mother and then back ;*

(*d*) *which of the answers alter if motion is viewed by one of the parents.*

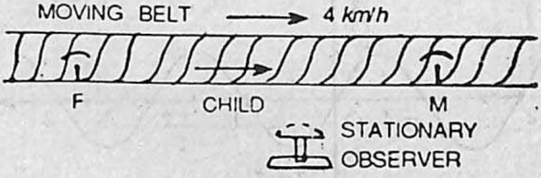

Fig. 3.7.

Solution.

(*a*) $V = V_1 + V_2 = 9 + 4 = 13$ **km/h**

(*b*) $V = V_1 - V_2 = 9 - 4 = 5$ **km/h**

(*c*) **Speed of child,**

$$V_1 = 9\ \text{km/h} = 9 \times \frac{5}{18} = 2{\cdot}5\ \text{ms}^{-1}$$

$$s = 50\ \text{m}$$

$$\therefore \qquad t = \frac{S}{V_1} = \frac{50}{2{\cdot}5} = 20\ \text{S in either direction.}$$

(*d*) If viewed by any one of the parents, the speed of the child is 9 km/h in either direction. The answer to part (*c*) remains unaltered.

Example 37. ***A police van moving on a highway with a speed of 30 km/h fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km/h. If the muzzle speed of the bullet is 150 ms⁻¹, with what speed does the bullet hit the thief's car ?***

Solution. Relative velocity of thief's car *w.r.t.*

police van $= (192 - 30)$

(Since car and van are moving in same direction)

$$= 162\ \text{km/h} = 162 \times \frac{5}{18} = 45\ \text{ms}^{-1}$$

Now the relative velocity of bullet w.r.t. car, with which it hits the car $=(150-45)\ 105\ \text{ms}^{-1}$.

(Again the bullet and car are moving in the same direction)

Example 38. *Figure 3·8 below gives the x-t plot of a particle executing one dimensional simple harmonic motion. Give the signs of position, velocity and acceleration variables of the particles at*

$$t=-1{\cdot}2\ s,\ 0{\cdot}3\ s,\ 1{\cdot}2\ s.$$

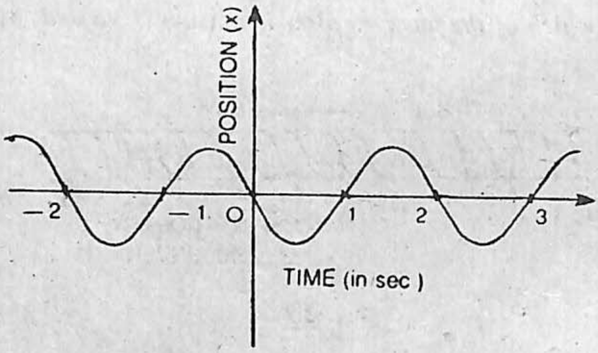

Fig. 3.8.

Solution. The simple harmonic motion has the characteristics that its acceleration is proportional to the displacement but in opposite direction.

(*i*) At $t=-1{\cdot}2$ S, it is evident from graph that x is also negative,

$$\therefore x < 0,\ v > 0 \left(\text{since } v=\frac{x}{t} \text{ and } x \text{ and } t \text{ both are negative}\right)$$

and $a > 0$ (since x and a are in opposite direction).

(*ii*) At $t=0{\cdot}3\ s$, $x=-$ive from the graph.

$\therefore\ x < 0,\ v < 0$ (since x is $-$ive and t is positive) and $a > 0$.

(*iii*) At $t=1{\cdot}2\ s$, $x=+$ive from the graph.

$\therefore\ x > 0,\ v > 0$ (since x and t both are positive) and $a < 0$.

Example 39. *Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 ms⁻¹ and 30 ms⁻¹. Verify that the following graph correctly represent the time variation of the relative positions of the second stone with respect to first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take g=10 ms⁻². Give the equation for the linear and curved parts of the plot.*

Solution. $a=-g=-10\ \text{ms}^{-2}$

By $s=v_i\, t+\frac{1}{2} a t^2$,

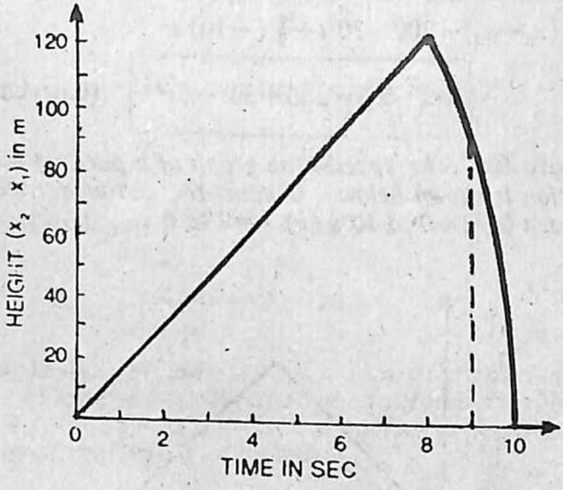

Fig. 3.9.

For the first stone, $x_1=15t-\frac{1}{2}\times 10t^2$

For the second stone $x_2=30t-\frac{1}{2}\times 10t^2$

$\therefore$ $\boxed{(x_2-x_1)=15t}$ (linear part)

When first stone reaches the bottom of the cliff,

$$s=200 \text{ m}, \; a=+g=10 \text{ ms}^{-2}$$

$$v_i=-15 \text{ ms}^{-1}$$

By $\quad s=v_i\, t+\frac{1}{2}\, at^2$

$$200=-15\, t+\frac{1}{2}\times 10\, t^2$$

$$t^2-3t-40=0$$

$$(t-8)\,(t+5)=0$$

$$t=8\, s \text{ (or } -5\, s \text{ which is not admissible)}$$

For second stone,

$$260=-30\, t+\frac{1}{2}\, 10\, t^2$$

$\therefore$ $t^2-6t-40=0$

$$(t-10)\,(t+4)=0$$

$$t=10\, s \text{ (or } -4\, s\text{, not admissible)}$$

It means (x_2-x_1) will increase linearly only up to 8 seconds when first stone reaches to the ground. Then (x_2-x_1) will decrease till 10 seconds and will become zero when the second stone also reaches to ground.

$\therefore$ For the relative positions of two stones between 8 and 10 s, By $$s=v_i\,t+\tfrac{1}{2}\,at^2$$

for second stone as first stone has already reached the ground.

$$(x_2-x_1)-200=30\,t+\tfrac{1}{2}\,(-10)\,t^2$$

$$\therefore\quad \boxed{(x_2-x_1)=200+30t-5t^2}\quad \text{(Curved part)}$$

Example 40. *The speed-time graph of a particle moving along a fixed direction is shown below. Obtain the distance traversed by the particle between (a) t=0 to 10 s (b) t=2 to 6 s.*

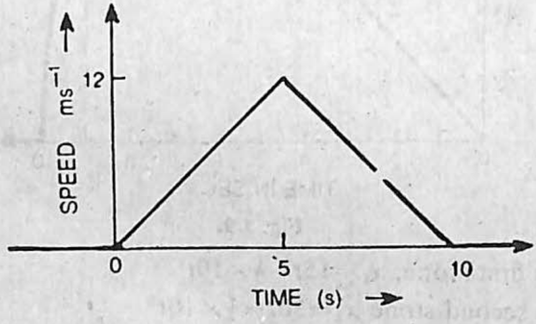

Fig. 3.10.

Solution. From Fig. 3·10 it is evident that the particle accelerates with $a=\frac{-12}{5}$ ms^{-2} for first 5 seconds and then decelerates with $a=\frac{\times 12}{5}$ ms^{-2} for next (10−5)=5 seconds.

(*a*) ∴ For first $t=5$ s,

By $$s=v_i t+\tfrac{1}{2}\,at^2$$

$$s_1=0+\tfrac{1}{2}\times\frac{12}{5}\,5^2$$

$$s_1=30\text{ m}$$

and for next $t=5$ seconds,

$$s_2=12\times 5+\tfrac{1}{2}\left(-\frac{12}{5}\right)5^2$$

$$s_2=30\text{ m.}$$

∴ Total distance traversed $=s_1+s_2$

$$=30+30$$
$$=60\text{ m.}$$

(*b*) For first $t=2s$,

$$s_3=0+\frac{1}{2}\left(\frac{12}{5}\right)2^2$$

$$s_3=4{\cdot}8 \text{ m}$$

For $(6-5)=1$ s in second half journey

$$s_4=12\times1+\tfrac{1}{2}\left(-\frac{12}{5}\right)1^2$$

$$s_4=10{\cdot}8 \text{ m}$$

∴ Total distance traversed from $t=2\ s$ to $t=6$ s

$$=(s_2-s_3)+s_4$$

$$=(30-4{\cdot}8)+10{\cdot}8$$

$$=36 \text{ m}$$

Example 41. *Figure 3·11 below shows the position-time graph of a particle of mass 4 kg. What is the (a) force on the particle for* $t<0, t>4\ s, 0<t<4\ s$*? (b) Impulse at* $t=0$ *and* $t=4\ s$*? (Consider one dimensional motion only).*

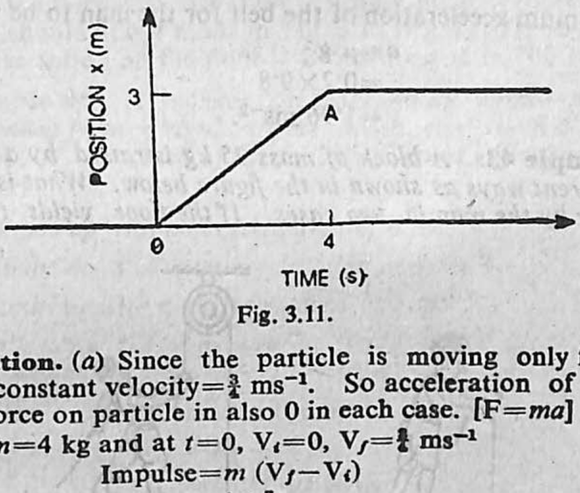

Fig. 3.11.

Solution. (*a*) Since the particle is moving only for 4 s and that with constant velocity$=\frac{3}{4}$ ms^{-1}. So acceleration of particle is 0, hence force on particle in also 0 in each case. [$F=ma$]

(*b*) $m=4$ kg and at $t=0$, $V_i=0$, $V_f=\frac{3}{4}$ ms^{-1}

$$\therefore \quad \text{Impulse}=m\,(V_f-V_i)$$

$$=4\times\tfrac{3}{4}$$

$$=3 \text{ kg ms}^{-1}$$

and at $t=4\ s$, $V_i=\frac{3}{4}$ ms^{-1}, $V_f=0$

$$\therefore \quad \text{Impulse}=\text{m}\,(V_f-V_i)$$

$$=4\left(0-\frac{3}{4}\right)$$

$$=-3 \text{ kg ms}^{-1}.$$

Example 42. *Figure 3·12 below shows a man standing stationary with respect to a horizontal conveyor belt that is accelerating with 1 ms*$^{-2}$*. What is the net force on the man? If the coefficient of* *static friction between the man's shoes and the belt is 0·2, upto what acceleration of the belt can the man continue to be stationary relative to the belt. (Mass of the man = 65 kg).*

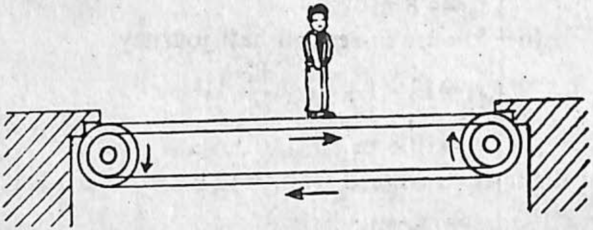

Fig. 3.12.

Solution. $m = 65$ kg, $a = 1$ ms^{-2}

∴ Net force on the man,

$$F = ma$$
$$= 65 \times 1$$
$$= 65 \text{ N}$$

Maximum acceleration of the belt for the man to be stationary,

$$a = \mu\, g$$
$$= 0{\cdot}2 \times 9{\cdot}8$$
$$= 1{\cdot}96 \text{ ms}^{-2}.$$

Example 43. *A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in the figure below. What is the action on the floor by the man in two cases. If the floor yields to a normal*

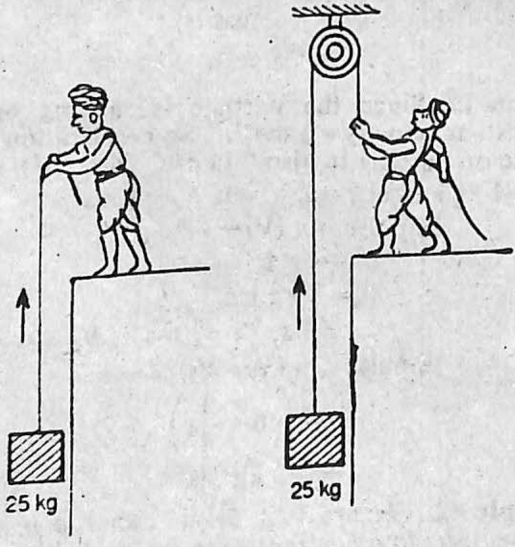

Fig. 3.13.

force of 700 N, which mode should the man adopt to lift the block without the floor yielding ?

Solution. For Fig. 3·13 (*left*) the weight of the man and the blocks both are acting on floor vertically downward,

$\therefore$ The action on the floor by the man $= mg + m'g$.

$$= (50+25)\ 9{\cdot}8$$

$$= 735\text{ N}$$

For the Fig. 3·13 (*right*) the weight of the man is acting vertically downward on the floor while weight of the block is acting vertically upward on the floor (through the string hold by the man).

$\therefore$ The action on floor by the man

$$= mg - m'g$$

$$= (50-25)\ 9{\cdot}8$$

$$= 245\text{ N}$$

Since the floor can yield only upto a normal force of 700 N, so the man should adopt mode in Fig. 3·13 (*right*) to raise the weight in which case action on the floor is 245 N (less than 700 N).

Example 44. *A monkey of mass 40 kg climbs on a rope freely suspended from a rigid support which can stand a maximum tension of 600 N. In which of the following cases will the rope break if the monkey :*

(*a*) *climbs up with an acceleration of 6 ms^{-2}.*

(*b*) *climbs down with an acceleration of 4 ms^{-2}.*

(*c*) *climbs up with a uniform speed of 5 ms^{-1}.*

(*d*) *falls down the rope nearly freely under gravity ?* (*Take $g = 10\ ms^{-2}$ and ignore the mass of the rope*).

Solution. (*a*) Tension on the rope

$$= mg + ma$$

$$= 40(10+6) = 640\text{ N}$$

(*b*) Tension on the rope

$$= mg - ma$$

$$= 40(10-4)$$

$$= 240\text{ N}$$

(*c*) Tension on the rope

$$= mg \qquad (\because\ a = 0)$$

$$= 40 \times 10$$

$$= 400\text{ N}$$

(*d*) Tension on the rope

$$=(mg-ma)$$
$$=m(g-g) \qquad (\because \text{ for freely fall } a=g)$$
$$=0.$$

It is in the first case (*a*) that the tension on the rope (640 N) is more than the maximum tension the rope can wear (600 N), so the rope will break.

Example 45. *The rear side of a truck is open ana a box of 40 kg mass is placed 5 m away from the open rear end of the truck. The coefficient of friction between the box and the surface below it is 0·15. On a straight road, the truck starts from rest and accelerate with 2 ms^{-2}. At what distance from the starting point does the box fall off the truck ? (Ignore the size of the block).*

Solution. The net force on the box moving in the truck,

$$F'=(F-f)$$

where $F=ma$ and f (frictional force)$=\mu N=\mu mg$

$$\therefore \quad F'=ma-\mu mg$$
$$F'=m(a-\mu g)$$

$\therefore$ The net acceleration of the box (in the direction of the motion of the truck),

$$a'=\frac{F'}{m}$$
$$=(a-\mu g)$$
$$=(2-{\cdot}15\times 9{\cdot}8)$$
$$=0{\cdot}53 \text{ ms}^{-2}$$

$\therefore$ For the motion of box in the direction opposite to that of truck,

$$v_i=0,\ a'=0{\cdot}53 \text{ ms}^{-2} \text{ and } s=5 \text{ m}$$

$\therefore$ By

$$s=v_it+\tfrac{1}{2}at^2,$$
$$s=0+\tfrac{1}{2}(0{\cdot}53)t^2 \qquad (\because \text{ Here } a=a')$$
$$t^2=\frac{5\times 2}{0{\cdot}53}$$
$$\therefore \quad t=4{\cdot}347 \ s$$

So box will fall 4·34 *s* after the start of the truck. During this time the truck will move the distance given by

$$s=v_it+\tfrac{1}{2}at^2$$
$$=0+\tfrac{1}{2}\times 2\times(4{\cdot}347)^2$$
$$=18{\cdot}896 \text{ m}.$$

Example 46. *A weight of 1000 N is supported by two chains as shown in the Fig. 3·14 (a). Determine the tension in each chain.*

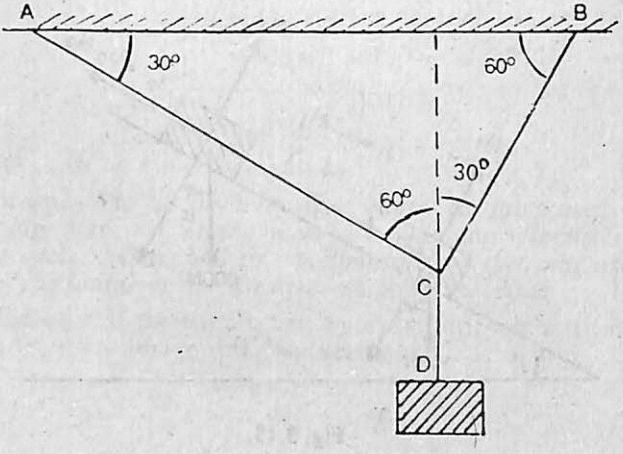

Fig. 3.14 (*a*)

Solution. Let T_1 and T_2 be the tensions in the two chains. Applying Lami's theorem at point C [Fig. 3·14 (*b*)].

$$\frac{T_1}{\sin 150} = \frac{T_2}{\sin 120}$$

$$= \frac{1000}{\sin 90}$$

$$\therefore \quad T_1 = 1000 \sin 150 \quad (\because \sin 90 = 1)$$

$$= 1000 \times \cdot 5$$

$$= 500 \text{ N}$$

and

$$T_2 = 1000 \sin 120$$
$$= 1000 \times \cdot 866$$
$$= 866 \text{ N.}$$

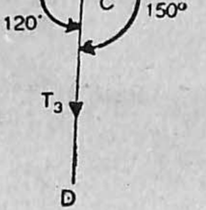

Fig. 3.14 (*b*)

Example 47. *A cord connects two bodies of weights 400 N and 800 N which are placed on an inclined plane, cord being parallel to inclined plane. The coefficient of friction between the inclined plane and the two weights are 0·15 and 0·4 respectively. Determine the inclination of the plane to the horizontal and the tension in the cord when the motion is about to take place down the inclined plane. The body weighing 400 N is below the body weighing 800 N.*

Solution. Different forces acting on the two bodies are shown in Fig. 3·15. *Forces acting on the first body* :

Resolving forces along the inclined plane,

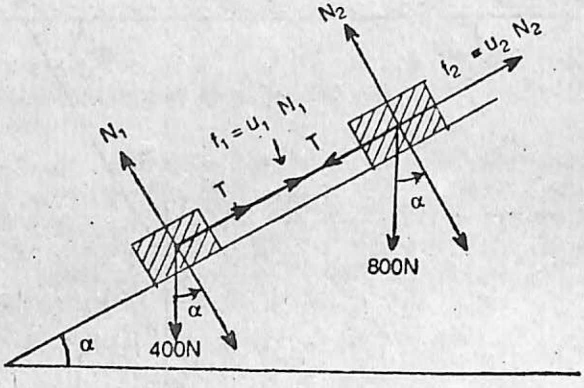

Fig. 3.15.

$$400 \sin \alpha = (T + f_1)$$
$$= (T + \mu_1 N_1)$$

$\therefore$ $400 \sin \alpha = (T + 0·15\ N_1)$

Resolving forces normal to the plane,

$$N_1 = 400 \cos \alpha$$

$\therefore$ $400 \sin \alpha = (T + 0·15 \times 400 \cos \alpha)$

$$= (T + 60 \cos \alpha)$$

$\therefore$ $T = 400 \sin \alpha - 60 \cos \alpha$ ...(1)

Forces acting on the second body :

Resolving forces along the plane,

$$800 \sin \alpha + T = f_2 = \mu_2 N_2$$
$$= 0·40\ N_2$$

Resolving forces normal to the plane

$$N_2 = 800 \cos \alpha$$

$\therefore$ $T = \mu_2\ N_2 - 800 \sin \alpha$

$$= 0·40 \times 800 \cos \alpha - 800 \sin \alpha$$

$T = (320 \cos \alpha - 800 \sin \alpha)$ ...(2)

$\therefore$ From eqns. (1) and (2),

$$400 \sin \alpha - 60 \cos \alpha = 320 \cos \alpha - 800 \sin \alpha$$
$$1200 \sin \alpha = 380 \cos \alpha$$

$\therefore$ $\tan \alpha = \frac{380}{1200} = 0·3166$

$$\therefore \qquad \boxed{\alpha = 17^\circ\ 34'}$$

Putting $\alpha = 17^\circ\ 24'$ in eqn. (1)

$$T = 400 \sin 17^\circ\ 34' - 60 \cos 17^\circ\ 34'$$
$$= 400 \times 0{\cdot}3017 - 60 \times 0{\cdot}9534$$

$$\therefore \qquad \boxed{T = 63{\cdot}48 \text{ N}}$$

Example 48. *A motor-cyclist wants to jump over a ditch as shown in Fig 3·16. Find the necessary minimum velocity 'v' at A of the motor cycle. Also find the inclination and the magnitude of the velocity of the motor cycle just after clearing the ditch.*

Solution. If the time taken by the motor-cycle to move from A to B is 't', then horizontal displacement,

$$4 = v \times t$$

$$\therefore \qquad v = \frac{4}{t} \qquad \text{...(1)}$$

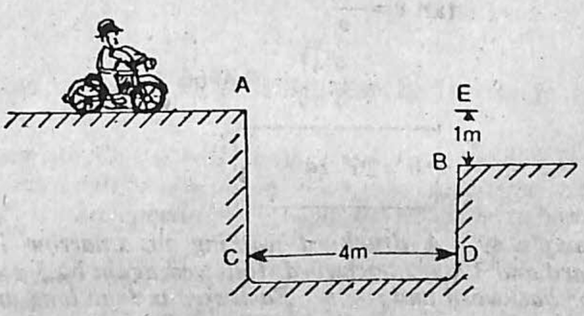

Fig. 3.16.

For the motion of the motor-cycle in vertical direction,

$$v_i = 0,\ a = +g = 9{\cdot}8 \text{ ms}^{-2},\ s = 1 \text{ m}$$

$\therefore$ By

$$s = v_i t + \tfrac{1}{2} a t^2$$
$$1 = 0 + \tfrac{1}{2}\, 9{\cdot}8\, t^2$$
$$t = \sqrt{\frac{2}{9{\cdot}8}} = 0{\cdot}45 \ s$$

$\therefore$ From eqn. (1)

$$v = \frac{4}{0{\cdot}45} = 8{\cdot}88 \text{ ms}^{-1}$$

$$\therefore \qquad \boxed{v = 8{\cdot}88 \text{ ms}^{-1}}$$

By
$$\begin{aligned} v_f &= v_i + at \\ &= 0 + 9{\cdot}8 \times 0{\cdot}45 \\ &= 4{\cdot}41 \text{ ms}^{-1} \end{aligned}$$

Since the velocity v and v_f at B are at right angle to each other, the resultant velocity of motor-cycle at B,

$$\begin{aligned} R &= \sqrt{(v^2 + v_f^2)} \\ &= \sqrt{(8{\cdot}88^2 + 4{\cdot}41^2)} \\ &= \sqrt{(79 + 19{\cdot}45)} \end{aligned}$$

$$\boxed{R = 9{\cdot}92 \text{ ms}^{-1}}$$

If the angle made by the resultant velocity at B with the horizontal direction is θ, then

$$\begin{aligned} \tan\theta &= \frac{v_f}{v} \\ &= \frac{4{\cdot}41}{8{\cdot}88} = 0{\cdot}4966 \end{aligned}$$

$$\therefore \qquad \boxed{\theta = 26^\circ\ 24'}$$

Example 49. *A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward and so on. Each step is 1 m long and requires 1 s. Plot the x-t graph of this motion. Determine graphically or otherwise how long the drunkard takes to fall in pit 13 m away from the start.*

Solution. $x-t$ graph of motion in question is shown in Fig. 3.17.

It is evident from graph that it will take 37 *s* to move 13 *m* away from the start and then to fall in pit.

The drunkard moves in each 5 *s*, 5 *m* forward and then in next 3 *s*, 3 *m* backward. So he moves in each $(5+3)=8$ *s* a distance $(5-3)=2$ *m*.

$\therefore$ 8 metres distance is travelled in $8 \times \frac{8}{2} = 32$ *s*.

Next 5 *m* he travels in next 5 *s*.

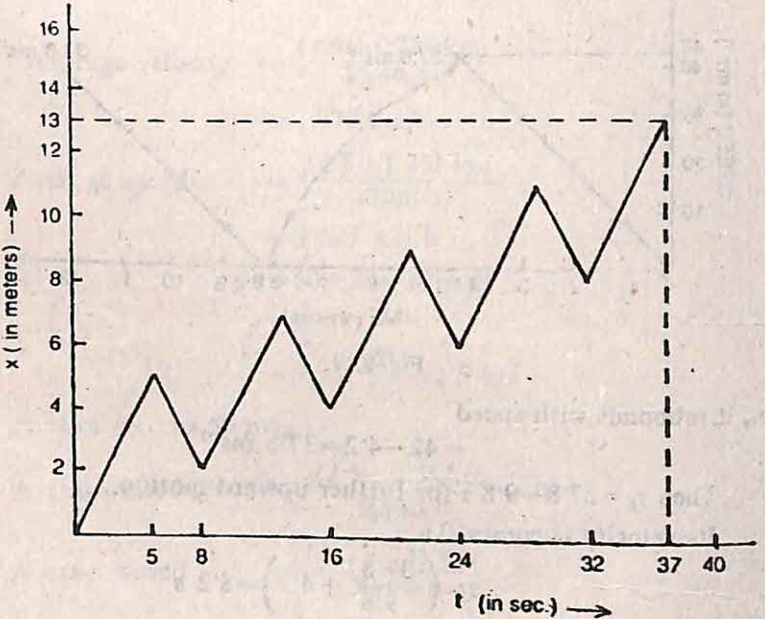

Fig. 3.17.

So the distance 13 m from start he travels in $(32+5)=37$ *s* and then he falls in pit.

Example 50. *A ball is dropped from a height of 9 m on a floor. At each collision with the floor, the ball loses one-tenth of its speed. Plot the speed-time graph of its motion between $t=0$ to $t=12$ s.*

Solution. (*i*) By $s=v_it+\frac{1}{2}gt^2$

$$90=0+\frac{1}{2}\times 9{\cdot}8\, t^2$$

$$t=\sqrt{\frac{2\times 90}{9{\cdot}8}}=\frac{30}{7}=4{\cdot}3\ s$$

and by $v_f=v_i+gt$

$$v_f=0+9{\cdot}8\ t$$

So $v_f \propto t$ upto $\frac{30}{7}$ s

and $v_f=9{\cdot}8\times\frac{30}{7}=42\ \text{ms}^{-1}$ at $4{\cdot}3$ *s*

$\therefore$ From $t=0$ to $t=4{\cdot}3$ *s*, $v-t$ graph is a straight line.

(*ii*) On collision with ground, it loses its velocity by

$$\frac{1}{10}\times 42=4{\cdot}2\ \text{ms}^{-1}$$

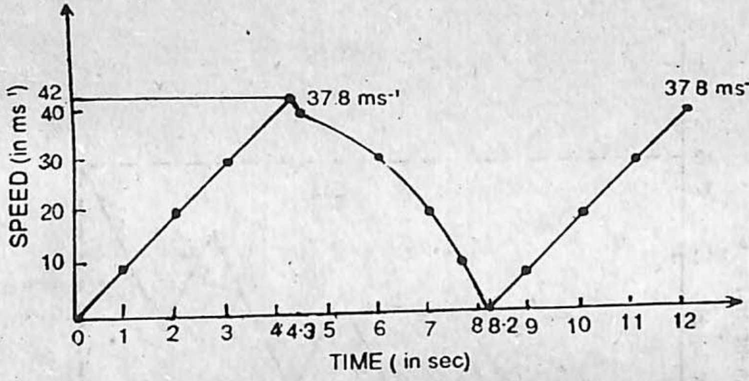

Fig. 3.18.

i.e., it rebounds with speed

$$=42-4{\cdot}2=37{\cdot}8\ \text{ms}^{-1}$$

Then $v_f=37{\cdot}8-9{\cdot}8\,t$ for further upward motion.

Its velocity becomes 0 at

$$t=\left(\frac{37{\cdot}8}{9{\cdot}8}+4{\cdot}3\right)=8{\cdot}2\ s$$

∴ From $t=4{\cdot}3\ s$ to $8{\cdot}2\ s$ its graph is a curve.

(*iii*) For further motion from $t=8{\cdot}2\ s$ to $12\ s$,

$$v_f=gt$$

∴ $v_f \propto t$ *i.e.*, $v-t$ graph is a straight line.

Example 51. *A man walks on a straight road from his home to a market 2·5 km away with a speed of 5 km/h. Finding the market closed, he instantly turns and walks back home with a speed of 7·5 km/h. What is the magnitude of average velocity and the average speed of the man over the interval of time :* (*a*) *0 to 30 min*, (*b*) *0—40 min*, (*c*) *0—50 min*.

Solution.

(*a*) $$t=\frac{S}{U}=\frac{2{\cdot}5}{5}h={\cdot}5\ h=30\ \text{min}$$

∴ For $t=0$ to 30 min, he is moving with the same speed and same velocity of 5 km/h.

(*b*) $$t=40-30=10\ \text{min}$$

$$S=U\times t=7{\cdot}5\times\frac{10}{60}=1{\cdot}25\ \text{km}$$

∴ For $t=0$ to 40 min.

$$\text{Average velocity} = \frac{(2{\cdot}5-1{\cdot}25)\text{ km}}{40/60\ h}$$

$$=1{\cdot}875\text{ km/h}$$

$$\text{Average speed} = \frac{(2{\cdot}5+1{\cdot}25)\text{ km}}{40/60\text{ h}}$$

$$=5{\cdot}625\text{ km/h}$$

(*c*) $t=50-30=20$ min

$$s=U\times t$$

$$=7{\cdot}5\times\frac{20}{60}=2{\cdot}5\text{ km}$$

$\therefore$ For $t=0$ to 50 min.

$$\text{Average velocity} = \frac{(2{\cdot}5-2{\cdot}5)\text{ km}}{50/60\text{ h}}=0$$

$$\text{Average speed} = \frac{(2{\cdot}5+2{\cdot}5)\text{ km}}{50/60\text{ h}}$$

$$=6\text{ km/h.}$$

Example 52. *A block of ice starts sliding down from the top of the inclined roof of a house (angle of inclination of roof with the horizontal=30°). The height and the lowest points of the roof are at heights 8·1 m and 5·6 m respectively from the ground. At what horizontal distance from the starting point will the block hit the ground. (Neglect friction).* (*IIT 1972*)

Solution. Let AB be the roof. The block of ice moves along AB and finally hit the ground at C.

$$AQ=(8{\cdot}1-5{\cdot}6)=2{\cdot}5\text{ m}$$

$$\sin 30=\frac{AQ}{AB} \Rightarrow AB=\frac{2{\cdot}5}{\frac{1}{2}}=5\text{ m}$$

$$\therefore \quad BQ=\sqrt{5^2-2{\cdot}5^2}=4{\cdot}33\text{ m}$$

$$\therefore \quad PR=4{\cdot}33\text{ m.}$$

For the velocity of block at B,

$$a=g\sin\theta,$$

$$V_i=0,\ s=5\ m$$

by $\quad V_f^2=V_i^2+2\,as$

$$=0+2\,g\sin\theta\times s$$

$$\therefore \quad V_f=\sqrt{2\times 9{\cdot}8\times\sin 30\times 5}$$

$$=7\text{ ms}^{-1}.$$

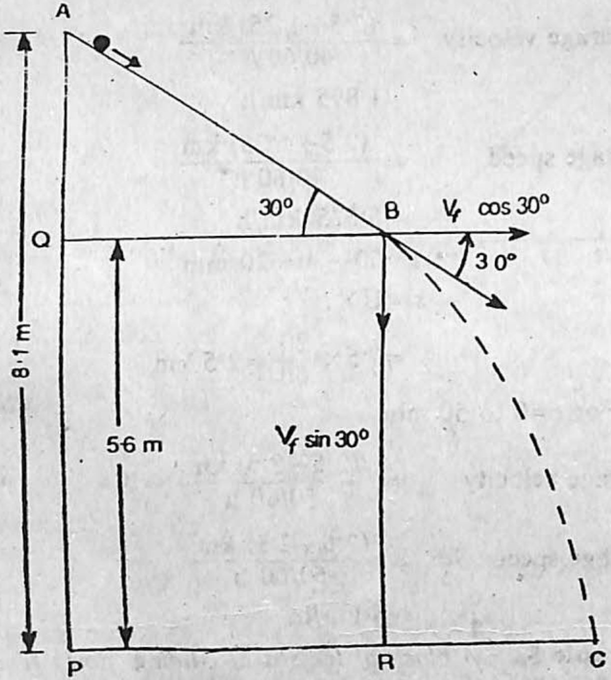

Fig. 3,19

From B, the block undergoes a projectile motion with horizontal component 7 cos 30 and vertical component 7 sin 30.

∴ For the motion of block in vertical downward direction,

by
$$s=v_it+\tfrac{1}{2}at^2$$
$$5{\cdot}6=7\sin 30\,t+\tfrac{1}{2}\times 9{\cdot}8\,t^2$$
$$7t^2+5t-8=0$$

On solving it, we get,
$$t=\frac{5{\cdot}4}{7}\ s$$

∴ Horizontal range,
$$RC=7\cos 30\times\frac{5{\cdot}4}{7} \qquad [\because R=U\times t]$$
$$=7\times\frac{\sqrt{3}}{2}\times\frac{5{\cdot}4}{7}$$
$$=4{\cdot}67\ m$$

∴ The total horizontal range,
$$R=PR+RC=4{\cdot}67+4{\cdot}33$$
$$=9\text{ m.}$$

EXERCISE 3

1. An automobile moving due north at the rate of 63 km/h is brought to rest in 10 seconds by application of brakes. Find the value of acceleration, assuming it to be constant. What is the distance moved by the automobile before it comes to rest ?

2. A ball of mass 0·1 kg moving horizontally with a speed of 20 ms^{-1} is stuck by a bat. After leaving the bat, the speed of the ball is 30 ms^{-1} in a direction opposite to the direction of motion. Calculate the impulse of the collision.

3. A batsman deflects a ball by an angle of 60° without changing its initial speed which is 25 ms^{-1}. What is the impulse imparted to the ball ? (Mass of the ball=0·12 kg).

4. A bullet of mass 0·03 kg moving with a speed of 400 ms^{-1} penetrates 12 cm into a fixed block of wood. Calculate the average force exerted by the wood on the bullet.

5. A boy whose mass is 50 kg stands on a spring weighing machine inside a lift. The lift starts to ascend with an acceleration of 2·45 ms^{-2}.

   (*a*) What is the reading of the machine ? (*b*) Find its reading when lift moves.

   (*i*) With uniform velocity (*ii*) With a retardation of 4·9 ms^{-2}

6. A neutron having mass of $1{\cdot}67 \times 10^{-27}$ kg and moving at 10^8 ms^{-1} collides with deutron at rest and sticks to it. Given that mass of deutron is $3{\cdot}34 \times 10^{-27}$ kg. Calculate the speed of the combination. [*D.B.S.S.E. 1978, A.I.S.S.E. 1981*]

7. A ball is dropped from a height of 16 m. If it loses 40% of its kinetic energy on striking the ground, what is the height to which it bounces ? How do you account for the loss in K.E. ?

8. While launching a rocket of mass 2×10^4 kg, a force of 6×10^5 N is applied for 30 *s*. What is the velocity attained by the rocket at the end of (*a*) 10 *s* (*b*) 30 *s* (*c*) 50 *s* ?

9. A truck of mass 3000 kg is moving at 10m/s and is acted by a forward force of 1000 N due to engine and a retarding force of 400 N due to friction. (*a*) What is the rate at which it is gaining the speed ? (*b*) How far will it travel in 10 *s* ? [*D.B.S.S.E. 1985*]

10 A car of mass 1000 kg is moving with a velocity of 10 m/s and is acted by a forward force of 1000 N due to engine and retarding force of 500 N due to friction. What will be its velocity after 5 seconds ? [*A.I.S.S.E. 1988*]

11. A food packet is released from a helicopter which is rising steadily at 2 ms^{-1}. After 2 *s* (*a*) what is the velocity of the packet, (*b*) how far is it below the helicopter ? [*D B.S.S.E. 1984, 1986*]

12. A car of mass 1000 kg is moving at a velocity of 10 $\mathrm{ms^{-1}}$ and is acted by a forward force of 1000 N due to engine and a retarding force of 500 N due to friction. What will be its velocity after 10 seconds ? *[A.I.S.S.C. 1981]*

13. While launching a rocket of mass 2×10^4 kg, a force of 5×10^5 N is applied for 20 *s*. What is the velocity attained by the rocket at the end of 20 *s* ? *[D.B.S.S.E. 1984; A.I.S.S.E. 1978]*

14. A person has to cross a river by a boat exactly on the other side of the river. In which direction should the person row the boat so as to finally arrive at the just opposite point ? Assume he can row at 6 km/h in still water. The velocity of river is 3 km/h.

15. A body is dropped from rest at a height of 80 *m* and simultaneously another body is dropped from rest from a point 45 *m* above the ground. What is their difference in height after they have fallen for (*a*) 1*s*, (*b*) 2*s*, (*c*) 3*s*. How does this difference in height vary with *b* time ?

16. An elevator's weight is 4000 kg. When the upward tension in supporting cables is 48000 N, what is the upward acceleration ? Starting from rest how far does it rise in 2 seconds.

17. A ship is sailing westwards at 8 m/s. While trying to fix a bolt at the top of the mast, a sailor drops the bolt. If the mast of the ship is 19·6 *m* high, where will the bolt hit the deck (*a*) if ship continues to move (*b*) if ship ceases to move as soon as bolt drops.

18. A shell is fired horizontally from a gun situated at a height of 44·1 *m* above a horizontal plane with a muzzle velocity of 300 $\mathrm{ms^{-1}}$ to hit a target on the horizontal plane.
    (*a*) How long does the shell take to hit a target ?
    (*b*) Where is the target situated ?
    (*c*) With what velocity does the shell strike the ground ?

19. A bomb is dropped from an aeroplane when it is directly above a target at a height of 1500 *m*. The aeroplane is going horizontally with a speed of 500 km/h. By how much distance will the bomb miss the target ?

20. From the top of a building 19·6 *m* high, a ball is projected horizontally. After how long does it strike the ground ? If the line joining the point of projection to the point where it hits the ground makes an angle of 45° with the horizontal, what is the initial velocity of the ball ?

21. A bomb is dropped from an aeroplane when it is directly above a target at a height of 4000 *m*. The aeroplane is flying horizontally with a speed of 600 km/h.
    (*a*) When will the bomb reach the ground ?
    (*b*) Will the bomb hit the target ? *[A.I.S.S.E. 1986]*

22. A bomb is dropped from an aircraft when it is directly above a target at a height of 1000 m. The aircraft is moving with a velocity of 500 km/h. Will the bomb hit the target ? If not by how much distance will be bomb miss the target ? [*D.B.S.S.E. 1983*]

23. Eight 50 paisa coins are put on top of each other on a table, each coin having the mass 1 mg. Give the magnitude and direction of

(*a*) the force on the 6th coin (counted from the bottom) due to all the coins on its top.

(*b*) the force on the 6th coin by the 7th coin.

(*c*) the reaction of the 5th coin on the 6th coin.

24. A ship of mass 3×10^7 kg initially at rest is pulled by a force of 5×10^4 N through a distance of 3 *m*. Assuming that the resistance due to water is negligible, calculate the speed of the ship. [*I.I.T. J.E.E. 1980*]

25. A stone is dropped from a rising balloon at a height of 91·8 *m* above the ground and it reaches the ground in 6 *s*. What was the velocity of the balloon just at the moment when the stone was dropped ?

26. A boy throws a ball to a height of 24 *m*, how long does it take to catch the ball back by the boy in his hands.

27. A stone is dropped from a rising balloon at a height of 300 *m* above the ground and it reaches the ground in 10 seconds. What was the velocity of the balloon at the moment the stone was dropped ?

28. From the top of a tower 100 *m* high a stone is thrown vertically upward with a velocity of 39·2 ms^{-1}. Calculate

(*a*) greatest elevation,

(*b*) the velocity with which it strikes the ground,

(*c*) the time it takes to reach the ground.

29. From the top of a tower 135 *m* high a stone is projected upward with a velocity of 30 ms^{-1}. Calculate the (*a*) greatest elevation (*b*) the velocity with which it strikes the ground (*c*) the total time it takes to reach the ground ($g = 10\ ms^{-2}$).

30. A stone is dropped from a rising ballon at a height of 477·4 *m*. If the balloon is rising at the speed of 19·6 ms^{-1}, when and with what velocity does it hit the ground ?

31. A ball of 0·1 kg makes an elastic collision with a ball of unknown mass that is initially at rest. If the 0·1 kg ball rebound to one-third of its original speed, what is the mass of the other ball ? [*D.B.S S.E. 1981*]

32. A body of mass 1·00 kg makes an elastic collision with another body at rest and continues to move in the original direction with a speed equal to one-fourth of its original speed. Find the mass of the second body.

33. A box of mass 4 kg is placed on a wooden plank of length 1·5 *m* which is lying on the ground. The plank is lifted from one end along its length so that it becomes inclined. It is noted that when plank from the ground becomes 0·5 m high the box begins to slide. Find the coefficient of friction between the box and the plank. [*A.I.S.S.E. 1981*]

34. A jet aeroplane travelling at the speed of 600 km/h ejects its products of combustion at the speed of 1400 km/h relative to the jet plane. What is the speed of the products to a stationary observer on the ground ?

35. A player throws a ball upwards with an initial speed of 58·8 ms^{-1}. To what height does the ball rise and after how long does the ball return to the player's hands ? ($g=9·8\ ms^{-2}$ and neglect air resistance).

36. A boy standing on a stationary lift (open from above) throws a ball upwards with the initial maximum speed he can equal to 50 ms^{-1}. How much time does the ball take to return to his hands ? If the lift starts moving up with a uniform speed of 10 ms^{-1} and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands. (Take $g=10\ ms^{-2}$).

37. A woman walks on a straight road from her home to a market 1·5 km away with a speed of 6 km/h. Finding the market closed, she instantly turns and walks back home with a speed of 3 km/h. What is the magnitude of average velocity and the average speed of the woman over the interval of time ?
(*a*) 0 – 15 min (*b*) 0—30 min (*c*) 0—45 min.

38. A bullet fired at an angle of 15° with the horizontal hits the ground 6 km away. By adjusting its angle of projection, can one hope to hit the target 12 km away. Assume the muzzle speed to be fixed and neglect air resistance.

39. A horizontal force of 700 N pulls two masses of 2 kg and 5 kg (lying on a frictionless table) connected by a light string. What is the tension in the string ? Does the answer depend, on which mass the pull is applied ?

40. Two masses of 5 kg and 9 kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration of masses and tension in the string when the masses are released.

41. Two billiard balls each of mass 0·04 kg moving in opposite directions with speed 5 ms^{-1} collides and rebound with the

same speed. What is the impulse imparted to each ball due to the other ?

42. A weight of 800 N is supported by two chains as shown in the figure below. Determine the tension in each chain.

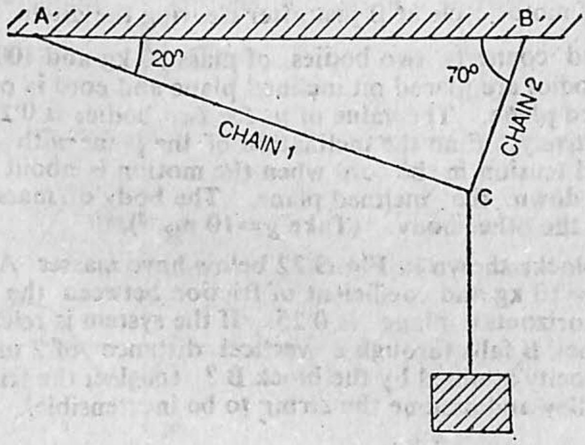

Fig. 3.20.

43. A mass of 50 kg is supported by two ropes as shown in the figure below. Determine the tension in each rope ($g=10\ \mathrm{ms}^{-2}$).

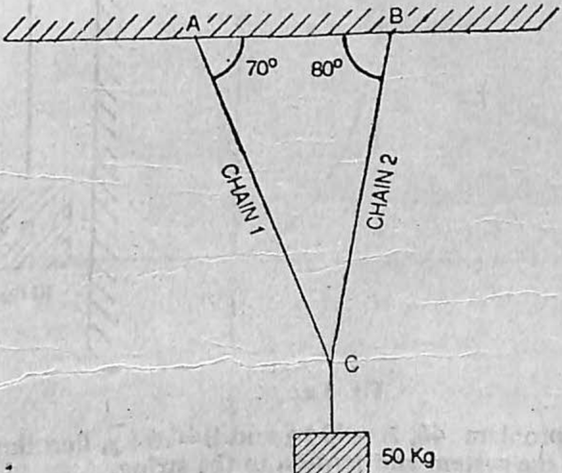

Fig 3.21.

44. A motor-cyclist wants to jumps over a ditch in Fig. 3·16 on page 61 if (*a*) CD=4 m & EB=2 m (*b*) CD=5 m and EB =2 m.

Find in each case the necessary minimum velocity of the motor cycle at A and also the direction and magnitude of velocity of motor cycle at B just after clearing the ditch.

45. A cord connects two bodies of mass 50 kg and 100 kg. The two bodies are placed on inclined plane and cord is parallel to inclined plane. The value of μ_s for two bodies is 0·2 and 0·4 respectively. Find the inclination of the plane with horizontal and tension in the cord when the motion is about to take place down the inclined plane. The body of mass 50 kg is below the other body. (Take $g=10 \text{ ms}^{-2}$).

46. Two blocks shown in Fig. 3·22 below have masses A=20 kg and B=10 kg and coefficient of friction between the block A and horizontal plane is 0·25. If the system is released and the block B falls through a vertical distance of 2 m, what is the velocity attained by the block B ? (neglect the friction in the pulley and assume the string to be inextensible).

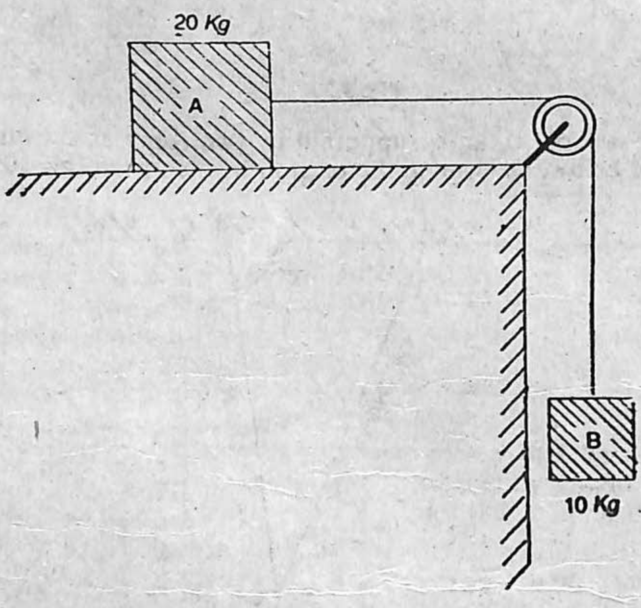

Fig. 3.22

47. If in problem 46, A=40 kg and B=20 kg, find the acceleration of the system and tension in the string.

48. Two masses of 1g and 4g are moving with equal kinetic energies. Calculate the ratio of the magnitude of their momenta. [*I.I.T. J.E.E. 1980*]

49. Two bodies of mass 6 kg and 4 kg connected to the two ends of the light inextensible string. The string is passing over a smooth pulley. Determine—
(*a*) the acceleration of the system
(*b*) the tension in the string. (Take $g=10\ \text{ms}^{-1}$)

50. Look at the graphs (*a*) to (*d*) in the Fig. 3.23 carefully and state with reasons, which of the following cannot possibly represent one dimensional motion of a particle.

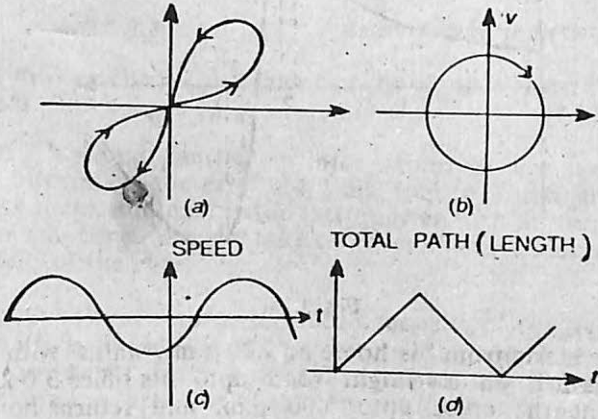

Fig. 3.23.

51. Figure 3.24 shows the $x-t$ plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t<0$ and on a parabolic path for $t>0$? If not, suggest a suitable context for this graph.

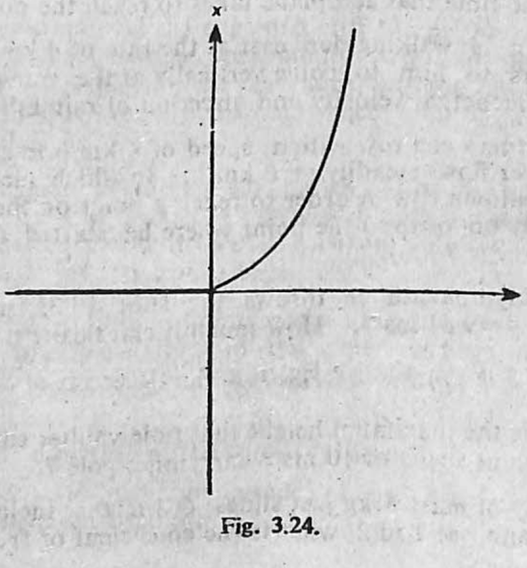

Fig. 3.24.

52. Figure 3.25 gives $x-t$ plot of a particle in one-dimensional motion. Three equal interval of times are shown. In which interval is the average speed greatest and in which it is least ?

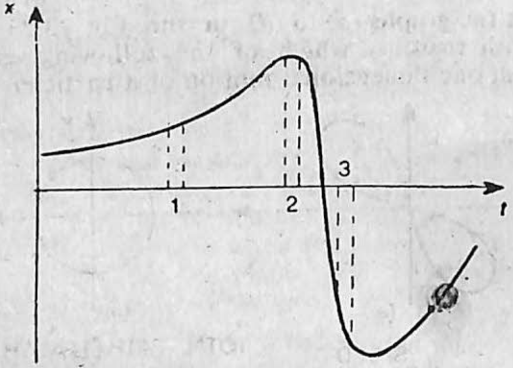

Fig. 3.25.

53. A man starts from his home at 9.30 a.m., walks with a speed of 6 km/h on a straight road upto his office 3·0 km away, stays at the office upto 5.00 p.m. and returns home by an autorickshaw with a speed of 36 km/h. Choose suitable scales and plot the $x-t$ graph of his motion.

54. An aeroplane wants to reach a point 120 km due south. If wind velocity is 100 km/h due east, in what direction must the aeroplane fly if the velocity of aeroplane is 260 km/h. Find also the time that aeroplane takes to reach the point.

55. A man is walking due east at the rate of 4 km/h. The rain appears to him to come vertically at the rate of $4\sqrt{3}$ km/h. Find the actual velocity and direction of rain fall.

56. A boatman can row with a speed of 8 km/h in still water. If the river flows steadily at 6 km/h, in which direction should the boatman row in order to reach a point on the other bank directly opposite to the point where he started. (the width of the river is 5 km).

57. If an olympic athlete throws the shot 18·31 m in Helsinki where $g=9{\cdot}81$ ms^{-1}. How much it can throw it on the moon where g is $\frac{1}{6}$ that in Helsinki ?

58. What is the maximum height that pole vaulter can clear if he can run at speed of 10 ms^{-1} carrying a pole ?

59. A body of mass 6 kg just slides down the inclined plane. If the plane rise 1 in 2, what is the coefficient of friction ?

60. Find the distance travelled by a body before coming to rest moving on a horizontal plane with velocity of 60 km/h and the coefficient of friction between the body and the ground is 0·2.

61. The displacement 'x' of a particle of mass 1 g moving in one-dimensional plane is under the action of a constant force is related to the time 't' by the equation

$t = \sqrt{x} + 3,$ (where x is in metres and t is in sec.)

Find (*a*) The displacement of the particle when its velocity is 0.
(*b*) Work done by the force in 6 *s*. [*I.I.T. J.E.E.* 1970]

62. A 2 hundred metres sprinter increases her speed from rest uniformly at the rate of 4/3 ms^{-2} up to three quarters of the total run and covers the last quarter with uniform speed. How much time does she take to cover the first half and the second half of the run ?

63. Two balls are thrown simultaneously. 'A' vertically upwards with a speed of 15 ms^{-1} from the ground and 'B' vertically downwards from a height of 30 m with the same speed and along the same line of motion. After how much time and at what point do the two balls meet ? (Take $g = 10\ ms^{-2}$)

64. A small sphere is suspended by a string from the ceiling of a car. If the car begins to move with a constant acceleration of 0·49 ms^{-2}, what will be the inclination of the string to the vertical ?

65. A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 18 ms^{-1}. Neglecting air resistance, find the time taken by the stone to reach the ground and speed with which it hits the ground.

66. An astronaut accidently get thrown out of his small space ship accelerating in intersteller space at a constant rate of 120 ms^{-2}. What is the acceleration of the astronaut the instant after he is outside the space ship ?

67. Two identical billiard balls strike a rigid wall with the same speed one normally and the other an angle of 40° with the normal to the wall and get reflected without any loss of speed what is

(*a*) the direction of the force on the wall due to each ball, and
(*b*) the ratio of the magnitudes of impulses imparted to the two balls by the wall ?

68. A rocket of initial mass 5000 kg ejects gases at a constant rate of 15 kgs^{-1} with constant relative speed of 8 kms^{-1}. What is the acceleration of the rocket 2 minute after the blast ?

69. A wooden block of mass 5 kg rests on a soft horizontal floor. When an iron cylinder of mass of 20 kg is placed on the top of the block, the floor yields steadily and the block and the cylinder together go down with an acceleration of 0·2 ms^{-2}. What is the action of the block on the floor before and after the floor yields ? (Take $g=10\ ms^{-2}$)

70. A body falling freely under gravity passes two points 30 m apart in 1 s. Find from what point above the upper point it begins to fall ? ($g=10\ ms^{-2}$) [*A.I.I.M.S.*, *1979*]

71. Obtain an equation for the path of a charged particle of mass *m* and having charge *q* when it is thrown vertically with a velocity V_0 in a uniform electric field E. Neglect gravity. [*Roorkee Engg.*, *1982*]

72. Particles P and Q of mass 20 gm and 40 gm respectively are simultaneously projected from points A and B on the ground. The initial velocities of P and Q make 45° and 135° angles respectively with the horizontal AB as shown in the Fig. 3.26. Each particle has an initial speed of 49 ms^{-1}. The separation AB is 24·5 m. Both particles travel in the same vertical plane and undergo a collision. After the collision, P retraces its path. Determine the position of Q when it hits the ground. How much time after collision does the particle Q take to reach the ground ? (Take $g=9{\cdot}8\ ms^{-2}$)

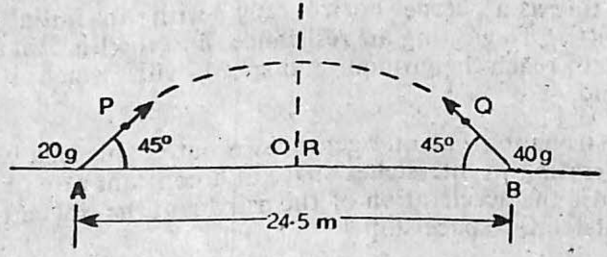

Fig. 3.26.

73. Figure 3.27 is the graph of displacement as a function of time of two particles of mass $m_1=3$ kg and $m_2=1$ kg, which are exerting force on each other. What are the initial energies of m_1 and m_2 ? What is the maximum potential energy ? Assume that the interaction at $t=0$ is negligible. [*Roorkee Ent. Exam.*, *1981*]

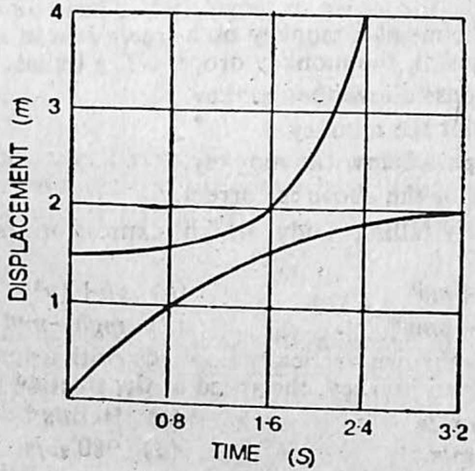

Fig. 3.27.

74. A bird is sitting at the top of a tree 35 m high and a shot is fired at a distance 70 m from the foot of the tree in such a direction it just hits the bird. Find the magnitude and direction in which the shot is fired.

OBJECTIVE TYPE QUESTIONS

75. An iron ball and a wooden ball of the same radius are released from same height '*h*' in vacuum, the time taken by both of them to reach the ground is

(*a*) roughly equal	(*b*) zero
(*c*) exactly equal	(*d*) unequal

[*Delhi P.M.T. 1982*]

76. Stone is dropped from a bridge reaches the bottom in 4 seconds. The height of the bridge is

(*a*) 78·4 m	(*b*) 64 ft.
(*c*) 2000 cm	(*d*) 260 m.

[*Delhi P.M.T. 1982*]

77. Select the correct answer of the following which is not the scaler quantity

(*a*) Time	(*b*) Momentum
(*c*) Volume	(*d*) Density.

[*Delhi P.M.T. 1983*]

78. Select the correct answer :
A rocket works on the principle of conservation of

(*a*) Linear momentum	(*b*) Mass
(*c*) Energy	(*d*) Angular momentum.

[*Delhi P.M.T. 1983*]

79. A hunter aims at a monkey on a tree. Just at the instant he fires his pistol, the monkey drops. The bullet

(*a*) will pass above the monkey.
(*b*) will hit the monkey.
(*c*) will pass below the monkey.
(*d*) none of the above is correct.

80. For a freely falling body which expression is a constant of motion

(*a*) $mgh+mv^2$ (*b*) $gh+\frac{1}{2}v^2$
(*c*) $mgh-\frac{1}{2}mv^2$ (*d*) $mgh-mv^2$

81. A body is thrown vertically upwards with a speed of 1000 m/s. In the return journey, the speed at the starting point will be

(*a*) 1000 m/s (*b*) 50 m/s
(*c*) 100 m/s (*d*) 980 m/s

[*Delhi P.M.T. 1985*]

82. Which of the following quantity is equivalent to work ?

(*a*) Power (*b*) Energy
(*c*) Force (*d*) Pressure [*A.F.M.C. 1985*]

83. If the tension in the cable of 1000 kg elevator is 1000 kg weight, the elevator

(*a*) is accelerating upwards.
(*b*) is accelerating downwards.
(*c*) may be at rest or accelerating.
(*d*) may be at rest or in uniform motion. [*A.F.M.C. 1984*]

84. When K.E. is increased by 300%, the momentum of the body increases by

(*a*) 20% (*b*) 50%
(*c*) 100% (*d*) 200% [*A.F.M.C. 1984*]

85. The length of a second's pendulum on earth where g is 980 cms^{-2} is 100 cm. The length of Second's pendulum on a planet where $g=490\ cms^{-2}$ is.........

(*a*) 100 cm (*b*) 200 cm
(*c*) 50 cm (*d*) 25 cm. [*D.S.S.E. 1988*]

86. A hole is drilled through the earth along the diameter and a stone is dropped into it. When the stone is at the centre of the earth, it has zero,.........

(*a*) Acceleration (*b*) Weight
(*c*) Mass (*d*) Kinetic energy

[*D.S.S.E. 1985*]

87. Newton per metre² is the unit of

(*a*) Energy (*b*) Force
(*c*) Momentum (*d*) Pressure [*C.M. P.T. 1988*]

88. A ball is allowed to fall from a height of 10 m. If there is 40% loss of energy due to impact. Then after one impact the ball will go up by

(*a*) 10 m (*b*) 8 m
(*c*) 64 m (*d*) 6 m [*C.P.M.T. 1988*]

89. A body of mass 2 kg is hung on a spring balance mounted vertically on a lift. If lift descends with an acceleration equal to acceleration due to gravity '*g*', the reading on the spring balance will be

(*a*) 2 kg (*b*) 0
(*c*) 2g (*d*) 9/2 kg [*C.P.M.T. 1988*]

90. A body of mass '*m*' moving with a constant velocity 'V' hits another body of same mass moving with the same velocity but in the opposite direction sticks to it. The velocity of compound body after collision is

(*a*) V (*b*) 0
(*c*) 2V (*d*) V/2. [*C.P.M.T. 1988*]

91. Newton second is the unit of

(*a*) Velocity (*b*) Momentum
(*c*) Angular momentum (*d*) Energy. [*C.P.M.T. 1988*]

92. The length of a cube is $2{\cdot}1\times10^{-2}$ m. The volume in significant figure will be

(*a*) $9{\cdot}261\times10^{-6}$ m^3 (*b*) $9{\cdot}2\times10^{-6}$ m^3
(*c*) $9{\cdot}26\times10^{-6}$ m^3 (*d*) $9{\cdot}3\times10^{-6}$ m^3
[*C.P.M.T. 1988*]

93. The mass of the lift is 100 kg which is hanging on a string. The tension in the string when the lift is moving with a constant velocity will be ($g=9{\cdot}8\ ms^{-2}$)

(*a*) 100 N (*b*) 980 N
(*c*) 980 kg (*d*) cannot be calculated
[*C.P.M.T. 1988*]

94. A bullet of 10 g is fired with a velocity of 400 m/s from a gun of 5 kg. The recoil velocity of the gun will be

(*a*) 0·8 m/s (*b*) 8 m/s
(*c*) 0·08 m/s (*d*) 800 m/s. [*C.P.M.T. 1988*]

95. The retarding force is applied to stop a train. If the speed doubled then the distance will be

(*a*) Same (*b*) Half
(*c*) Doubled (*d*) Four times [*C.P.M.T. 1988*]

96. From the top of a tower of a height 1960 m a stone is horizontally projected with a velocity of 10 ms^{-1}. It will strike the ground after

(*a*) 10 S (*b*) 5 S
(*c*) 20 S (*d*) cannot be calculated.
[*C.P.M.T. 1988*]

97. A ball is just let fall from the window of a moving train, the ball will hit the ground
(a) Straight line path (b) Circular path
(c) Parabolic path (d) Hyperbolic path
[C.P.M.T. 1988]

98. In a collision of two balls in general the following parameters are conserved
(a) Kinetic energy (b) Momentum
(c) Kinetic energy & momentum
(d) None of the above. *[C.P.M.T. 1988]*

99. Which of the following is a vector quantity
(a) Electric charge (b) Charge density
(c) Electric potential (d) Electric field intensity
[Delhi P.M.T. 1983]

100. Newton's law of gravitation
(a) does not hold outside our solar system
(b) can be verified in the laboratory
(c) accounts for the spin of the planets *[A.I.I.M.S. 1982]*

101. In order to find time, the astronaut orbiting in an earth satellite should use
(a) pendulum clock (b) a watch having hair spring
(c) either pendulum clock or a watch
(d) neither a pendulum clock nor a watch. *[M.A.M.C. 1982]*

102. A simple pendulum of period T has a metal bob which is negatively charged. It is allowed to oscillate above a positively charged metal plate, its period will
(a) remain same (b) become less than T
(c) become greater than T (d) become infinite.
[M.A.M.C. 1982]

103. A stone dropped from a bridge reaches the bottom in 4s. The height of the bridge is
(a) 7840 cm (b) 64 ft.
(c) 2000 cm (d) 265 ft. *[M.A.M.C. 1982]*

104. An example of vector quantity is
(a) volume (b) displacement
(c) temperature (d) speed. *[M.A.M.C. 1982]*

105. A ball thrown vertically upwards with an initial velocity 19.6 ms^{-1} return in 4 s. The maximum height attained by the ball is
(a) 19.6 m (b) 26.7 m
(c) 10 m (d) none. *[M.A.M.C. 1983]*

106. A body of mass *m* moved to a height *h* equal to the radius of the earth R. The increase in potential energy is
(*a*) $\frac{1}{2}$ mgR (*b*) 2 mgR
(*c*) mgR (*d*) $\frac{1}{4}$ mgR. [*M.A.M.C. 1983*]

107. The energy of a particle moving at 5 ms^{-1} is 125 J, its mass is
(*a*) 4 kg (*b*) 6 kg
(*c*) 10 kg (*d*) 25 kg. [*B.H.U. 1983*]

108. At what position the value of *g* is minimum.
(*a*) at the centre of the earth
(*b*) on the surface of the earth
(*c*) in an aeroplane at a height of 2 km
(*d*) in a mine which is 2 km below earth's surface.
[*B.H.U. 1983*]

109. The height above the ground of a child on a swing varies from 2 ft at his lowest point to 5 ft at the highest point. The maximum speed of the child is (approximately)
(*a*) 11 ft s^{-1} (*b*) 14 ft/s
(*c*) 18 ft s^{-1}
(*d*) dependent on the mass of the child. [*B.H.U. 1983*]

110. The motion in two dimensional plane is given by equations

$$x=5t^2+4$$
$$y=2t^2+5$$

The trajectory of the body is
(*a*) parabola (*b*) circle
(*c*) straight line (*d*) can't be decided.

111. You are moving in a lift. Your weight will be zero when
(*a*) lift moves up with $a=9{\cdot}8$ ms^{-2}
(*b*) lift moved down with $a=9{\cdot}8$ ms^{-2}
(*c*) lift moves up with uniform velocity.
(*d*) lift moves down with uniform velocity.

112. A body is thrown vertically upwards with a velocity of 40 ms^{-1}. It returns back in thrower's hand with velocity
(*a*) 20 ms^{-2} (*b*) zero
(*c*) 40 ms^{-1} (*d*) -40 ms^{-1}.

113. The momentum of a body increases by 20%, the percentage increase in K.E. will be
(*a*) 36 (*b*) 22
(*c*) 44 (*d*) 12.

□□

4

Work, Energy and Power

IMPORTANT FORMULAE

1. Work done by a constant force,

$$W = F.S. \cos\theta$$

where F = Force applied

S = Displacement

and θ = Angle between directions of F and S

2. Gravitational potential energy.

$$P.E. = mgh$$

3. Potential energy of a spring,

$$P.E. = \frac{1}{2} k x^2$$

where k = Spring constant

i.e., restoring force per unit displacement ;

x = extension or compression in spring.

4. Kinetic energy,

$$K.E. = \frac{1}{2} mv^2$$

5. Power,

$$P = \frac{W}{t}$$

or

$$P = F \times v$$

SOLVED EXAMPLES

Example 1. *A body is displaced by 5 m by applying a constant force of 20 N. What is the work done if the angle between the direction of force and that of displacement is (a) 0, (b) 60°, (c) 90°.*

Solution. $F = 20$ N, $S = 5$ m

(*a*) $\theta = 0$

$$\therefore \quad W = F.S. \cos\theta$$

$$= 20 \times 5 \times \cos\theta$$

$$= 100 \times 1$$

$$= 100 \text{ J}$$

(*b*)

$$W = 20 \times 5 \times \cos 60$$

$$= 100 \times \frac{1}{2}$$

$$= 50 \text{ J}$$

(*c*)
$$\begin{aligned} W &= 20\times5\times\cos 90 \\ &= 100\times0 \\ &= 0 \end{aligned}$$

Example 2. *A bullet of mass 0·02 kg moving with a speed of 100 ms^{-1} penetrates 4 cm into fixed block of wood. Calculate the average force exerted by the wood on the bullet.*

Solution. The kinetic energy of the bullet just before striking the block of the wood,

$$\begin{aligned} E &= \tfrac{1}{2}\, mv^2 \\ &= \tfrac{1}{2}\times 02\times100^2 \\ &= 100 \text{ J} \end{aligned}$$

The entire kinetic energy is used in doing work against the average resisting force F of the wood.

$$\therefore \quad E = W = F\times S$$
$$100 = F\times0{\cdot}04$$
$$\therefore \quad F = 2{\cdot}5\times10^3 \text{ N.}$$

Example 3. *A body of mass 5 kg initially at rest is subjected to a force of 20 N. What is the kinetic energy acquired by the body at the end of 10 s ?*

Solution. $m=5$ kg, $F=20$ N
$V_i=0$ and $t=10$ s

$\therefore$ By
$$\begin{aligned} F &= ma \\ 20 &= 5\,a \\ a &= 4 \text{ ms}^{-2} \end{aligned}$$

$\therefore$ By
$$\begin{aligned} V_f &= V_i+at \\ &= 0+4\times10 \\ V_f &= 40 \text{ ms}^{-1} \end{aligned}$$

$$\begin{aligned} \therefore \quad K.E. &= \tfrac{1}{2}\, m\, V_f^2 \\ &= \tfrac{1}{2}\times5\times40^2 \\ &= 4000 \text{ J.} \end{aligned}$$

Example 4. *An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV and the second with 100 keV Which is faster, the electron or the proton ? Obtain the ratio of their speeds. Electron mass= $9{\cdot}11\times10^{-31}$ kg, proton mass= $1{\cdot}67\times10^{-27}$ kg, 1 eV=$1{\cdot}60\times10^{-19}$ J).*

Solution. For the electron,

$$\begin{aligned} K.E. &= 10 \text{ KeV} \\ &= 10\times10^3\times1{\cdot}6\times10^{-19} \text{ J.} \\ &= 1{\cdot}6\times10^{15} \text{ J} \end{aligned}$$

$$\therefore \quad \tfrac{1}{2}\, m_e v_e^2 = 1{\cdot}6\times10^{-15}$$

$$v_e=\sqrt{\frac{2\times1{\cdot}6\times10^{-15}}{9{\cdot}11\times10^{-31}}}$$
$$=5{\cdot}93\times10^{7}\ \text{ms}^{-1}$$

For the proton,

$$\text{K.E.}=100\ \text{KeV}$$
$$=1{\cdot}6\times10^{-14}\ \text{J}$$
$$\therefore\quad \tfrac{1}{2}\,m_pv_p^2=1{\cdot}6\times10^{-14}$$
$$\therefore\quad v_p=\sqrt{\frac{2\times1{\cdot}6\times10^{-14}}{1{\cdot}67\times10^{-27}}}$$
$$=4{\cdot}38\times10^{6}\ ms^{-1}$$
$$\therefore\quad \frac{v_e}{v_p}=27:2$$

∴ The electron is faster than the proton.

Example 5. *An automobile moving horizontally at a speed of 54 kmh^{-1} reaches the foot of an inclined smooth plane and the engine is switched off. How much distance does the automobile go up the incline before coming to rest ? The inclination of the plane to the horizontal is 40°.*

Solution.
$$V=54\ \text{km/h}$$
$$=54\times\frac{5}{18}=15\ \text{ms}^{-1}$$

The kinetic energy of the automobile is used to do working against the component of the gravitational force on the automobile down the plane.

$$\therefore\quad F\times S=\tfrac{1}{2}\,mv^2$$
or
$$S=\frac{mv^2}{2F}$$
$$=\frac{mv^2}{2\,(ma)}$$
$$\therefore\quad S=\frac{V^2}{2a}$$
Now
$$a=g\sin\theta=9{\cdot}8\sin40=6{\cdot}3\ \text{ms}^{-2}$$
$$\therefore\quad S=\frac{15^2}{2\times6{\cdot}3}$$
$$=17{\cdot}86\ \text{m}.$$

[**Note.** The students may note that it does not matter how much is the mass of the automobile you may get the same answer by $2\,aS=V_f^2-V_i^2$].

Example 6. *A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half of its original height, it* *attains its maximum (terminal) speed and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey ?*

Solution. Accleration of falling rain drop, through air,

$$a=g-\frac{(B+R)}{m}$$

where viscous force, $R=6\pi\eta rv$,

η=viscosity of air

Force of Buoyancy $B=\frac{4}{3}\pi r^3\rho_0\, g$,

ρ_0=density of air

and mass of drop, $m=\frac{4}{3}\pi r^3\rho$, (ρ=density of water)

$$\therefore\quad a=g-\left(\frac{\rho_0}{\rho}g+\frac{9}{2}\;\frac{\eta r}{v^2\rho}\right)$$

Now since $\frac{\rho_0 g}{\rho}=\frac{1{\cdot}3}{1000}g={\cdot}0013\times9{\cdot}8=0{\cdot}012$ ms^{-2} is small and $\frac{9}{2}\;\frac{\eta r}{v^2\rho}$ will also be too small as η for air is $17{\cdot}2\times10^{-6}$ PaS

$\therefore$ Neglecting them,

$a=g$, for each half journey

$\therefore$ Work done in each half journey

$$\begin{aligned}
&=F\times S\\
&=mg\times S\\
&=\left(\frac{4}{3}\pi r^3\,\rho\right)g\times S\\
&=\left[\frac{4}{3}\times\frac{22}{7}\,(2\times10^{-3})^3\,10^{-3}\right]9{\cdot}8\times250\\
&=0{\cdot}082\text{ J}
\end{aligned}$$

Example 7. *A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction 0·1. Calculate the*

(*a*) *Work done by the applied force in 10 S.*

(*b*) *Work done by friction in 10 S.*

(*c*) *Work done by the net force on the body in 10 S.*

(*d*) *Charge in K.E. of the body in 10 S.*

Solution. The force which moves the body,

$F=(F'-f_k)$ where f_k= kinetic frictional force and $F' = \text{applied force}$

Now $f_2 = \mu_2 N = \mu_2 mg$

$= 0{\cdot}1 \times 2 \times 9{\cdot}8 = 1{\cdot}96\ N$

$\therefore$ $F = (7 - 1{\cdot}96) = 5{\cdot}04\ N$

$\therefore$ By $F = ma$

$5{\cdot}04 = 2 \times a$

$\therefore$ $a = 2{\cdot}52\ ms^{-2}$

$V_i = 0$ and $t = 10\ S$

$\therefore$ By $S = v_{i_1}t + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times 2{\cdot}52 \times 10^2$

$= 126\ m$

(*a*) Work done by the applied force,

$W = F' \times S$

$= 7 \times 126$

$= 882\ J$

(*b*) Work done by frictional force

$W' = f_2 \times S$

$= -1{\cdot}96 \times 126$

$= -246{\cdot}96\ J$

(*c*) $\therefore$ Work done by the net force on the body,

$W'' = W + W'$

$= 882 - 246{\cdot}96$

$= 635{\cdot}04\ J$

(*d*) Since whatever work done by net force the same will be change in K.E. of the body,

So the change in K.E. of body

$= 635{\cdot}04\ J$

Example 8. *A car of mass 2000 kg is lifted up a distance of 30 m by a crane in 1 min. A second crane does the same job in 2 min. Do the cranes consume the same or different amounts of fuel ? What is the power supplied by each crane ? Neglect dissipation of power against friction.*

Solution. Since the two cranes lift the same car by same distance, the work done by each of them will be

$W = F \times S$

$= mg\ S$

$= 2000 \times 9{\cdot}8 \times 30$

$= 5{\cdot}88 \times 10^5\ J$

Since the work done by each crane is $5{\cdot}88 \times 10^5$ J, so they consume the same amount of fuel.

Power of first crane,

$$\begin{aligned} P_1 &= \frac{W}{t_1} \\ &= \frac{5{\cdot}88\times 10^5\ \text{J}}{60\ \text{s}} \\ &= 9{\cdot}800\times 10^3\ \text{watt} \\ &= 9{\cdot}800\ \text{kW} \end{aligned}$$

Power of second crane,

$$\begin{aligned} P_2 &= \frac{W}{t_2} \\ &= \frac{5{\cdot}88\times 10^5\ \text{J}}{2\times 60\ \text{s}} \\ &= 4{\cdot}9\ \text{kW}. \end{aligned}$$

Example 9. *A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m³ in 15 min. If the tank is 40 m above the ground and the efficiency of the pump is 30%, how much electric power is consumed by the pump ?*

Solution.

$$\text{Efficiency, } \eta = \frac{\text{Output work } (W_1)}{\text{Imput work } (W)} \times 100$$

$$\therefore \quad W = \frac{W_1 \times 100}{\eta}$$

Now

$$\begin{aligned} W_1 &= mgh \\ &= (V\rho)\, gh \\ &= (30\times 1000)\ 9{\cdot}8\times 40 \\ & \qquad [\because\ \rho = \text{density of water} = 1000\ \text{kg m}^{-3}] \\ &= 1176\times 10^4\ \text{J} \end{aligned}$$

$$\therefore \quad W = \frac{1176\times 10^4 \times 100}{30}\ \text{J}$$

$$W = 392\times 10^5\ \text{J}.$$

$\therefore$ Electric power consumed by the engine,

$$\begin{aligned} P &= \frac{W}{t} \\ &= \frac{392\times 10^6\ \text{J}}{15\times 60\text{s}} \\ &= 43{\cdot}6\times 10^3\ \text{watt} \\ &= 43{\cdot}6\ \text{kW}. \end{aligned}$$

Example 10. *A motor boat is moving at a steady speed of 36 km h⁻¹. If the water resistance of the motion of the boat is 5000 N, calculate the power of the engine.*

Solution. $F = 5000$ N, $v = 36 \times \frac{5}{18} = 10 \text{ ms}^{-1}$

Power of the engine,

$$\begin{aligned} P &= F \times v \\ &= 5000 \times 10 \\ &= 5 \times 10^4 \text{ watt.} \end{aligned}$$

Example 11. *A train weighing 10^6 kg is moving upan inclined plane rising 1 in 200, with a uniform speed of 72 km h^{-1}. If the frictional resistance amount to 0·4 kg per metric ton, find the power of the engine.*

Solution. $m = 10^6 \text{ kg} = 10^3$ metric ton

Frictional resistance,

$$\begin{aligned} f_k &= 0{\cdot}4 \times 10^3 \text{ kg wt.} \\ &= 0{\cdot}4 \times 9{\cdot}8 \times 10^3 \text{ N} \\ &= 3920 \text{ N} \end{aligned}$$

Resultant force moving the train up the inclined plane,

$$\begin{aligned} F &= (f_k + mg \sin\theta) \\ &= \left(3920 + 10^6 \times 9{\cdot}8 \ \frac{1}{200}\right) \\ &= (3920 + 49000) \\ F &= 52920 \text{ N} \end{aligned}$$

$$v = 72 \times \frac{5}{18} = 20 \text{ ms}^{-1}$$

∴ Power of the engine,

$$\begin{aligned} P &= F \times v \\ &= 52920 \times 20 \\ &= 1058400 \text{ watt} \\ &= 1058{\cdot}4 \text{ kW.} \end{aligned}$$

Example 12. *A bullet of mass 0·012 kg and horizontal speed 70 ms^{-1} strikes a block of wood of mass 0·4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wire. Calculate the height to which the block rises. Also estimate the amount of heat produced in the block.*

Solution. From principle of conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1v_i + m_2v_f$$

$$0{\cdot}012 \times 70 + 0{\cdot}4 \times 0 = 0{\cdot}012\, v + 0{\cdot}4 \times v$$

$$\therefore \quad v = \frac{{\cdot}012 \times 70}{0{\cdot}412} = \frac{210}{103} \text{ ms}^{-1}$$

Since the bullet must have embedded into block after striking it, by principle of conservation of energy,

$$\tfrac{1}{2}(m_1+m_2)\, v^2=(m_1+m_2)\, gh$$

$$h=\frac{v^2}{2g}=\frac{1}{2\times 9{\cdot}8}\left(\frac{210}{103}\right)^2 \text{ m}$$

$$=21{\cdot}2 \text{ cm.}$$

Amount of heat produced = Loss of energy in collision

= [K.E. of bullet before collision—K.E. of bullet and block after collision]

$$=[\tfrac{1}{2}\, m_1u_1^2-\tfrac{1}{2}(m_1+m_2)\, v^2]$$

$$=\frac{1}{2}\left[\, 0{\cdot}12\times 70^2-0{\cdot}412\left(\frac{210}{103}\right)^2\right]$$

$$=28{\cdot}55 \text{ J}$$

Example 13. *A body of mass 0·3 kg is taken up an inclined plane of length 10 m and height 5 m and then allowed to slide down the bottom again. The coefficient of friction between the body and the plane is 0·15. What is the*

(*a*) *Work gone by the gravitational force over the round trip.*

(*b*) *Work done by the applied force for the upward journey.*

(*c*) *Work done by frictional force over the round trip.*

(*d*) *Kinetic energy of the body at the end of the trip ?*

Solution.

(*a*) $\sin\theta=\frac{h}{l}=\frac{5}{10}=\frac{1}{2}$

$\therefore\quad \theta=30°$

Work done to move the body up the inclined plane against gravitational force,

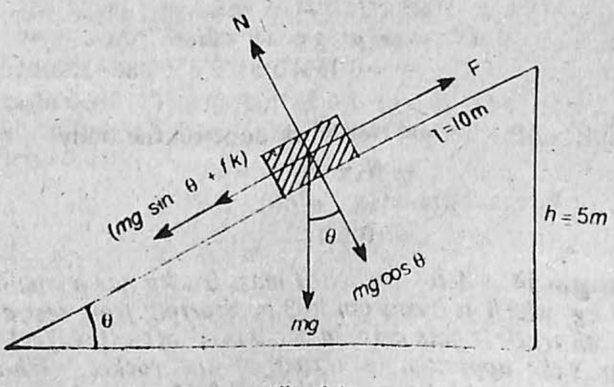

Fig. 4.1.

$$
\begin{aligned}
W &= F\times S\\
&= mg\sin 0\times l\\
&= -0{\cdot}3\times 9{\cdot}8\times\tfrac{1}{2}\times 10\\
&= -14{\cdot}7\ \text{J}
\end{aligned}
$$

Work done by the gravitational force to bring the body down the inclined plane,

$$
\begin{aligned}
W' &= mg\sin\theta\times l\\
&= 14{\cdot}7\ \text{J}
\end{aligned}
$$

∴ Net work done by the gravitational force over the round trip

$$
\begin{aligned}
&= W+W' = -14{\cdot}7+14{\cdot}7\\
&= 0
\end{aligned}
$$

(*b*) The applied force (see figure),

$$
\begin{aligned}
F &= (mg\sin\theta+f_k)\\
&= mg\sin\theta+\mu N\\
&= mg\sin\theta+\mu mg\cos\theta\\
&= mg(\sin\theta+\mu\cos\theta)\\
&= 0{\cdot}3\times 9{\cdot}8\,(\tfrac{1}{2}+0{\cdot}15\times\cos 30)\\
&= 2{\cdot}90\,(0{\cdot}5+0{\cdot}15\times 0{\cdot}866)\\
&= 1{\cdot}85\ \text{N}
\end{aligned}
$$

∴ Work done by the applied force,

$$
\begin{aligned}
W_1 &= F\times S\\
&= 1{\cdot}85\times 10\\
&= 18{\cdot}5\ \text{J.}
\end{aligned}
$$

(*c*) Work done by the frictional force on the body over the round trip,

by $W = F\times S$

$$
\begin{aligned}
W_2 &= -f_k\times 2l\\
&= -\mu mg\cos\theta\times 2l\\
&= -0{\cdot}15\times 0{\cdot}3\times 9{\cdot}8\times 0{\cdot}866\times 2\times 10\\
&= --7{\cdot}6\ \text{J.}
\end{aligned}
$$

(*d*) K.E. of the body = Net work done on the body

$$
\begin{aligned}
&= W_1+W_2\\
&= (18{\cdot}5-7{\cdot}6)\\
&= 10{\cdot}9\ \text{J.}
\end{aligned}
$$

Example 14. *A toy rocket of mass 0·1 kg has a small fuel of mass 0·02 kg which it burns out in 3 s. Starting from rest on a horizontal smooth track it gets a speed of 20 ms^{-1} after the fuel is burnt out. What is the approximate thrust of the rocket? What is the energy content per unit of mass of the fuel?* (*Ignore the small mass variation of the rocket during fuel burning.*)

Solution. Thrust of the rocket,

$$F = ma$$
$$= \frac{m(v_f - v_i)}{t}$$
$$= 0{\cdot}1\left(\frac{20-0}{3}\right)$$
$$= \tfrac{2}{3}\ \text{N}$$

The kinetic energy the rocket gained in 3 seconds

$$= \tfrac{1}{2}\ mv^2$$
$$= \tfrac{1}{2} \times 0{\cdot}1 \times 20^2$$
$$= 20\ \text{J}$$

∴ The energy content per unit mass of the fuel

$$= \frac{20\ \text{J}}{{\cdot}02\ \text{J}} = 1000\ \text{J/kg}.$$

Example 15. *A bolt of mass 0·3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 ms^{-1}. It hits the floor of the elevator and does not rebound length of the elevator is 3 m. What is the heat produced by the impact? Would your answer be different if the elevator were stationary?*

Solution. It is the potential energy of the bolt which converts into heat when the bolt hits the floor.

∴ Heat produced $= mgh$

$$= 0{\cdot}3 \times 9{\cdot}8 \times 3$$
$$= 8{\cdot}82\ \text{J}$$

Heat produced will still be 8·82 J if the elevator is even stationary.

Example 16. *The bob of the pendulum is released from a position when the string holding the bob is straight and horizontal. If the length of the pendulum is 1·5 m, what is the speed with which the bob arrives at the lower most point, given that it dissipates 5% of its initial energy against air resistance.*

Solution. The potential energy of the bob converts into K.E. when bob reaches at the lower most point. Since 5% energy get dissipated,

∴ K.E. = 95% of potential energy

$$\therefore \quad \frac{1}{2}\ mv^2 = \frac{95}{100}\ mgh$$
$$v^2 = 2 \times \frac{19}{20} \times 9{\cdot}8 \times 1{\cdot}5$$
$$v^2 = 27{\cdot}93$$
$$\therefore \quad v = 5{\cdot}3\ \text{ms}^{-1}.$$

Example 17. *A trolley of mass 200 kg moves with a uniform speed of 36 kmh^{-1} on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 ms^{-1} relative to the trolley's motion and jumps out of the trolley. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?*

Solution. $36 \text{ km/h} = 36 \times \frac{5}{18} = 10 \text{ ms}^{-1}$

The child gives an impulse to the trolley to start running on the trolley. If v is the increase in the velocity of the trolley due to this impulse, then by principle of conservation of momentum,

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$
$$200(10+v) + 20(10+v-4)$$
$$= 200 \times 10 + 20 \times 10$$

(Since the boy before starting running on the trolley, he is moving along with the trolley at the speed of 10 ms^{-1},

i.e. $u_2 = 10 \text{ ms}^{-1}$)

On solving the above equation, we get

$$v = \frac{4}{11} \text{ ms}^{-1}$$
$$= 0{\cdot}36 \text{ ms}^{-1}$$

∴ The new speed of the trolley,

$$v' = (10 + 0{\cdot}36)$$
$$= 10{\cdot}36 \text{ ms}^{-1}$$

The time taken by the child to cross the trolley,

$$t = \frac{s}{v}$$
$$= \frac{10}{4}$$
$$= 2{\cdot}5 \ s$$

∴ The distance moved by trolley during this time,

$$s = v' \times t$$
$$= 10{\cdot}36 \times 2{\cdot}5$$
$$= 25{\cdot}9 \text{ m.}$$

Example 18. *Estimate the amount of energy released in the nuclear fusion reaction :*

$${}_1H^2 + {}_1H^2 \longrightarrow {}_2He^3 + {}_0n^1$$

(*Given that mass of* ${}_1H^2 = 2{\cdot}0141\ u$

mass of ${}_2H^3 = 3{\cdot}0160\ u$

mass of ${}_0n^1 = 1{\cdot}0087\ u$)

Solution. Total mass of reactants

$$= 2 \times 2{\cdot}0141\ u$$
$$= 4{\cdot}0282\ u$$

Total mass of products

$$\begin{array}{r} =3{\cdot}0160\ u \\ +1{\cdot}0087\ u \\ \hline 4{\cdot}0247\ u \end{array}$$

$\therefore$ Loss of mass in reaction

$$=4{\cdot}0282\ u-4{\cdot}0247\ u$$
$$=0{\cdot}0035\ u$$

But on loss of mass of 1 u, 933 MeV energy is released.

$\therefore$ Energy released in the reaction,

$$E=0{\cdot}0035\times 933\ \text{MeV}$$
$$=3{\cdot}2655\ \text{MeV}.$$

Example 19. *A truck tows a trailer of 1500 kg at a steady speed of 54 km/h on a level road. The tension in the coupling is 800 N. Find the tension in the coupling when truck ascends a road having inclination 1 in 5. What is the power expended on trailer on the (a) level road (b) inclination ?*

Solution. $m=1500$ kg, $v=54$ km/h

$$=54\times\frac{5}{18}=15\ ms^{-1}$$

Initial tension $T_0=800$ N

Additional tension in coupling on the inclination,

$$\Delta T=mg\sin\theta$$
$$=1500\times 9{\cdot}8\times\frac{1}{5}$$
$$=2940\ \text{N}$$

$\therefore$ Total tension in the coupling,

$$T=T_0+\Delta T$$
$$=(800+2940)$$
$$=3740\ \text{N}$$

(*a*) Power expended to trailer on the level road

$$P_0=T_0\times v \qquad [\because\ P=F\times v]$$
$$=800\times 15$$
$$=1200\ \text{watt}$$
$$=1{\cdot}2\ \text{kW}$$

(*b*) Power expended to the trailer on inclination,

$$P=Tv$$
$$=3740\times 15$$
$$=56100\ \text{watt}$$
$$=56{\cdot}1\ \text{kW}$$

Example 20. *Calculate the energy equivalent to one atomic mass unit.*

Solution. 1 atomic mass unit.

$$1u = 1{\cdot}661 \times 10^{-27}\ \text{kg}$$

$\therefore$ Energy due to 1u,

by $E = mc^2$,

$$1u = (1{\cdot}661 \times 10^{-27})\,(2{\cdot}998 \times 10^{8})^2\ \text{J}$$

$$= 14{\cdot}929 \times 10^{-11}\ \text{J}$$

$$= \frac{14{\cdot}929 \times 10^{-11}}{1{\cdot}6 \times 10^{-13}}\ \text{MeV}$$

$\therefore$ $1v = 933$ MeV

Example 21. *A boy of mass 40 kg sits in a swing by a rope of 6 m long. A person pulls the swing to a side so that the rope makes an angle of 60° with the vertical. What is the gain in potential energy of the boy?*

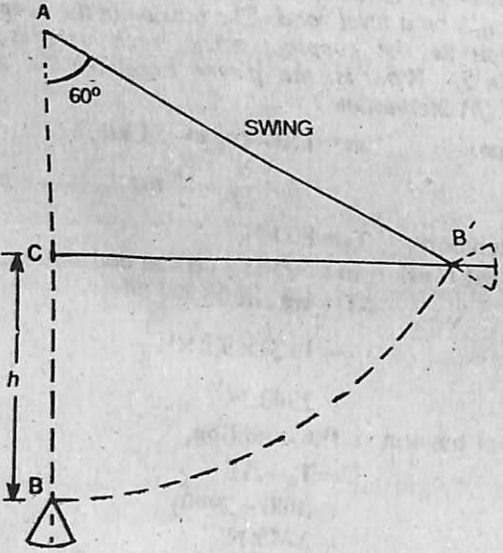

Fig. 4.2.

Solution. The height through which the boy is raised

$$= AB - AC$$

$$= 6 - 6\cos 60$$

$$= 6(1 - \tfrac{1}{2})$$

$\therefore$ $h = 3$ m

The gain in P.E. of the boy = mgh

$$= 40 \times 9{\cdot}8 \times 3$$

$$= 1176\,J$$

Example 22. *A spring fixed at one end stretches by 2 cm when 2 kg weight is suspended from its lower end. Calculate (a) the force constant (b) the work done to stretch it further from 2 cm to 5 cm ($g=10\ ms^{-2}$.)*

Solution. The force constant of the spring (spring constant ,

$$K=\frac{F}{\xi}$$
$$=\frac{mg}{\xi}$$
$$=\frac{2\times10}{\cdot02}$$
$$=10^3\ Nm^{-1}$$

The potential energy of a spring,

$$E=\frac{1}{2}K\xi^2$$

∴ The work done$=E_2-E_1$

$$=\frac{1}{2}K\xi_2^2-\frac{1}{2}K\xi_1^2$$
$$=\frac{1}{2}K(\xi_2^2-\xi_1^2)$$
$$=\frac{1}{2}\times10^3(\cdot05^2-\cdot02^2)$$
$$=1\cdot05\ J$$

EXERCISE 4

1. An engine pulls a train 2 km over a level track. Calculate the work done, given that the frictional force is 4×10^5 N.
2. A bullet of mass 0·03 kg moving with a speed of 500 ms^{-1} penetrates 12 cm into a fixed block of wood. Calculate the average force exerted by the wood on the bullet.
3. A shot travelling at the rate of 100 m/s is just able to pierce a planck 4 cm thick. What velocity is required to pierce the plank 0·25 cm thick ?
4. A weight lifter carries 20 kg weight over his head through a distance of 50 m in (*a*) the horizontal direction, (*b*) the vertical direction. Calculate the work done in each case.
5. A boy of mass 50 kg sits in a swing suspended by a rope 5 m long. It is pulled by a person so as to make an angle of 30° with the vertical What is the gain in gravitational potential energy of the boy ?
6. A train weighing 10^5 kg is moving up an inclined plane rising 1 in 100, with a uniform speed of 72 km/h. If the frictional

resistance amounts to 0·5 kg wt. per metric ton, find the power of the engine. [*H.P.S.S.E. 1988*]

7. An automobile moving horizontal with a speed of 72 kmh^{-1} reaches the foot of a smooth incline when the engine is switched off. How much distance does the automobile go up the incline before coming to rest ? The inclination of the plane to the horizontal is 30° and the acceleration due to gravity at the place is 9·8 ms^{-2}.

8. A car weighing 10^3 kg moving with a velocity of 36 kmh^{-1} is having a uniform acceleration of 2 ms^{-2} for 30 s. Find the work done.

9. The driver of a truck of mass 10 metric ton, moving at a speed of 72 kmh^{-1}, finds a child at a distance 102 m. He applies the brakes to stop the truck within 2m of the child. Calculate the work done

10. A body of mass 1 kg is attached to a string 2 m long. The body is held so that the string is horizontal and it is then allowed to fall. Calculate the K.E. of the body when the string makes an angle of (*a*) 60° (*b*) 0° with the vertical.

11. The mass of a pendulum bob is 100 g and the string is 1 m long. The bob is held so that the string is horizontal and it is then allowed to fall. Find the K.E. when the string makes an angle (*a*) 0° and (*b*) 30° with the vertical [*I.I.T. J.E.E. 1963*]

12. What is the energy released when 1 g of a substance is lost ?

13. Calculate the energy equivalent to 1 amu (1 amu$=1{\cdot}66\times10^{-27}$ kg & $c=3\times10^{8}$ ms^{-1}).

14. What mass must be converted to get 1 MJ of energy ?

15. A truck tows a trailor at a steady speed of 20 ms^{-1} on a level road. The tension in the coupling is 1000 N. Calculate the power expended on the trailer.

16. In Olympic a weight lifter lifts 300 kg weight over his head by 2 m in 2 minutes What is the power of the man ?

17. A truck tows a trailer of mass 1200 kg at a steady speed of 10 ms^{-1} on a level road. The tension in the coupling is 1000 N. Find the tension in the coupling when the truck ascend a road having an inclination of 1 in 6. Calculate the power expended on the trailer (*a*) on the level road, (*b*) on the inclined road.

18. A motor boat is moving at a steady speed of 20 ms^{-1}. If the water resistance of the motion of the boat is 6000 N. Calculate the power of the engine.

19. A pump set is used to lift water to a reservoir of 6000 litres capacity over an average height of 20 m. If it takes 1 hour

to fill the reservoir completely, calculate the work done and the power supplied to the pump if its efficiency is 60%.

20. An electric motor is used to lift an elevator and its load, total being 1500 kg to a height of 20 m. The time taken in the job is 20 S. The efficiency of the motor is 75 per cent. What is the work done and the power of the motor ?

21. A pump is required to lift 600 kg of water per minute from a well 25 m deep and to eject it with a speed of 50 ms^{-1}. Calculate the

    (*a*) work done per minute in lifting the water.

    (*b*) work done in imparting K.E. to water.

    (*c*) the power of the engine.

22. A crane is used to lift 10 metric ton of coal from a mine 100 m deep. Calculate the total work done, if the time taken by the crane is 1 hour. Calculate the power of the crane, assuming the efficiency of the crane 80%.

23. An electron strikes the screen of a cathode ray tube with a velocity of 10^6 ms^{-1}. Calculate the K.E. (mass of electron = 9×10^{-31} kg).

24. A spring fixed at one end is stretched by 5 cm when a weight of 30 N is suspended from its other end. The spring is further stretched such that its elongation increases from 5 cm to 8 cm. Find (*a*) the force constant and (*b*) the work done.

25. A 2 kg mass on a floor is connected to a 5 kg by a string passing over a pulley. Obtain the speed of the masses after they are released when the 5 kg mass just reaches the floor. Establish that gain in K.E. of the system equals the loss in its potential energy. The 5 kg mass is initially at a height of 3 m above the ground.

26. A body of mass 5 kg moving with a speed of 4 ms^{-1} on a frictionless table impinges on a mounted spring of force constant 5×10^6 Nm^{-1}. What is the compression of the spring when the body comes to test ?

27. A house is fitted with 5 electric lamps each of 40 W and other 5 lamps each of 60 W. If all the lamps are lighted for 8 hours in a day. What is the bill of the electricity in the month of November at the rate of 30 paise per unit ?

28. A railway carriage of mass 8,000 kg moving with a speed of 72 km/h strikes a stationary carriage of the same mass. After the collision the carriages get coupled and move together. What is their common speed after collision ?

29. A rain drop of radius 4 mm falls from a height of 200 m above the ground. It falls with decreasing acceleration due to viscous forces of the air until at one-fourth of its original

height it attains the terminal velocity and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second part of the journey.

30. A body of mass 1·5 kg initially at rest moves under the action of an applied force of 8 N for 5 seconds. If coefficient of kinetic friction is 0·2, calculate the
    - (*a*) work done by the applied force
    - (*b*) work done by friction
    - (*c*) work done by the net force
    - (*d*) change in K.E. of the body.

31. A body of mass 2·5 kg initially moving with velocity 5 ms^{-1} is acted upon by a force of 2 kg wt for 12 S. If coefficient of kinetic friction is 0·15, calculate the above quantities from (*a*) to (*d*) in Q. No. 30. (Take $g=10$ ms^{-2}).

32. A ball of mass 200 g dropped from the top of a lift 2·5 high hits it floor and does not rebound. If the lift is moving with a uniform speed of 8 ms^{-1}, what is the heat produced by the impact ?

33. The bob of a pendulum is released from a horizontal position keeping its string straight. If the length of the pendulum is 1·2 m, what is the speed with which the bob arrives at lower most point given that 1% energy is dissipated against air resistance.

34. A rocket model of a mass 0·2 kg has a fuel 0·05 kg which it burns in 4 s. Starting from rest on a horizontal smooth track it gets a speed of 12 ms^{-1} after the fuel is burnt out What is the approximate thrust of the rocket ? What is the energy content per unit mass of the fuel ?

35. A trolley of mass 150 kg moves with a uniform speed of 12 ms^{-1} on a frictionless track. A boy of mass 40 kg runs on the trolley from one end to the other 4 m away with a speed of 5 ms^{-1} relative to the trolley in the same direction of the motion of trolley and jumps out of the trolley (*a*) What is the final speed of the trolley ? How much has the trolley moved from the time the child begins to run ? (*b*) If the boy runs with the same relative speed in the direction opposite to the motion of the trolley, recalculate both the values of part (*a*).

36. Calculate the energy released in MeV in the following nuclear reaction

$$_0n^1+{}_3Li^6 \longrightarrow {}_2He^4+{}_1H^3$$

(masses of $_0n^1=1{\cdot}008665$ amu, $_3Li^6=6{\cdot}015126$ amu, $_2He^4=4{\cdot}002604$ amu and $_1H^3=3{\cdot}016049$ amu).

37. 200 MeV energy is released in the fission of a single nucleus of $_{92}U^{235}$. How many fissions must occur per minute to produce a power of 2 kW ?

38 When four hydrogen atoms combine, they form a helium atom along with two positrons ($_{+1}e^{\circ}$) each of mass 0·0005494 *u*. Calculate the energy released (mass of $_2He^4$ atom$=4·002604u$).

39. The fission of single $_{92}U^{235}$ nucleus releases 180 MeV energy. How many fissions must occur per hour to produce a power of 3 kW ?

40. Calculate the binding energy for $_1H^2$ nucleus in MeV.
(mass of proton$=1·007825$ a.m.u.
mass of neutron$=1·008665$ a.m.u.
mass of $_1H^2=2·014103$ a.m.u.
and 1 a.m.u.$=$ 931 MeV) [*A.I.S.S.E. 1989*]

[**Hint**—Binding energy is the energy released due to mass defected.]

OBJECTIVE TYPE QUESTIONS

41. The unit of power is
(*a*) Joule (*b*) Joule per second only
(*c*) Joule per second and watt both (*d*) Watt only.
[*C.P.M.T 1988*]

42. If the velocity of a particle is doubled then the kinetic energy will be
(*a*) Doubled (*b*) Remains the same
(*c*) Four times (*d*) $\frac{1}{4}$th. [*C.P.M.T. 1980, 1988*]

43. State if each the following statement is true or false :
(*a*) In an elastic collision of two bodies, the momentum and energy of each body is conserved ;
(*b*) Total energy of the system is always conserved ;
(*c*) Work done in the motion of a body over a closed loop is zero for every force in nature ;
(*d*) In an inelastic collision, the final K.E. is always less than the initial K.E. of the system.

44 A ball is allowed to fall from a height of 10 m. If there is 40% loss of energy due to impact. Then after one impact ball will go up
(*a*) 10 m (*b*) 8 m (*c*) 4 m (*d*) 6 m [*C.P.M.T. 1988*]

45. A light and heavy body both have equal momentum which one has greater K.E.
(*a*) The light body (*b*) Both have equal K.E.
(*c*) The heavy body (*d*) The data given is incomplete.
[*C.P.M.T. 1988*]

46. A light and heavy body have equal kinetic energy. Which one has greater momentum

(*a*) The light body (*b*) Both have equal momentum
(*c*) The heavy body (*d*) The data are in sufficient.
[*C.P.M.T. 1988*]

47. The momentum of a body decreases by 20%, the percentage decrease in its K.E. will be
(*a*) 36 (*b*) 48 (*c*) 40 (*d*) None of these.

48. The power required to raise a body of weight 368 N vertically at a velocity of 4 ms^{-1} is equal to
(*a*) 92 Watt (*b*) 372 Watt
(*c*) 92 H.P. (*d*) 2 H.P.

49. A particle executes simple harmonic motion with a frequency '*f*' The frequency with which its kinetic energy oscillates is :
(*a*) $\frac{f}{2}$ (*b*) f
(*c*) $2f$ (*d*) 4 [*I.I.T. J.E.E. 1987*]

50. Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants K_1 and K_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of amplitude of vibrations of M to that of N is
(*a*) k_1/k_2 (*b*) $\sqrt{k_1/k_2}$
(*c*) $k\ /k_1$ (*d*) $\sqrt{k_2/k_1}$. [*I.I.T. J.E.E 1988*]

51. A uniform chain of length L and mass M is lying on a smooth table and one-third of its length is hanging vertically down over the edge of the table. If '*g*' is acceleration due to gravity, the work required to pull the hanging part on to the table is
(*a*) Mg L (*b*) Mg L/3
(*d*) Mg L/9 (*d*) Mg L/18. [*I.I.T. J.E.E. 1985*]

52. Two masses of 1 g and 4 g are moving with equal K.E. the ratio of the magnitudes of their linear momenta is
(*a*) 4 : 1 (*b*) 2 : 1
(*c*) 1 : 2 (*d*) 1 : 16. [*I.I.T. J.E.E. 1988*]

53. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time '*t*' is proportional to
(*a*) $t^{1/2}$ (*b*) $t^{3/4}$
(*c*) $t^{3/2}$ (*d*) t^2

54. Two boys of same mass climb the same ladder in different interval of time. Which of the following quantities is different for two boys
(*a*) Work (*b*) Power
(*c*) Work and power both (*d*) Weight. ☐☐

5

Rotational Motion

IMPORTANT FORMULAE

1. Centripetal force, centrifugal force and tension in a string rotating in a circular path,

$F=\frac{mv^2}{r}$;

m=mass of the body
v=speed of the body
r=radius of circular path

2. $v=2\pi nr$;

n=No. of revolutions per sec.

3. The banking angle (θ) of a path at the turning, is given by

$\tan\theta=\frac{v^2}{rg}$;

v=speed of vehicle
r=radius of circular path at the turning.

4. The velocity of a vehicle on a banked road of radius 'r' is given by

$v^2=rg\frac{(\mu+\tan\theta)}{(1-\mu\tan\theta)}$;

θ=banking angle
μ=coeffi. of friction

5. Rotational K.E.$=\frac{1}{2}I\omega^2$;

I=moment of inertia
ω=Angular velocity.

6. Angular momentum,

$L=mvr=I\omega$

7. Torque, $\tau=F.r=I\omega$;

ω=Angular acceleration

8. Work done by Torque,

$W=I\theta$;

θ=angle of rotation

9. Equations of rotational motion of a body,

$\omega=\omega_0+\dot{\omega}t^2$;
$\theta=\omega_0 t+\frac{1}{2}\dot{\omega}t^2$
$2\dot{\omega}\theta=\omega^3-\omega_0^2$

ω=Final angular velocity
ω_0=Initial angular velocity
$\dot{\omega}$=Angular acceleration
θ=angle of rotation
t=time.

10. The moment of inertia of a body,

$I=\sum_i m_i r_i^2$

11. **Principle of Parallel Axis**. The moment of intertia of a body about any axis is equal to the moment inertia about a parallel axis through its centre of mass plus the product of the mass of the body and square of distance 'a' between the two axes, *i.e.*

$I=I_g+Ma^2$.

12. **Principle of Perpendicular Axis.** The moment of inertia of a plane lamina about an axis z perpendicular to the plane of the lamina is equal to the sum of the moments of inertia I_x and I_y of the lamina about two axes x and y at right angles to each other in its own plane, intersecting each other at the point where perpendicular axis passes through it

$$I_z = I_x + I_y.$$

13. Moment of Inertia of Some Common Solids

S. No.	*Shape of the Rigid Body*	*Dimensions*	*Axis of Rotation*	*Moment of Inertia*
1.	Thin Uniform rod	Length l	(*a*) Through the centre and perpendicular to length	$\frac{Ml^2}{12}$
			(*b*) Through one end of the rod and perpendicular to its length	$\frac{Ml^2}{3}$
2.	Thin Circular ring	Radius r	(*a*) Through the centre and perpendicular to its plane	Mr^2
			(*b*) Through the centre about one of its diameters	$\frac{1}{2}Mr^2$
3.	Rectangular lamina	length l and width b	Through the centre and perpendicular to its plane	$\frac{M(l^2+b^2)}{12}$
4.	Uniform cylinder	Radius r	Through its centre and perpendicular to its plane surface	$\frac{1}{2}Mr^2$
5.	Sphere	Radius r	Through its centre	$\frac{2}{5}Mr^2$
6.	Circular disc	Radius r	About a diameter	$\frac{1}{4}Mr^2$

SOLVED EXAMPLES

Example 1. *A body rotating in a circular path is making 300 rpm. Calculate the angular velocity of the body. What is the speed of the bady if the radius of the circular path is 70 cm ?*

Solution. $n = 300$ *rpm* (*rpm*=revolution per minute)

$$= \frac{300}{60}$$

$$= 5 \text{ rev. per sec.}$$

Angular velocity,

$$\omega = 2\pi n = 2\pi \times 5 = 10\pi = 31{\cdot}4 \text{ rad s}^{-1}$$

Speed of the body

$$V = \omega r$$

$$= 10\pi \times 70 \text{ cm s}^{-1}$$

$$= 22 \text{ ms}^{-1}.$$

Example 2. *A mass of 0·5 kg is whirled in a circle by a string 0·5 m in length. If the maximum tension the string can bear in 25 N, what is the fastest speed with which mass can be revolved without breaking the string ?*

Solution. Here $m = 0{\cdot}5$ kg, $r = 0{\cdot}5$ m and $F = 25$ N

By
$$F = \frac{mv^2}{r}$$

$$25 = \frac{0{\cdot}5\, v^2}{0{\cdot}5}$$

$$v = 5 \text{ ms}^{-1}$$

Example 3. *A body weighing 1·4 kg is whirled in a vertical circle making 3 revolutions per second. If the radius of the circle is 0·7 m, find the tension in the string when a body is (a) at the bottom of the circle (b) at the top of the circle*

Solution. $m = 1{\cdot}4$ kg, $n = 3$ rev./s $r = 0{\cdot}7$ m

By $v = 2\pi nr,$

$$v = 2 \times \frac{22}{7} \times 3 \times 0{\cdot}7$$

$$= 13{\cdot}2 \text{ ms}^{-1}$$

(*a*) The tension in the string at the bottom of the circle,

$$T_1 = \left(\frac{mv^2}{r} + mg\right)$$

$$= \left[\frac{1{\cdot}4 \times (13{\cdot}2)^2}{0{\cdot}7} + 1{\cdot}4 \times 9{\cdot}8\right]$$

$$= 362{\cdot}2 \text{ N}$$

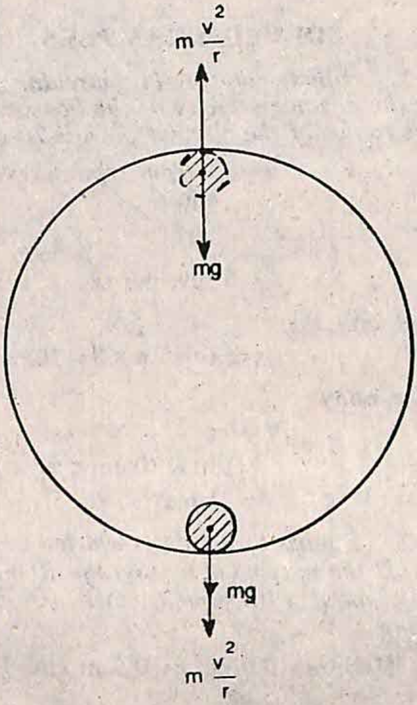

Fig. 5.1.

(*b*) The tension in the string at the top of circle,

$$T_2 = \left(\frac{mv^2}{r} - mg \right)$$

$$= \left[\frac{1{\cdot}4 \times (13{\cdot}2)^2}{0{\cdot}7} - 1{\cdot}4 \times 9{\cdot}8 \right]$$

$$= 334{\cdot}76 \text{ N}$$

Example 4. *A spherical cage is erected in a circus for performing a motor-cycle feat of going in a verticle circle. If the radius of the spherical cage is 5 m, what is the maximum speed with which he has to perform the feat so that he may not fall ?*

Solution. For not to fall when going around in a vertical circle in a cage,

$$\frac{mv^2}{r} \geqslant mg$$

$$v \geqslant \sqrt{rg}$$

$$\geqslant \sqrt{5 \times 9{\cdot}8}$$

$$\geqslant 7 \text{ ms}^{-1}$$

∴ The minimum speed of motor cycle must be 7 ms^{-1}.

Example 5. *A train has to negotiate a curve of 250 m. By how much should the outer rail be raised with respect to the inner rail for a speed of 54 kmh^{-1} ? The distance between the rails is 1·2 m.*

(*Take* $g=10\ ms^{-2}$)

Solution. $v=54\ \text{km/h}^{-1}=54\times\frac{5}{18}=15\ \text{ms}^{-1}$

$$\tan\theta=\frac{v^2}{rg}$$

$$=\frac{225}{250\times10}$$

$$=\cdot09$$

If outer rail is raised by h metre in comparison to the inner rail,

$$\tan\theta=\frac{h}{d}$$

$$\therefore\quad \cdot09=\frac{h}{1\cdot2}$$

$$\therefore\quad h=\cdot108\ \text{m}.$$

Example 6. *The total mass of a motor-cycle and the rider is 150 kg. The rider has to negotiate a curve of radius 40 m at a speed of 45 kmh^{-1}. If the coefficient of friction is 0·4, will the rider be able to go around the curve successfully ? At what angle should he learn over so as to avoid a spill.*

Solution. $v=45\times\frac{5}{18}\ \text{ms}^{-1}=12\cdot5\ \text{ms}^{-1}$

The centripetal force required to go around the curve,

$$F=\frac{mv^2}{r}$$

$$=\frac{150\times(12\cdot5)^2}{40}$$

$$=585\cdot9\ \text{N}$$

The maximum force of friction,

$$f_{ms}=\mu N=\mu mg$$

$$=0\cdot4\times150\times9\cdot8$$

$$=588\ \text{N}$$

$\because\ f_{ms}>F$, the necessary centripetal force will be provided by the frictional force and the motorcyclist will go around the curve successfully. If θ is the angle he should lean from the vertical, then

$$\tan\theta=\frac{v^2}{rg}$$

$$=\frac{(25/2)^2}{40\times 9{\cdot}8}$$

$$=0{\cdot}3986$$

$$\therefore \qquad \theta=21°44'$$

Example 7. *An aeroplane travelling at a speed of 540 kmh^{-1} banks at an angle of 45° when it takes a turn. What is the radius of the curve ?*

Solution. $v=540\times\frac{5}{18}\ \text{ms}^{-1}=150\ \text{ms}^{-1}$

$$\tan\theta=\frac{v^2}{rg}$$

$$\therefore \qquad \tan 45=\frac{150^2}{r\times 9{\cdot}8}$$

$$\therefore \qquad r=\frac{22500}{9{\cdot}8} \qquad [\because\ \tan 45=1]$$

$$=2295{\cdot}92\ \text{m}$$

Example 8. *A 60 kg man stands in contact against the wall of a cylindrical drum of radius 2 m rotating about its vertical axis with 100 rpm. The coefficient of friction between the wall and his clothing is 0·14. What is the minimum rotational speed of the cylinder to enable the man to remain struck to the wall without falling when the floor is suddenly removed ?*

Solution. The required centripetal force is provided by the horizontal reaction 'N' of the wall of rotating cylinder on the man.

$$\therefore \qquad N=\frac{mv^2}{r}$$

The frictional force acting vertically upwards balances the weight of the man.

∴ The man will not fall down even after the floor is removed if

$$f_k \geqslant mg$$

or

$$\mu N \geqslant mg$$

or

$$\frac{\mu mv^2}{r} \geqslant mg$$

$$\therefore \qquad v \geqslant \sqrt{\frac{gr}{\mu}}$$

$$\sqrt{\frac{9{\cdot}8\times 2}{0{\cdot}14}}$$

$$v \geqslant 11{\cdot}83\ \text{ms}^{-1}$$

∴ Minimum Rotational Speed

$$=\frac{v}{r}=\frac{11{\cdot}83}{2}$$
$$=5{\cdot}92 \text{ rad. s}^{-1}$$

Example 9. *A thin circular wire of radius r rotates about its vertical diameter with an angular frequency 'ω'. Show that a small bead on the wire remains at its lower most point for* $\omega \leqslant \sqrt{g/r}$*, what is the angle made by radius vector joining the centre to the bead with the vertical downward direction for* $\omega=\sqrt{\frac{2g}{r}}$ *?*

Solution. The normal reaction on the bead is equal to the centrifugal force.

$$\because \quad N=\frac{mv^2}{r}$$
$$=m\omega^2 r$$

where ω=angular frequency

It is evident from the figure,

$N\cdot\cos\theta=mg$

$$\therefore \quad \cos\theta=\frac{mg}{N}$$
$$=\frac{mg}{m\omega^2 r}$$
$$\cos\theta=\frac{g}{\omega^2 r}$$

We know $|\cos\theta|<1$

∴ The bead remains at the lower most point for

$$\omega\leqslant\sqrt{\frac{g}{r}}$$

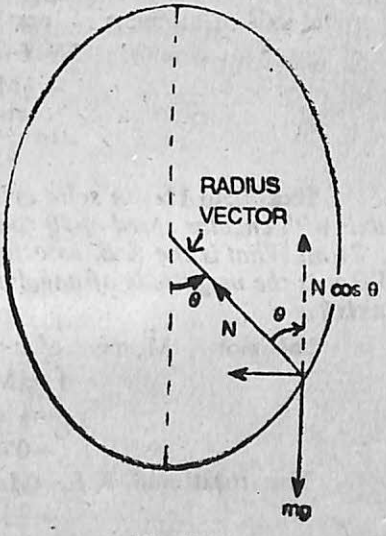

Fig. 5·2.

For $\omega=\sqrt{\frac{2g}{r}}$,

$$\cos\theta=\frac{g}{\omega^2 r} \text{ becomes}$$

$$\cos\theta=\frac{g}{\frac{2g}{r}\times r}$$
$$=\frac{1}{2}$$
$$\therefore \quad \theta=60^\circ$$

Example 10. *If the moment of inertia of a uniform circular disc of mass M and radius r about an axis passing through its centre and perpendicular to its plane in $\frac{1}{2}Mr^2$. What is the moment of inertia of the disc (a) about any of its diameter (b) an axis passing through a point on its edge and normal to the disc ?*

Solution. (*a*) Let $I=\frac{1}{2}Mr^2$ and moment of inertia about any diameter be I_1.

$\therefore$ By principle of perpendicular axis of moment of inertia,

$$I=I_1+I_1 \qquad [I_z=I_x+I_y]$$

$$\therefore \quad I_1=\tfrac{1}{2}I$$

$$=\tfrac{1}{2}(\tfrac{1}{2}Mr^2)=\tfrac{1}{4}Mr^2$$

(*b*) Let I_2 be the moment of inertia about an axis passing through a point on its edge and normal to disc. By principle of parellel axis of moment of inertia,

$$I_2=I+Mr^2 \qquad [I=I_c+Ma^2]$$

$$=\tfrac{1}{2}Mr^2+Mr^2$$

$$=\frac{3}{2}Mr^2$$

Example 11. *A solid cylinder of mass 15 kg rotates about its axis with angular speed of 50 rads⁻¹. The radius of the cylinder is 0·24 m. What is the K.E. associated with the rotation of the cylinder ? What is the magnitude of angular momentum of the cylinder about its axis ?*

Solution. Moment of inertia of the cylinder about its axis,

$$I=\tfrac{1}{2}Mr^2$$

$$=\tfrac{1}{2}\times 15\times(\cdot 24)^2$$

$$=0{\cdot}432 \text{ kgm}^2$$

The rotational $\text{K.E.}=\tfrac{1}{2}I\omega^2$

$$=\tfrac{1}{2}\times 0{\cdot}432\times 50^2$$

$$=540 \text{ J}$$

The angular momentum of the cylinder,

$$L=I\omega=0{\cdot}432\times 50$$

$$=21{\cdot}6 \text{ kgm}^2\text{s}^{-1}$$

Example 12. *A child stands at the centre of a turning table with his two arms outstretched. The turntable is set rotating with an angular speed of 60 rpm. (a) How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 3/4 times the initial value ? Assume that the turntable rotates without friction. (b) Show that the child's new K.E of rotation is more than the initial K.E. of rotation. How do you account for this increase in K.E. ?*

Solution. (*a*) By principle of conservation of angular momentum,

$$I_2\omega_2 = I_1\omega_1$$

$$\frac{3}{4} I_1\omega_2 = I_1 \times 60$$

$$\therefore \quad \omega_2 = 80 \text{ rpm}$$

(*b*) Initial R.K.E. $= \frac{1}{2} I_1\omega_1^2$

$$= \tfrac{1}{2} I_1 60^2$$

$$= 1800 I_1$$

New R.K.E. $= \frac{1}{2} I_2\omega_2^2$

$$= \frac{1}{2} \times \frac{3}{4} I_1 \times 80^2$$

$$\therefore \quad = 2400 \, I_1$$

The increase in K.E. is due to the internal energy that child uses in folding his hands back from his stretched position.

Example 13. *A rope is wound round a hollow cylinder of mass 2 kg and radius 25 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N ? What is the linear acceleration of the rope ?*

Solution. Moment of inertia of the cylinder

$$= \frac{1}{2} Mr^2$$

$$= \frac{1}{2} \times 2 \times {\cdot}25^2$$

$$= 0{\cdot}0625 \text{ kg m}^2$$

Now moment of the force *i.e.* torque

$$I\dot{\omega} = F.r$$

$$\dot{\omega} = \frac{F \times r}{I}$$

$$= \frac{30 \times {\cdot}25}{{\cdot}0625}$$

$\therefore$ Angular acceleration,

$$\dot{\omega} = 120 \text{ rad s}^{-1}$$

Linear acceleration, $a = \dot{\omega} r$

$$= 120 \times {\cdot}25$$

$$= 30 \text{ ms}^{-2}$$

Example 14. *On the applications of constant torque, a wheel is turned from rest through an angle of 200 radian 8 S. (a) What is its angular acceleration and (b) Angular velocity of the wheel after 12 S from start.*

Solution. By $\theta = \omega_0 t + \frac{1}{2} \dot{\omega} t^2$

$$200=0\times 8+\frac{1}{2}\dot{\omega}\times 8^2$$

$$\omega=\frac{200}{32}$$

$$=6{\cdot}25 \text{ rad s}^{-2}$$

By
$$\omega=\omega_0+\dot{\omega}t$$

$$=0+6{\cdot}25\times 12$$

$$=75{\cdot}00 \text{ rad s}^{-1}$$

Example 15. *A cylinder of mass 4 kg and radius 20 cm and free to rotate about its axis, receives an angular impulse of 4 kg m^2s^{-1} initially followed by a similar impulse after every 5 s. What is the angular speed of the cylinder 44 s after the initial impulse ? The cylinder is at rest initially.*

Solution. Moment of inertia of the cylinder,

$$I=\frac{1}{2}Mr^2$$

$$=\frac{1}{2}\times 4\times (2)^2$$

$$={\cdot}08 \text{ kg m}^2$$

Angular Impulse,

$$J=I\,\omega_0$$

$$\therefore \quad \omega_0=\frac{J}{I}$$

$$=\frac{4}{{\cdot}08}$$

$$=50 \text{ rad s}^{-1}$$

This angular velocity 50 rad s^{-1} is obtained by the cylinder in 5 S from the rest.

$$\therefore \quad \omega=\frac{50}{5}=5 \text{ rad s}^{-3}$$

Since one impulse each is received after every 5 S, out of 44 S, the velocity will increase only during 40 S. (During this time, 8 impulses will be received and 9th impulse will be received only after 45 S).

By
$$\omega=\omega_0+\omega t$$

$$=50+5\times 40$$

$$=250 \text{ rad s}^{-2}$$

Example 16. *A solid cylinder rolls down an inclined plane. Its mass is 2 kg and radius 0·1 m. If the height of the inclined plane in 3 m, what is the rotation K.E. when it reaches the foot of the plane ? Assume that the surfaces are smooth.*

Solution. When the cylinder reaches from top A of the inclined plane to its foot B, the potential energy of the cylinder converts into the rotational K.E. and linear K.E. So by principle of conservation of energy,

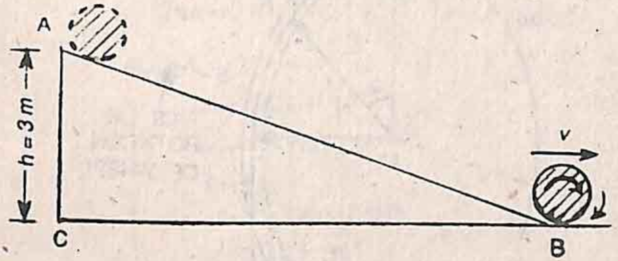

Fig. 5·3.

$$mgh=\tfrac{1}{2}\,I\omega^2+\tfrac{1}{2}\,mv^2$$

But $\quad I=\tfrac{1}{2}\,mr^2$ for the cylinder

and $\quad \omega=\dfrac{v}{r}$

$$\therefore \quad mgh=\tfrac{1}{2}(\tfrac{1}{2}mr^2)\left(\frac{v}{r}\right)^2+\tfrac{1}{2}mv^2$$

$$\therefore \quad v^2=\frac{4}{3}\,gh$$

Now rotational K.E. of the cylinder,

$$\begin{aligned}\text{R.K.E.}&=\tfrac{1}{2}\,I\omega^2\\&=\frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2\\&=\tfrac{1}{4}\,mv^2\\&=\tfrac{1}{4}m\,(\tfrac{4}{3}\,gh)\\&=\tfrac{1}{3}mgh\\&=\tfrac{1}{3}\times2\times9{\cdot}8\times3\\&=19{\cdot}6\text{ J}\end{aligned}$$

Example 17. *A cord is wound around the circumference of a wheel of diameter 0·4 m. The axis of the wheel is horizontal. 1·5 kg mass is attached at the end of the chord and it is allowed to fall from rest. If the weight falls 2·4 m 's 5 s. What is the angular acceleration of the wheel? Also find out the moment of inertia of the wheel.*

Solution. When the weight falls from B to B′ by 2·4 m, a point 'A' on the string moves from A to A′ by 2·4 m in 5 s.

$\therefore$ Arc AA′ of the wheel, $l=2{\cdot}4$ m

radius of the wheel, $r=0{\cdot}4$ m $\qquad$ (OA=OA′=r)

$$\therefore \quad \theta=\frac{l}{r}=\frac{2{\cdot}4}{0{\cdot}4}=6\text{ rad.}$$

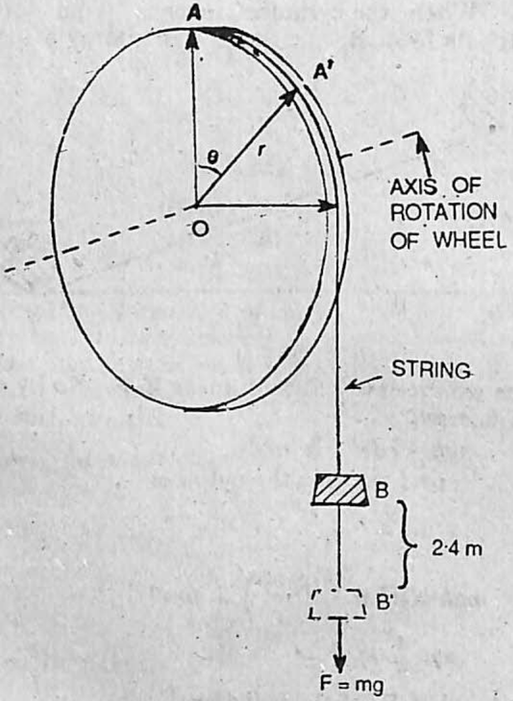

Fig. 5.4.

Now by

$$\theta=\omega_0 t+\tfrac{1}{2}\dot{\omega}t^2$$

$$6=0\times 5+{}^{1}\dot{\omega}5^2$$

$$\dot{\omega}=\frac{6\times 2}{25}$$

$$=0{\cdot}48 \text{ rad s}^{-2}$$

Now moment of the force *i.e.* torque,

$$I\times\dot{\omega}=F\times r$$

$$I=\frac{mg\times r}{\dot{\omega}}$$

$$=\frac{1{\cdot}5\times 9{\cdot}8\times 0{\cdot}4}{0{\cdot}48}$$

$$=12{\cdot}25 \text{ kgm}^2$$

Example 18. *A grindstone is in the form of a flat circular disc of diameter 0·4 m and mass 14 kg. What constant torque should be applied so that it attains an angular velocity of 420 rpm in 10 s. What is the power at the end of this time ?*

Solution.

$$\omega=2\pi n=2\times\frac{22}{7}\times\left(\frac{420}{60}\right)=44 \text{ rad s}^{-1}$$

By $\omega=\omega_0+\dot{\omega}t$

$$44=0+\dot{\omega}\times 10$$

$\therefore$ $\dot{\omega}=4{\cdot}4 \text{ rad s}^{-2}$

Now

$$\begin{aligned}I&=\tfrac{1}{2}mr^2\\&=\tfrac{1}{2}\times 14\times(0{\cdot}2)^2\\&=0{\cdot}28 \text{ kgm}^2\end{aligned}$$

Here power,

$$\begin{aligned}P&=\text{Torque}\times\text{Angular Velocity}\\&=I\dot{\omega}\times\omega\\&=0{\cdot}28\times 4{\cdot}4\times 44\\&=54{\cdot}208 \text{ watt.}\end{aligned}$$

Example 19. *A solid disc and a ring both of radius 14 cm are placed on a horizontal table simultaneously with initial angular speed $15\pi\ s^{-1}$. If coefficient of kinetic friction is 0·25 which of the two will start to roll earlier ?*

Solution. Frictional force accelerates the body from its initial linear zero velocity

$\therefore$ $V_i=0$, and $a=\dfrac{F}{m}=\dfrac{\mu_k mg}{m}=\mu_k g$

$\therefore$ The linear velocity of the body after the time '*t*' will be

$$v_f=v_i+at$$

$$v_f=0+\mu_k gt$$

$\therefore$ $v_f=\mu_k gt$

Rolling will begin when angular velocity becomes

$$\omega=\frac{v_f}{r}$$

$\therefore$

$$\omega=\frac{\mu_k gt}{r} \qquad \text{...(1)}$$

Now Torque, $F\times r=I\dot{\omega}$

$\therefore$

$$\dot{\omega}=\frac{Fr}{I}=\frac{-\mu_k mgr}{I} \qquad \text{...(2)}$$

(Since frictional torque causes retardation in initial angular velocity)

$\therefore$ By $\omega=\omega_0+\omega t$

$$\frac{\mu_k gt}{r}=\omega_0-\frac{\mu_k mgr}{t}t \qquad \text{[By Eqn. (1) and (2)]}$$

Solving this eqn., we get

$$t=\frac{\omega_0}{\mu_k g\left(\frac{1}{r}+\frac{mr}{I}\right)}$$

Now for a ring, $I=mr^2$

∴ Eqn. (3) becomes,

$$t=\tfrac{1}{2}\,\frac{\omega_0 r}{\mu_k g}=\frac{15\pi\times{\cdot}14}{2\times{\cdot}25\times9{\cdot}8}=1{\cdot}35\text{ s}$$

For a disc, $I=\tfrac{1}{2}mr^2$

$$\therefore\quad t=\frac{1}{3}\,\frac{\omega_0 r}{\mu_k g}\qquad\text{...[Eqn. (3)]}$$

$$=\frac{1}{3}\,\frac{15\pi\times0{\cdot}14}{0{\cdot}25\times9{\cdot}8}=0{\cdot}95\text{ s}$$

Thus the disc begins to roll earlier than the ring.

Example 20. *A solid sphere of mass 6 kg and radius 5 cm rotating with initial angular speed of 120 rad s^{-1} is placed lightly on a horizontal table, without any translational push. If coefficient of kinetic friction is 0·15, calculate*

(*a*) *After how long does the sphere start rolling ?*

(*b*) *What is the initial and final value of* (*i*) *translational energy* (*ii*) *rotational energy.*

(*c*) *What is the loss of total energy ? Where does this energy disappear ?*

Solution. (*a*) Proceed as in last example to get Eqn. (3)

$$t=\frac{\omega}{\mu_k g\left(\frac{I}{r}+\frac{mr}{I}\right)}$$

For sphere $\quad r=\frac{2}{5}mr^2$

$$\therefore\quad t=\frac{2}{7}\,\frac{\omega_0 r}{\mu_k g}$$

$$=\frac{2}{7}\times\frac{120\times{\cdot}05}{{\cdot}15\times9{\cdot}8}$$

$$=1{\cdot}166\text{ s}$$

(*b*) (*i*) Initial translational energy,

$$E_0=\tfrac{1}{2}mv_i^2=0$$

Final translational energy

$$E=\tfrac{1}{2}mv_f^2$$

$$=\tfrac{1}{2}\times m\times(\mu_k g t)^2$$

$$=\tfrac{1}{2}\times6\times({\cdot}15\times9{\cdot}8\times1{\cdot}166)^2$$

$$=8{\cdot}8\text{ J}$$

(*ii*) Initial rotational energy

$$\begin{aligned} E_0' &= \frac{1}{2} I\omega_0^2 \\ &= \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega_0^2 \\ &= \frac{1}{5} \times 6 \times ({\cdot}05)^2\ 120^2 \\ &= 43{\cdot}2 \text{ J} \end{aligned}$$

Final rotational energy,

$$\begin{aligned} E' &= \frac{1}{2} I\omega^2 \\ &= \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{\mu_k g t}{r}\right)^2 \\ &= \frac{1}{5} m(\mu_k g t)^2 \\ &= \frac{1}{2} \times 6 \times ({\cdot}15 \times 9{\cdot}8 \times 1{\cdot}166)^2 \\ &= 3{\cdot}52 \text{ J} \end{aligned}$$

(*c*) Initial total energy

$$\begin{aligned} &= E_0 + E_0' \\ &= 0 + 43{\cdot}2 \\ &= 43{\cdot}2 \text{ J} \end{aligned}$$

Final total energy

$$\begin{aligned} &= E + E' \\ &= 8{\cdot}8 + 3{\cdot}52 \\ &= 12{\cdot}32 \text{ J} \end{aligned}$$

$\therefore$ Loss in Energy

$$\begin{aligned} &= (43{\cdot}2 - 12{\cdot}32) \\ &= 30{\cdot}88 \text{ J} \end{aligned}$$

This loss of energy is dissipated against friction before rolling motion begins.

Example 21. *A ring, a cylinder and a sphere all of the same radius and mass roll down inclined plane from the same height. Which of the three reaches the bottom (a) latest (b) earliest.*

Solution. By principle of conservation of energy (see Fig. 5·3 on page 111).

$$mgh = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$$

or

$$mgh = \frac{1}{2}\left(I\frac{v^2}{r^2} + mv^2 \right) \qquad \left[\because\ \omega = \frac{v}{r}\right]$$

or

$$mgh = \frac{1}{2} v^2 \left(\frac{I}{r^2} + m\right) \qquad \ldots($$

If 'a' is the acceleration of the body moving down the inclined plane by

$$2aS = V_f^2 - V_i^2$$

$$a = \frac{V_f^2 - 0^2}{2AB}$$

$$\therefore \quad v_f^2 = 2a.AB = 2a.\frac{h}{\sin\theta}$$

∴ Eqn. (1) becomes

$$mgh = \frac{1}{2} \times \frac{2ah}{\sin\theta}\left(\frac{1+mr^2}{r^2}\right)$$

$$\therefore \quad a = g\sin\theta\left(\frac{mr^2}{1+mr^2}\right) \qquad \text{...(2)}$$

Now for a ring, $I = mr^2$

∴ By eqn. (2),

$\because \quad a = g\sin\theta\,(\tfrac{1}{2}) = 0{\cdot}5\,g\sin\theta$

For a cylinder, $I = \frac{1}{2}mr^2$

$$a = g\sin\theta\left(\frac{2}{3}\right) = 0{\cdot}67\,g\sin\theta$$

For a sphere, $I = \frac{2}{5}mr^2$

$$\therefore \quad a = g\sin\theta\left(\frac{5}{7}\right) = 0{\cdot}71\,g\sin\theta$$

(*a*) Since the value of acceleration down the inclined plane is minimum for ring, it will reach the bottom latest.

(*b*) Since the value of acceleration down the inclined plane is maximum for the sphere, it will reach the bottom earliest.

Example 22. *A cyclist is riding with a speed of 36 km/h. As she approaches a circular path of radius 50 m, she applies brakes and reduces her speed at a constant rate of 0·5 ms⁻¹. What is the magnitude and direction of the net acceleration of the cyclist on the circular path?*

Solution. Linear acceleration of cyclist $a_1 = 0{\cdot}5\ \text{ms}^{-2}$

$$v = 36\ \text{km/h} = 36 \times \frac{5}{18} = 10\ \text{ms}^{-1}$$

and $r = 50$ m

$$\therefore \text{ centripetal acceleration, } a_2 = \frac{v^2}{r}$$

$$a_2 = \frac{10 \times 10}{50} = 0{\cdot}2\ \text{ms}^{-2}$$

The direction of a_1 is along the tangent of the circular pa and that of a_2 is along the radius of the circular path. So a_1 and th are at right angles to each other and hence net acceleration will be a

$$a=\sqrt{a_1^2+a_2^2}$$
$$=\sqrt{0{\cdot}5^2+0{\cdot}2^2}$$
$$=0\ 539\ \text{ms}^{-2}$$

If direction of 'a' with a_1 is θ, then

$$\tan\theta=\frac{a_2}{a_1}=\frac{2}{5}=0{\cdot}4$$

$\therefore$ $\theta=21{\cdot}8°$ with the direction of velocity 'a_1'.

EXERCISE 5

1. A body is rotating with an angular velocity of 6 rad. s^{-1}. After 8 seconds, the angular velocity of the body becomes 70 rad s^{-1}. Determine the angular acceleration of the body.
2. What is the angular velocity in rad s^{-1} of a flywheel making 150 rpm ?
3. A wheel rotating about a fixed axis at 30 rpm is uniformly accelerated for 50 s, during which it makes 40 revolutions. Find (*a*) angular velocity after this interval of time (*b*) time required for the speed to reach 80 revolutions per minute.
4. A fly wheel starts rotating from rest and is given an acceleration of 1·5 rad s^{-2}. (*a*) Find the angular velocity and speed in rpm after 60 s. (*b*) If the flywheel is brought to rest with a uniform angular retardation of 0·5 rad^{-2}, determine the time taken by the flywheel to come to rest.
5. The angle of rotation of a body is given by the equation

   $$\theta=t^3-4t^2+10t+5$$

   where θ is in radian and t is in second. Determine angular velocity and angular acceleration of the body at $t=0$ and $t=5$ s.
6. The spin drier of a washing machine revolving at 900 rpm slows down uniformly to 300 rps by making 50 revolutions. Find the angular accelaration and time required in the process.
7. How many turns does a flywheel make in 5 s from rest if it revolves with an angular acceleration of 20 rad s^{-2}. How many turns does it make during fifth second ?
8. A flywheel 50 cm in radius is rotating at 120 rpm. Calculate (*a*) the angular velocity in rad s^{-1} (*b*) the linear velocity of a point on its rim.
9. The angle θ covered by a body is given by the equation

   $$\theta=8t+5t^2+t^3$$

   Calculate the instantaneous angular velocity and angular acceleration at $t=3$ s.
10. An automobile of mass 3000 kg has to make a turn on a circular path of radius 50 m. What centripetal acceleration and centripetal force act on the automobile if it is moving at a speed of 36 km h^{-1} ?

11. A boy swings a pile of water in a vertical circle of radius 1 m. What is minimum number of revolutions per second if water does not spill out ?

12. A bend in a level road has a radius of 30 m. Find the maximum speed which a car turning this bend may have without skidding. The coefficient of kinetic friction between the car and road is 0·75.

13. The radius of a spherical cage meant for motor-cycle feat in a circus is 8 m. Calculate the minimum speed with which the motor-cyclist has to perform the feat without falling.

14. What is the angular momentum of a body of mass 0·5 kg moving in a horizontal circle of radius 2 m and making 60 rpm ?

15. A thin metal of hoop of radius 0·25 m and mass 3 kg starts from rest and rolls down an inclined plane. If the linear velocity on reaching the foot of the plane is 4 ms^{-1}. What is the rotational K.E. at that instant ?

16. A solid cylinder rolls down on inclined plane. Its mass is 1·5 kg and radius 0·2 m. If the height of the inclined plane is 5 m, (*a*) What is its rotational K.E. when it reaches just at the foot of the plane ? (*b*) What will be its value if cylinder is replaced by (*i*) a ring, (*ii*) a sphere of same mass and radius ?

17. The moon is $3{\cdot}84 \times 10^8$ m from the earth. It circles the earth once in every 27·3 days, calculate its centripetal acceleration.

18. A body of mass 0·5 kg is whirled in a circle with velocity of 2 ms^{-1} by using a 0·5 m long string which can withstand a tension of 5 N. Neglecting the force on the body, predict whether or not the string will break ? Give reasons for your predictions. [*D.B.S.S.E., 1979*]

19. A body of mass 2·5 kg is tied to a string of length 1·5 m and revolved in a horizontal circle. Calculate the tension in the string if the body makes 210 rpm.

20. A string 1 m long is used to whirl a body of mass 0·5 kg in a vertical circle. What is the tension in the string when the body is at the top of the circle moving with 5 ms^{-1} ?

21. A string 1·5 m long is used to whirl a 1·5 kg body in a vertical circle so that its velocity at the top is 6 ms^{-1}, what is the tension in the string when it is horizontal ?

22. A body weighing 0·4 kg is whirled in a vertical circle making 2 revolutions per second. If the radius of the circle is 1·2 m, find the tension in the string when the body is (*a*) at the top of the circle (*b*) at the bottom of the circle.

23. A body of mass 0·5 kg is whirled in a vertical circle by a string 1 m long calculate the minimum speed it must have at the bottom of the circle so that the string may not slacken when the body reaches at the top.

24. An aeroplane is travelling at a speed of 500 km h^{-1} banks at an angle of 30° as it takes a turn. What is the radius of the curve ?

25. At what angle must a railway track with a bend of radius 200 m be banked for the safe running of trains at a speed of 36 km/h ? (Take $g=10$ ms^{-2}).

26. A train has to negotiate a curve of radius 400 m. By how much should the outer rail be raised with respect to the inner rail for speed of 48 km h^{-1} ? The distance between the rails is 1 m.

27. A cyclist takes a turn round a circular road of radius 5 m with a speed of 9 km h^{-1}. What is his inclination to the vertical ? (Take $g=10$ ms^{-2}).

28. A solid sphere rolls down an inclined plane. Its mass is 2 kg and radius is 0·1 m. If the height of the inclined plane is 3·5 m, what is its linear K.E. when it reaches the foot of the plane ?

29. On the applications of a constant torque, a wheel is turned from rest through an angle of 200 rad. in 8 s. What is (*a*) its angular acceleration and (*b*) the angular velocity of the wheel after 16 s from the start ?

30. Starting from rest the flywheel of a motor attains an angular velocity of 60 rad s^{-1} in 5 s. What is (*a*) the angular acceleration and (*b*) the angular displacement of the start ?

31. A uniform circular disc of mass 1 kg and radius 0·1 m is acted upon by a torque comprising two equal and opposite forces applied tagentially to the rim from the extremities of a diameter. The disc is free to rotate about an axis passing through its centre and perpendicular to the plane of the disc. What should be the magnitude of the each force to produce an angular acceleration of 20 rad. s^{-2} ? What is the K.E. transferred to disc in 5 s ?

32. A cord is wound around the circumference of a wheel of diameter 0·3 m. The axis of the wheel is horizontal. A mass of 2·5 kg is attached at the end of the cord and it is allowed to fall from the rest. If the weight falls 1·5 m in 4 s, what is (*a*) the angular acceleration of the wheel and (*b*) the moment of inertia of the wheel ?

33. A flywheel of mass 25 kg has a radius of 0·2 m. It is making 240 rpm. What is the torque necessary to bring it to rest in 20 s ? If the torque is due to a force applied tagentially on the rim of the flywheel, what is the magnitude of the force ?

34. A grind stone is in the form of a flat circular disc of diameter 0·5 m and mass 16 kg. What constant torque should be applied so that it attains an angular velocity of 120 rpm in 8 s ? What is the power at the end of this time ?

35. What is moment of inertia of a sphere about an axis parallel to its diameter and at a distance R/2 from the diameter ? It is given that the moment of inertia about its any of the diameter is $\frac{2}{5}$ MR^2, where 'M' is the mass of the sphere and 'R' its radius.

36. What is the moment of inertia of a cylinder of radius r and mass M about an axis (*a*) passing through its central and normal to the cylinder (*b*) passing a point on its edge and normal to the cylinder ? The moment of inertia of the cylinder about its any of diameter is given as $\frac{1}{4}$ Mr^2.

37. The moment of inertia of a uniform rod of mass M and the length '*l*' about an axis through its centre and perpendicular to the length of the rod is $\frac{1}{12}$ Ml^2. Calculate the moment of inertia of the same rod about an axis through one end and perpendicular to the length of the rod.

38. A solid sphere of mass 32 kg rotates about an axis passing through its centre with angular speed of 50 rad s^{-1}. The radius of the sphere is 10 cm. What is the magnitude of rotational K.E. and angular momentum ?

39. One end of a string of length 2 m is tied to a stone of mass 0·5 kg and the outer end to a small pivot on the smooth vertical board. What is the minimum speed of the stone required at the lower most point so that the string does not slack at any point in its motion along the vertical circle ? (Take $g=10$ ms^{-2}).

40. An aircraft executes a horizontal loop at a speed of 900 km/h with its wings banked at 12°. What is the radius of the loop ?

41. An aeroplane moving with a speed of 630 km h^{-1} has to negotiate a turn of radius 15 km. At what angle the wings should be banked by pilot ?

42. A train rounds an unbanked circular bend of radius 60 m at a speed of 63 km/h. The mass of the train is 125·0 metric tonnes. What provides the centripetal force required for this purpose ? What is the angle of banking required to prevent wearing out of the rails ?

43. A long playing record revolves with a speed of 21 rpm and a radius of 14 cm. Two coins are placed at 5 cm and 10 cm away from the centre of the record. If the coefficient of friction between the coins and the record is 0·12, will both the coins revolve with the record ?

44. A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 6 rad s^{-1}. (*a*) How much is the angular speed of the child if he folds his hands back and thereby reducing his

moment of inertia to $\frac{3}{7}$ of its initial value. Assume that the turntable rotates without friction, (*b*) Calculate also initial and final rotational K.E. if moment of inertia of turntable is 5 kg m^2. How will you account in increase in K E. ?

45. A student of 50 kg stands in contact against the wall of a cylindrical drum of radius 1·5 m rotating about its vertical axis with 120 rpm. The coefficient of friction between the wall and the cloth of the student is 0·20. What is the minimum speed of the cylinder to enable the student to remain struck to the wall without falling when the floor is suddenly removed ?

46. A rope wound round a hollow cylinder of mass 4 kg and radius 32 cm is pulled by a force of 25 N. What is the linear acceleration of the rope ?

47. A solid sphere and a ring both of radius 7 cm are placed on a horizontal table simultaneously with initial angular speed 10 rad s^{-1}. If coefficient of kinetic friction is 0·18 which of the two will start to roll earlier ?

48. A ring of mass 2 kg and radius 25 cm rotating with initial angular speed of 100 rad s^{-1} is placed gently on a horizontal table without any translational push. If coefficient of kinetic friction is 0·12, calculate

    (*a*) After how long does tne ring starts rolling ?

    (*b*) What is the initial and final value of (*i*) translational energy (*ii*) rotational energy ?

    (*c*) What is the loss of total energy and where does this energy dissipate ?

49. If instead of ring in above question, we take a cylinder of mass 10 kg and radius 20 cm rotating with initial angular speed of 150 rad s^{-1}. Calculate (*a*), (*b*) and (*c*) in above Question No. 48 [Take μ_k=0·20].

50. An automobile moves on a road with a speed of 45 km h^{-1}. The mass and radius of the wheel (in the form of disc) is 20 kg and 0·4 m respectively. What is the average torque transmitted by its brakes to wheels if the vehicle is brought to rest in 25 s ?

51. An aircraft executes a horizontal loop of radius 2 km with a steady speed of 720 km/h. Compare its centripetal acceleration with the acceleration due to gravity.

52. A motor cyclist is driving with a speed of 54 km/h. As he approaches a circular path on the road of radius 90 m, he applies the brakes and reduces his speed at the constant rate of 2·5 m/s. What is the magnitude and direction of the net acceleration of the motor-cyclist on the circular path ?

OBJECTIVE TYPE QUESTIONS

53. A stone of mass 'm' is whirled in a verticle circle of radius 'r'. If the speed of the stone and the tension in the string at the lowest point be v_1 and T_1 respectively, what is the net force at this point in the vertical downward direction

(*a*) $(mg+T_1)$ (*b*) $(mg-T_1)$

(*b*) $\left(mg+T_1-\frac{mv_1^2}{r}\right)$ (*d*) $\left(mg-T_1-\frac{mv_1^2}{r}\right)$

54. Moment of inertia can be taken as a

(*a*) scaler quantity (*b*) vector quantity

(*c*) neither vector nor scaler

55. Moment of inertia of a body does not depend upon

(*a*) its mass (*b*) its radius

(*c*) its angular speed (*d*) position of axis of rotation.

56. The rotational analogue of mass in linear motion is

(*a*) moment of inertia (*b*) weight

(*c*) angular momentum (*d*) mass itself.

57. The rotational analogue of force in linear motion is

(*a*) force itself (*b*) weight

(*c*) angular momentum (*d*) torque.

58. If no torque acts on a body which is rotating, what physical quantity remains constant even if its angular speed changes

(*a*) linear momentum (*b*) angular momentum

(*c*) linear K.E. (*d*) angular K.E.

59. A body with the moment of inertia 5 kg m^2 and rotating with angular speed 4 rad s^{-1} will has same K.E. as a mass 20 kg moving with a speed of

(*a*) 4 ms^{-1} (*b*) -2 ms^{-1}

(*c*) 2 ms^{-1} (*d*) 1 ms^{-1}.

60. A boy comes running and sits on a rotating platform. What is conserved

(*a*) Linear K.E. (*b*) Rotational K.E.

(*c*) Linear momentum (*d*) Angular momentum.

61. A particle moving in a uniform circular motion has an acceleration which is

(*a*) constant in magnitude but directed toward the centre.

(*b*) constant in magnitude but away from the centre.

(*c*) varying in magnitude but towards the centre.

(*d*) varying in magnitude but away from the centre.

[*B.H.U. 1983*]

62. A 1 kg stone at end of 1 m long string is whirled in a vertical circle with a constant speed of 4 ms^{-1}. The tension in the string is 6 N when the stone is
(*a*) at the top of the circle (*b*) at the bottom of the circle
(*c*) half way down (*d*) none of the above.
[Take $g=10\ \text{ms}^{-1}$] [*B.H.U. 1983*]

63. Which of the following of same mass and radius has highest moment of inertia
(*a*) disc (*b*) ring
(*c*) rods forming a square $(l=2r)$ (*d*) None.
[*B.H.U. 1983*]

64. Which of the following with the same mass has the highest moment of inertia about an axis passing through the centre of
(*a*) a circular plate of radius '2 *r*'
(*b*) a circular ring of radius '*r*'
(*c*) a sphere of radius '2·5 *r*'
(*d*) a thin rod of length '2 *r*'.

65. A stone of mass *m* is tied to a string of length L is rotated in a circle with a constant speed V. If the string is released the stone flies
(*a*) radially outward (*b*) radially inward
(*c*) tangentially outward (*d*) with an acceleration $\frac{mV^2}{L}$

66. The kinetic energy of rotation is given by
(*a*) $\frac{1}{2} mv^2$ (*b*) $\frac{1}{2} Iw^2$
(*c*) $\frac{1}{2} m\omega^2$ (*d*) $\frac{1}{2} Iv^2$. [*C.P.M.T. 1988*]

□ □

6

Gravitation

IMPORTANT FORMULAE

1. Newton's universal gravitational law :

$$F=G\,\frac{m_1 m_2}{x^2}\ ;$$

where F = Force of attraction

m_1 and m_2 = mass of bodies

x = Distance between their centres of mass.

G = Gravitational const.

$=6{\cdot}67\times10^{-11}\ \text{Nm}^2/\text{kg}^2$

2. Acceleration due to gravity

(*a*) At the surface of the earth,

$$g=\frac{\text{G.M.}}{R^2}\ ;$$

M = Mass of the earth

R = Radius of the earth

(*b*) Inside the surface of the earth.

$$g=\frac{4}{3}\,\pi\,G\rho R$$

where ρ = mean density of the earth.

3. Gravitational field,

$$f=\frac{\text{G.M.}}{x^2}$$

4. Gravitational P.E.,

$$\phi=-\frac{GMm}{R}$$

5. Gravitational Potential,

$$v=-\frac{GM}{R}$$

6. Orbital velocity,

$$V=\sqrt{\frac{GM}{(R+h)}}=\sqrt{g\,(R+h)}$$

7. Escape velocity,

$$V=\sqrt{\frac{2\,GM}{R}}=\sqrt{2gR}$$

8. Kepler's law :

$$\frac{T^2}{a^3}=a\text{ constant}$$

T = period of revolution of planet
a = mean distance of planet from the sun.

9. Mass of a heavier body (around which a lighter body is revolving)

$$M=4\pi^2/G\times a^3/T^2$$

10. Cavendish's experiment,

$$G=\frac{\alpha\,\theta\,r^2}{Mml},$$

where

α = torsion const. of fibre
θ = angle of twist of fibre
r = distance between each pair of heavy and light spheres.
M = mass of heavy sphere
m = mass of light sphere
l = length of the rod.

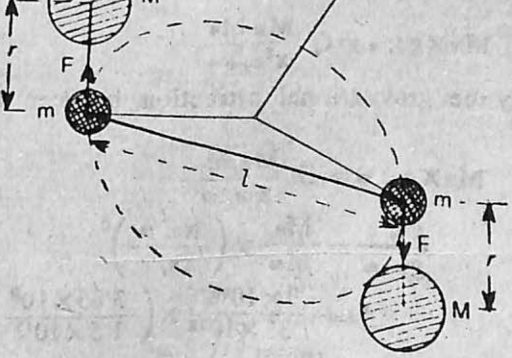

Fig. 6·1.

SOLVED EXAMPLES

Example 1. *Calculate the gravitational force of attraction between the Sun and the Jupiter. The mass of the Sun and the Jupiter is* 2×10^{30} *kg and* 2×10^{27} *kg respectively and the mean distance between is* $7{\cdot}8\times 10^{11}$ *m.*

Solution. By

$$F=G\frac{m_1 m_2}{x^2}$$

$$=6{\cdot}67\times 10^{-11}\frac{2\times 10^{30}\times 2\times 10^{27}}{(7{\cdot}8\times 10^{11})^2}$$

$$=4{\cdot}385\times 10^{23}\text{ N}.$$

Example 2. *Calculate the mass of the earth from the following data :*

$$g=9{\cdot}8\ ms^{-2},$$
$$R=6{\cdot}38\times 10^{6}\ m$$
$$G=6{\cdot}67\times 10^{-11}\ Nm^2/kg^2$$

[*A.I.S.S.E. 1982*]

Solution. By

$$g=\frac{GM}{R^2}$$

$$M=\frac{gR^2}{G}$$

$$=\frac{9{\cdot}8\times(6{\cdot}38\times 10^{6})^2}{6{\cdot}67\times 10^{-11}}$$

$$=5{\cdot}98\times 10^{24}\text{ kg}.$$

Example 3. *Compare the gravitational acceleration of the earth due to attraction of the sun with that due to attraction of the moon. Given that mass of sun*$=2\times 10^{30}$ *kg, mass of moon*$=7{\cdot}35\times 10^{22}$ *kg, mean distance of the earth from the sun*$=1{\cdot}5\times 10^{11}$ *m and the mean distance of the moon from the earth*$=3{\cdot}85\times 10^{8}$ *m.*

Solution. The gravitational attraction between the sun and the earth,

$$M_E\times g_{E,\,S}=G\,\frac{M_E\,M_S}{x^2_{E,\,S}}$$

Similarly the gravitational attraction between the earth and the moon,

$$M_E\times g_{E,\,m}=G\,\frac{M_E\,M_m}{x^2_{E,\,m}}$$

$$\therefore\qquad \frac{g_{E,\,S}}{g_{E,\,m}}=\frac{M_S}{M_m}\times\left(\frac{x_{E,\,m}}{x_{E,\,S}}\right)^2$$

$$=\frac{2\times 10^{30}}{7{\cdot}35\times 10^{22}}\times\left(\frac{3{\cdot}85\times 10^{8}}{1{\cdot}5\times 10^{11}}\right)^2$$

$$=179{\cdot}2.$$

Example 4. *Assuming that a satellite is to orbit 300 km above the surface of the earth having radius 6400 km and mass 6×10^{24} kg, calculate the necessary speed with which is should be put in orbit. Also determine the period of revolution of the satellite.*

Solution. By

$$V=\sqrt{\frac{GM}{(R+h)}}$$

$$=\sqrt{\frac{6{\cdot}67\times10^{-11}\times6\times10^{24}}{(6400+300)\ 10^{3}}}$$

$$\therefore \qquad V=7{\cdot}729\times10^{3}\ \text{ms}^{-2}$$

The circumference of the orbit,

$$S=2\pi\ (R+h)$$

$$=2\ \times\frac{22}{7}=(6400+300)\times10^{3}$$

$$=421{\cdot}143\times10^{5}\ \text{m}$$

∴ Time of revolution of the satellite around the earth,

$$t=\frac{S}{V}$$

$$=\frac{421{\cdot}143\times10^{5}}{7{\cdot}729\times10^{3}}$$

$$=54{\cdot}49\times10^{2}\ \text{s}$$

$$=90\ \text{min}\ 49\ \text{s}.$$

Example 5. *A satellite is raised to a height of 400 km. (a) What is the acceleration due to gravity at this point ? (b) With what velocity should this body be projected so that it describes a circle about the earth.*

(Radius of earth=6600 km, $g=9{\cdot}8$ ms^{-2} at the surface of the earth) *[A.I.S.S.E. 1986]*

Solution. $$g=\frac{GM}{R^{2}}$$

and $$g'=\frac{GM}{(R+h)^{2}}$$

$$\therefore \qquad g'=g\ \frac{R^{2}}{(R+h)^{2}}$$

$$=9{\cdot}8\left(\frac{6600}{(6600+400)}\right)^{2}$$

$$=8{\cdot}712\ \text{ms}^{-2}$$

Orbital velocity,

$$\begin{aligned} V &= \sqrt{g'(R+h)} \\ &= \sqrt{8{\cdot}712\,(6600+400)\times 10^3} \\ &= 7{\cdot}8\times 10^3\ \text{ms}^{-1} \\ &= 7{\cdot}8\ \text{km s}^{-1}. \end{aligned}$$

Example 6. *Prove that orbital velocity of a satellite revolving very near to the earth's surface iv $V=\sqrt{gR}$.* [*A.I.S.S.E. 1988*]

Solution. A satellite will revolve in a orbit, if necessary centripetal force to revolve it in circular orbit is provided by gravitation force of attraction between the satellite and the earth.

$$\therefore \qquad \frac{mV^2}{R} = G\frac{Mm}{R^2} \qquad ...(1)$$

But weight of the satellite,

$$mg = G\frac{Mm}{R^2} \qquad ...(2)$$

Comparing eqn. (1) and (2), we have

$$\frac{mV^2}{R} = mg$$

$$\therefore \qquad V^2 = gR$$

or

$$\boxed{V = \sqrt{gR}}$$

Example 7. *A projectile is projected vertically upward from the earth's surface with a speed of 1 km/s. How high will it rise above the earth? ($R=6400$ km and $g=9{\cdot}8\ ms^{-1}$).*

Solution. Let the projectile be fired from the surface of earth with a velocity 'v' and rises in space to the height 'x'. Then by principle of conservation of energy,

$$\tfrac{1}{2}mv^2 + U_0 = 0 + U$$

[$\because$ Final velocity of projectile is zero]

$$\text{or} \qquad \tfrac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = \left(-\frac{GMm}{x}\right)$$

$$\therefore \qquad \tfrac{1}{2}mv^2 = \frac{GMm}{R}\left(1-\frac{R}{x}\right)$$

$$\text{or} \qquad \tfrac{1}{2}v^2 = \frac{GM}{R}\left(1-\frac{R}{x}\right) \qquad ...(1)$$

$$\text{But} \qquad g = \frac{GM}{R^2} \qquad ...(2)$$

$\therefore$ Dividing 1st eqn. by 2nd, we have

$$\frac{v^2}{2g} = R\left(1-\frac{R}{x}\right)$$

$$\therefore \quad \left(1-\frac{R}{x}\right)=\frac{v^2}{2\,gR}$$

$$\therefore \quad \frac{R}{x}=\left(1-\frac{v^2}{2\,gR}\right)$$

$$\therefore \quad \frac{R}{x}=\left[1-\frac{(10^3)^2}{2\times 9{\cdot}8\times 6{\cdot}4\times 10^6}\right]$$

$$\frac{R}{x}=0{\cdot}992$$

$$\therefore \quad x=\frac{R}{0{\cdot}992}=\frac{6400}{0{\cdot}992}=6451{\cdot}6 \text{ km.}$$

Example 8. *Find the escape velocity on a planet whose diameter is 1·5 times earth's diameter and mass is also 1·5 times the mass of the earth. If the maximum temperature on the planet is 800 K, deduce whether oxygen gas molecules would be available on the planet.* (*Given mass of oxygen molecule*$=5{\cdot}3\times 10^{-26}$ *kg.*)

(*Roorkee Ent. Exam. 1984*)

Solution. Escape velocity on the planet,

$$V=\sqrt{\frac{2\,GM}{R}}$$

$$=\sqrt{\frac{2G\,(1{\cdot}5\,M_e)}{(1{\cdot}5\,R_e)}}$$

$$=\sqrt{\frac{2\,GM_e}{R_e}}$$

$=$Escape velocity on earth

$=11{\cdot}16$ km s^{-1}

The velocity of oxygen gas molecules at the planet,

$$C_{rms}=\sqrt{\frac{3\,RT}{N\,m}}$$

$$=\sqrt{\frac{3\times 8{\cdot}3\times 800}{6{\cdot}023\times 10^{26}\times 5{\cdot}3\times 10^{-26}}}$$

$$=24{\cdot}98 \text{ ms}^{-1}=0{\cdot}25 \text{ km s}^{-1}$$

The oxygen gas molecules will be available on the planet as the velocity (0·25 km s^{-1}) of gas molecules is less than the value of escape velocity (11·16 km s^{-1}) on the planet.

Example 9. *Compare the period of rotation of planet Mars about the Sun with that of the earth around it. The mean distance of the Mars from the Sun is 1·52 A.U.*

Solution. We know

$$\frac{a^3}{T^2} \text{ is same for all planets.}$$

$$\frac{a_E^3}{T_E^2}=\frac{a_m^3}{T_m^2}$$

$$\therefore \quad \left(\frac{T_m}{T_E}\right)^2=\left(\frac{a_m}{a_E}\right)^3$$

$$\therefore \quad \frac{T_m}{T_E}=\left(\frac{a_m}{a_E}\right)^{3/2}=(1{\cdot}52)^{3/2}$$

$$=1{\cdot}874 : 1.$$

Example 10. *What is the gravitational potential and gravitational potential energy of a body of 0·2 kg at a height 1600 km above the surface of the earth.* ($G=6{\cdot}67\times10^{-11}$ *Nm²/kg², mass of the earth* $=6\times10^{24}$ *kg and radius of the earth=6400 km).*

Solution. Gravitational potential,

$$V=-\frac{GM}{(R+h)}$$

$$=-\frac{6{\cdot}67\times10^{-11}\times6\times10^{24}}{(6400+1600)\ 10^3}$$

$$=5\times10^7\ \text{J kg}^{-1}$$

∴ Gravitational potential energy,

$$=-\frac{GMm}{(R+h)}$$

$$=Vm$$

$$=5\times10^7\times{\cdot}2$$

$$=10^7\ \text{J}.$$

Example 11. *The escape velocity of a projectile on earth's surface is 11·2 km s⁻¹. A body projected from the earth's surface reaches far away from the earth i.e. at infinity with a velocity of 19·4 km s⁻¹. With what velocity the body was projected ?*

Solution. If 'v_0' is the velocity of projection of the body and 'v' its velocity at infinity, by principle of conservation of energy,

$$\tfrac{1}{2}\,mv_0^2+\left(-\frac{Gmm}{R}\right)=\tfrac{1}{2}\,mv^2+0$$

But $\dfrac{GMm}{R}=\tfrac{1}{2}\,m\,v_e^2$ where v_e is the escape velocity of the body.

$$\tfrac{1}{2}\,mv_0^2-\tfrac{1}{2}\,mv_e^2=\tfrac{1}{2}\,mv^2$$

$$\therefore \quad v_0=\sqrt{v^2+v_e^2}$$

$$=\sqrt{19{\cdot}4^2+11{\cdot}2^2}$$

$$=22{\cdot}4\ \text{km s}^{-1}.$$

Example 12. *If the earth completes its one revolution in 365·25 days revolving around the sun in a circular orbit of radius* $1{\cdot}5\times10^{11}$ *m, calculate the mass of the sun.* (*Given* $G=6{\cdot}67\times10^{-11}$ *Nm²/kg²*).

Solution. Mass of the sun,

$$M=\frac{4\pi^2}{G}\ \frac{a^3}{T^2}$$

$$=\frac{4\times(3{\cdot}14)^2}{6{\cdot}67\times10^{-11}}\ \frac{(1{\cdot}5\times10^{11})^3}{(365{\cdot}25\times24\times60\times60)^2}$$

$$=2\times10^{30}\ \text{kg.}$$

Example 13. *A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expanded to rocket the satellite out of the earth's gravitattonal field. (Given mass of the satellite 200 kg, mass of the earth*$=6{\cdot}0\times10^{24}$ *kg,* $G=6{\cdot}67\times10^{-11}$ Nm^2/kg^2 *and radius of the earth*$=6{\cdot}4\times10^6$ *m).*

Solution. The orbital velocity of a satellite is given by

$$v=\sqrt{\frac{GM}{(R+h)}}$$

∴ K.E. of the satellite, orbiting around the earth,

$$E_k=\tfrac{1}{2}\,mv^2$$

or

$$E_k=\tfrac{1}{2}m\,\frac{GM}{(R+h)} \qquad ...(1)$$

$$=\frac{\frac{1}{2}\times200\times6{\cdot}67\times10^{-11}\times6\times10^{24}}{(6{\cdot}4+{\cdot}4)\times10^6}$$

$$=5{\cdot}9\times10^9\ \text{J.}$$

Gravitational potential energy of the satellite at this height,

$$E_p=-\frac{GMm}{(R+h)}$$

$$=-2E_k \qquad \text{[Eqn. (1)]}$$

$$=-2\times5{\cdot}9\times10^9$$

$$=-11{\cdot}8\times10^9\ \text{J}$$

∴ Total energy of the satellite,

$$E=E_k+E_p$$

$$=(5{\cdot}9-11{\cdot}8)\times10^9\ \text{J}$$

$$=-5{\cdot}9\times10^9\ \text{J.}$$

So the energy expanded to rocket the satellite out of the earth's gravitational field$=5{\cdot}9\times10^9$ J.

Example 14. *A rocket is fired from the earth towards the sun. At what point on its path is the gravitational force on the rocket is zero? (Given mass of the sun*$=2\times10^{30}$ *kg, mass of the earth*$=6\times10^{24}$ *kg and orbital radius*$=1{\cdot}5\times10^{11}$ *m). Neglect the effect of the other planets etc.*

Solution. Let a point x metre away from the earth's surface the gravitational force on the rocket is zero. Then

Gravitational pull due to earth on the rocket

=Gravitational pull due to sun on the rocket

$$\therefore \quad \frac{GM_e m}{x^2}=\frac{GM_s m}{(R-x)^2} \text{ where R=orbital radius}$$

$$\therefore \quad \left(\frac{R-x}{x}\right)^2=\frac{M_s}{M_e}$$

$$=\frac{2\times 10^{30}}{6\times 10^{24}}$$

$$\therefore \quad \left(\frac{R}{x}-1\right)=\sqrt{10^6/3}$$

$$\therefore \quad \left(\frac{R}{x}-1\right)=577{\cdot}35$$

$$\therefore \quad \frac{R}{x}=578{\cdot}35$$

$$x=\frac{R}{578{\cdot}35}=\frac{1{\cdot}5\times 10^{11}}{578{\cdot}35}=2{\cdot}59\times 10^{8}\text{ m}$$

Example 15. *Three mass points each of mass m are placed at the vertices of an equilateral triangle of side 'l'. What is the gravitational field and potential due to three masses at the centroid of the triangle.*

Solution.

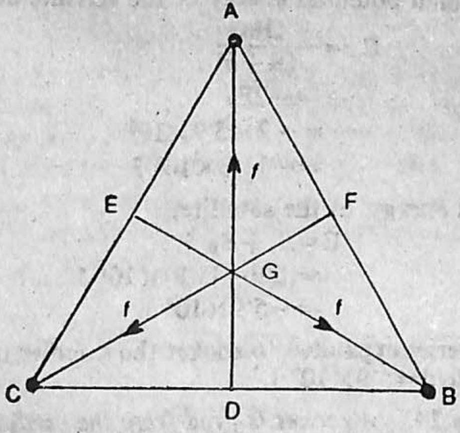

Fig. 6.2.

Since in the above figure,

$$AB=BC=CA=l$$

Now centroid 'G' divides each median AB, BC and CA in the ratio 2 : 1. So if FG=x, then AG=$2x$.

$\therefore$ In rt. $<\triangle AFG$,

$$AG^2 = AF^2 + FG^2$$

$$(2x)^2 = \left(\frac{l}{2}\right)^2 + x^2$$

$$4x^2 - x^2 = \frac{l^2}{4}$$

$$x = \frac{l}{2\sqrt{3}}$$

$$\therefore \quad AG = BG = CG = 2x$$

$$= \frac{2l}{2\sqrt{3}} = \frac{l}{\sqrt{3}}$$

$\therefore$ The magnitude of gravitational field at G due to each of mass 'm' at each of the three vertex A, B and C will be,

$$f = \frac{Gm}{(l/\sqrt{3})^2}$$

$$= \frac{3\,Gm}{l^2} \text{ each along GA, GB and GC}$$

Evidently the resultant of these three equal fields inclined at equal angles with each other will be zero.

$\therefore$ Resultant gravitational field at G$=0$.

Total gravitational potential at G due to the mass 'm' at each vertex at A, B and C will be

$$V = -\frac{Gm}{l/\sqrt{3}} - \frac{Gm}{l/\sqrt{3}} - \frac{Gm}{l/\sqrt{3}}.$$

$$= -3\sqrt{3}\,\frac{Gm}{l}$$

Example 16. *Two spheres one of mass 100 kg and the other of 50 kg are placed 1 m apart on a horizontal table. What is the gravitational field and potential at a point 0·4 m away from 50 kg on the line joining the centre of the two spheres.*

Solution. Given BP$=0{\cdot}4$ m

$$\therefore \quad AP = 1 - 0{\cdot}4$$

$$= 0{\cdot}6 \text{ m}$$

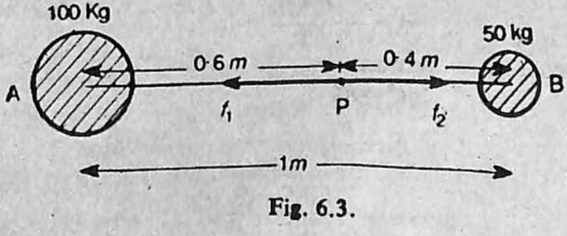

Fig. 6.3.

∴ Gravitational field at P due to sphere at A,

$$f_1=\frac{Gm}{x^2}$$
$$=\frac{6{\cdot}67\times10^{-11}\times100}{{\cdot}6^2}$$
$$=1{\cdot}85\times10^{-8}\ \text{ms}^{-2}\ \text{along}\ \overrightarrow{PA}$$

and field at P due to sphere at B,

$$f_2=\frac{6{\cdot}67\times10^{-11}\times50}{{\cdot}4^2}$$
$$=2{\cdot}08\times10^{-8}\ \text{ms}^{-2}\ \text{along}\ \overrightarrow{PB}$$

∴ Resultant gravitational field at P,

$$f=f_2-f_1$$
$$=(2{\cdot}08-1{\cdot}85)\ 10^{-8}\ \text{ms}^{-8}$$
$$=0{\cdot}23\times10^{-8}\ \text{ms}^{-1}\ \text{along}\ \overrightarrow{PB}$$

Gravitational potential at P due to masses at A and B will be

By $$V=-\frac{Gm}{x}$$
$$V=-\frac{6{\cdot}67\times10^{-11}\times100}{0{\cdot}6}-\frac{6{\cdot}67\times10^{-11}\times50}{0{\cdot}4}$$
$$=-11{\cdot}116\times10^{-9}-8{\cdot}337\times10^{-9}$$
$$=-19{\cdot}453\times10^{-9}\ \text{J kg}^{-1}.$$

Example 17. *Light from a massive star suffers gravitational red shift (a) Derive a formula for red shift. (b) Estimate the magnitude of the red shift for light of wavelength 5000 Å for a star of mass 10^{32} kg and radius 10^6 km. (Given $G=6{\cdot}67\times10^{-11}$ Nm²/kg² and $c=3\times10^8\ ms^{-1}$).*

Solution. (*a*) Due to gravitational attraction of the star, the wavelength of the light emitted by star changes toward the red end. If a photon of energy $E=h\nu$ and mass 'm' is emitted by star, its new energy 'E'' due to gravitational attraction will be given by

$$E'=E+\phi$$

where ϕ (gravitational potential energy of photon near star)

and $$\phi=-\frac{GMm}{R}$$

Now energy of photon,

$$mc^2=h\nu$$

$$\therefore\quad m=\frac{h\nu}{c^2}$$

$$\therefore\quad \phi=-\frac{GM}{R}\ \frac{h\nu}{c^2}$$

and $$E'=h\nu'$$

$\therefore$ $$h\nu'=h\nu-\frac{GM}{R}\,\frac{h\nu}{c^2}$$

$$h\nu'=h\nu\left(1-\frac{GM}{Rc^2}\right)$$

or $$\frac{c}{\lambda'}=\frac{c}{\lambda}\left(1-\frac{GM}{Rc^2}\right)$$

$\therefore$ $$\lambda'=\lambda\left(1-\frac{GM}{Rc^2}\right)^{-1}$$

or $$\lambda'=\lambda\left(1+\frac{GM}{Rc^2}\right)$$ (by Binomial theorem)

$\therefore$ Red shift $(\lambda'-\lambda)=\dfrac{\lambda GM}{Rc^2}$

(*b*) Red shift $$=\frac{\lambda GM}{Rc^2}$$

$$=\frac{(5000)\,(6{\cdot}67\times10^{-11})\,(10^{32})}{(10^{9})(3\times10^{8})^{2}}\ \text{Å}$$

$$=0{\cdot}37\ \text{Å}.$$

EXERCISE 6

1. Compute the gravitational force of attraction between two 10 kg balls 1 m apart. ($G=6{\cdot}67\times10^{-11}$ Nm^2/kg^2). [*A.I.S.S.E. 1983*]

2. The mass of the planet Mars is $6{\cdot}37\times10^{23}$ kg and that of the sun is 2×10^{30} kg. The mean distance of the sun from Mars is $2{\cdot}28\times10^{11}$ m.
   (*a*) Calculate the gravitational force which the sun exerts on Mars.
   (*b*) Assuming the Mars moves in a circular orbit around the sun, calculate the speed of Mars.

3. Calculate the value of radius of the earth from the following data:—(Given+Mass of the earth$=6\times10^{24}$ kg, $G=6{\cdot}67\times10^{-11}$ Nm^2/kg^2 and $g=9{\cdot}8$ ms^{-2}.

4. The radius of the moon is $1{\cdot}7\times10^{6}$ m and its mass is $7{\cdot}35\times10^{22}$ kg.
   (*a*) What is the acceleration due to gravity on the surface of the moon ?
   (*b*) Calculate the escape velocity from its surface.

5. The mass of Mars in $0{\cdot}1065$ times the mass of the earth. The diameter of Mars is $6{\cdot}880\times10^{6}$ m and that of earth is

12·800 $\times 10^6$ m. Calculate the value of acceleration due to gravity of Mars if g on earth is 9·8 ms^{-2}.

6. A satellite revolves around a planet mercury in an orbit just above the surface of the planet. Taking $G=6{\cdot}67\times10^{-11}$ Nm^2/kg^2 and mean density of planet$=5{\cdot}4\times10^3$ kg m^{-3}, find the period of the satellite. *[H.P.S.S.E. 1988]*

7. A body is raised to a height of 500 km. What is the acceleration due to gravity at this point? With what velocity should this body be projected so that it describes circle about the earth? (Radius of the earth=6500 km, $g=9{\cdot}8$ ms^{-2}). What will be the period of revolution of the body about the earth? *[I I.T. J.E.E. 1985]*

8. Calculate the escape velocity for an atmospheric particle 1000 km above the earth's surface. Radius of the earth$=6{\cdot}4\times10^6$ m, 'g' on the surface of the earth$=9{\cdot}8$ ms^{-2}. *[Indian School Certificate Exam. 1978]*

9. Calculate the mass of the moon, given that the gravitational constant $G=6{\cdot}67\times10^{-11}$ Nm^2/kg^2. the radius of the moon $R=1{\cdot}7\times10^6$ m and the acceleration due to gravity $g=1{\cdot}7$ ms^{-2}.

10. What is the minimum velocity a satellite should have in order to pursue a suitable orbit? (Assume $g=10$ ms^{-2} and radius of the earth$=6{\cdot}4\times10^3$ km). *[A.I.S.S.E. 1980]*

11. Calculate the value of escape velocity on the surface of the earth. Given $g=9{\cdot}8$ ms^{-2} and the radius of the earth=6400 km. *[D.S.S.E. 1982 ; A.I.S.S.E. 1984]*

12. If the escape velocity from the surface of earth is 11·3 km/s, find escape velocity from the surface of another heavenly body whose mass and diameter are twice to that of earth. *[D.S.S.E. 1981, 1985]*

13. Assuming that a satellite is to orbit 350 km above the surface of the earth :

    (*a*) Calculate the necessary speed with which it should be put in orbit. (Radius of earth=6400 km, mass of the earth $=6\times10^{24}$ kg.)

    (*b*) Also determine the period of revolution of the satellite.

14. The moon's radius is 27% of earth's radius and its mass is 1·2% of the earth's mass. What will be the weight of the body at the moon if its weight on the earth is 70 kg?

15. How many revolutions per day should the earth make in order that the weight of the body at the earth may become zero (g at earth$=9{\cdot}8$ ms^{-2})?

16. The mean distance of Mars from Sun is 1·524 times the distance of the earth from the sun. Calculate the period of revolution of Mars.

17. Calculate the value of g at on a hypothetical planet whose mass and radius both are one-third of the earth. (Value of g at earth $=9{\cdot}8\ \text{ms}^{-2}$).

18. A satellite is to be put into orbit around the moon just above its surface. Assuming that the moon's radius is half that of the earth and that the acceleration due to gravity is one-sixth of that exists at the earth, find the speed of the satellite.

19. Calculate the value of acceleration due to gravity and escape velocity on lunar surface. Given that mean density of the moon $=3340\ \text{kg m}^{-3}$; radius of moon $=0{\cdot}27\times6371\times10^{6}$ m and $G=6{\cdot}67\times10^{-11}\ \text{Nm}^2/\text{kg}^2$. *[M.A.M.C. 1982]*

20. Orbital period of moon is $27\frac{1}{3}$ days and its orbital radius is $3{\cdot}85\times10^{8}$ m, find the mass of the earth.

21. Taking moon's period of revolution about the earth as 30 days (and neglecting the effect of sun and the other planets on its motion), calculate its distance from the earth. (Given $G=6{\cdot}67\times10^{-11}\ \text{Nm}^2/\text{kg}^2$ and mass of the earth $=6\times10^{24}$ kg). *[Roorkee Engg. Ent, Exam. 1984]*

22. With how much velocity a body must be projected vertically upward from earth's surface to reach a height 8 times the radius of the earth? ($g=9{\cdot}8\ \text{ms}^{-2}$, $R=6{\cdot}4\times10^{6}$ m)

23. Calculate the period of revolution of Neptune around the sun given that the diameter of its orbit is 30 times the diameter of the earth's orbit around the sun, both the orbits being assumed to be circular.

24. Calculate the mass of the sun, given that the distance between sun and the earth is $1{\cdot}5\times10^{11}$m, $G=6{\cdot}67\times10^{-11}\ \text{Nm}^2/\text{kg}^2$ and one year $=365$ days.

25. A satellite goes around the earth in 90 minute in a circular orbit. Calculate the height of the satellite above the surface of the earth, taking the earth to be a sphere of radius 6370 km. The value of g at the orbit of the satellite $=9{\cdot}83\ \text{m}^{-2}$. *[D.S S.E. 1984]*

26. How long will the earth take to rotate about the sun, if its distance becomes half the present distance from the sun?

27. Compare the period of rotation of Jupiter about sun with that of earth. The mean distance of Jupiter from the sun is $5{\cdot}2$ A U.

28. What is the gravitational potential and gravitational potential energy of a body of mass 5 kg at a height of 300 km above the surface of the earth? ($G=6{\cdot}67\times10^{-11}\ \text{Nm}^2/\text{kg}^2$; mass of the earth $=6\times10^{24}$ kg and radius of the earth $=6{\cdot}37\times10^{6}$ m).

29. A body is projected out with a speed of 20 km^{-1} from the surface of the earth. What is the speed of the body far away

from the earth. The escape velocity of a projectile from the surface of the earth is 11 km s^{-1}.

30. If a body projected with a speed of 25 km s^{-1} from the surface of the earth reaches at infinite distance with a speed of 22·35 kms^{-1}. What is the escape velocity of the body at the earth's surface ?

31. A body projected from the surface of earth reaches at infinite distance with a speed of 15 km s^{-1}. If the escape velocity of the body at the surface of earth is 11·2 km s^{-1}. Calculate the velocity with which the body was projected ?

32. The radius of the moon is about 0·27 times that of the earth and the mass of the moon is about 0·0127 times the mass of the earth. What is the value of acceleration due to gravity at moon if its value at earth is 9·8 ms^{-2} ?

33. The mean distance of the earth from the sun is $1·5\times10^{11}$ m and its period of revolution around the sun is 365·25 days. If the distance of planets Mars and Venus from the sun is $2·28\times10^{11}$ m and $1·08\times10^{11}$ m respectively, calculate their period of revolution round the sun.

34. The mean distance of the earth from the sun is $1·5\times10^{11}$ m and its period of revolution around the sun is 365·25 days. If period of revolution of Jupiter and Saturn around the sun is 4333 days and 10760 days respectively, calculate their mean distance from the sun.

35. In an experiment using the cavandish balance, the smaller spheres have a mass 10 g each and the larger spheres have a mass 6 kg each. The length of the rod is 1 m and the distance between the centre of each pair is 12 cm. If the angle of twist of the fibre is 0·437°, calculate the value of gravitational constant. (Given torsion constant of the fibre is $3·50\times10^{-8}$ Nm rad^{-1}).

36. The peak of a hill is 3200 m above sea level. Calculate the value of acceleration due to gravity at this height, given that value of '*g*' at earth is 9·8 ms^{-2}. (Take radius of earth =6400 km).

37. The satellite revolves in a circular orbit at a height of 630 km about the earth's surface. What is (*a*) the orbital velocity and (*b*) period of revolution of the satellite ? (Given that the radius of the earth $6·370\times10^{6}$ m and acceleration due to gravity on the surface of the earth is 9·8 ms^{-2}).

38. A body weighs 64 kg on the surface of the earth. What is the gravitational force on it due to the earth at a height to one-third the radius of the earth ?

39. A man weighs 60 kg on the surface of the earth. What will be his weight at a height equal to the radius of the earth from its surface ?

40. A pluto year is 247 times the earth. How far is the pluto from the sun if the earth is $1{\cdot}496 \times 10^8$ km away from the sun ?

41. A projectile is fired upward with a speed of 8 km s^{-1}. How high will it rise above the earth ? (Radius of the earth=6400 km and $g=9{\cdot}8$ ms^{-2}).

42. A rocket is fired vertically with a speed 4 kms^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth ? (Mass of the earth$=6{\cdot}0\times10^{24}$ kg, mean radius of the earth$=6{\cdot}4\times10^6$ m $G=6{\cdot}67\times10^{-11}$ Nm^2/kg^2).

43. If a body is projected vertically upwards from the earth's surface to reach a height equal to ten times the radius of the earth, with what velocity the body should be projected ? ($R=6{\cdot}0\times10^6$ m and $g=9{\cdot}8$ ms^{-2}).

44. Assuming the earth to be a sphere of uniform density, how much would a body weigh one-fourth way down to the centre of the earth, if it weighed 24 kg on the surface.

45. A satellite of mass 200 kg orbits the earth at a height of 430 km from its surface. Calculate its (*a*) kinetic energy, (*b*) potential energy and (*c*) the total energy. (Given mass of the earth$=6{\cdot}0\times10^{24}$ kg, radius of the earth$=6{\cdot}37\times10^6$ m and $G=6{\cdot}67\times10^{-11}$ m).

46. A satellite of mass 400 kg orbits at a height of 330 km from the surface of the earth around it. If the value of acceleration due to gravity at this height is $9{\cdot}8$ ms^{-2}, calculate the satellite's (*a*) kinetic energy (*b*) potential energy and (*c*) total energy. (Given the radius of the earth$=6{\cdot}37\times10^6$ m).

47. A rocket is fired vertically from the surface of Mars with a speed of 3 km s^{-1}. If 25% of its initial energy is lost due to atmospheric resistance, how far will the rocket go from the surface of Mars before returning to it ? (Given mass of Mars $=6{\cdot}4\times10^{23}$ kg, radius of Mars=3395 km and $G=6{\cdot}67\times10^{-11}$ Nm^2/kg^2).

48. A rocket is fired from the earth towards the moon. At what point on its path is the gravitational force on the rocket zero. (It is the point where the rocket will be weightless). Given mass of the earth$=6\times10^{24}$ kg, mass of the moon$=7{\cdot}36\times10^{22}$ kg and orbital radius of the moon$=3{\cdot}85\times10^8$ m).

49. Four mass points each of mass '*m*' are placed at the vertices of a square of side '*l*'. What is the gravitational field and potential at the centre of the square due to the four masses ?

50. **Two heavy spheres each of mass 20 kg are placed 0·4 m apart on a horizontal table. What is the gravitational field and potential at the mid-point of the line joining the centre of spheres ?**

51. Two spheres one of mass 60 kg and the other 40 kg are placed 80 cm apart from each other on a horizontal table. What is the gravitation field and potential at a point 60 cm away from 60 kg on the line joining the centre of the two spheres ?

52. Light from the sun suffers a red shift. Estimate the magnitude of the red shift for light of wavelength 4500 Å from the sun. (Given mass of the sun$=2\times10^{30}$ kg, radius of the sun$=7\times10^{8}$ m, G$=6{\cdot}67\times10^{-11}$ Nm/kg^2 and c$=3\times10^{8}$ ms^{-1}).

53. Calculate the height of a artificial satellite moving in a circular orbit if it is always above a certain place on the earth's surface. Take a radius of the earth=6370 km.

OBJECTIVE TYPE QUESTIONS

54. The escape velocity of a projectile from the earth is approximately :
(*a*) 7 km/s (*b*) 11·2 km/s
(*c*) 1·1 km/s (*d*) 112 km/s *[C.P.M.T. 1982]*

55. A satellite goes round the earth along a circular path of radius R. The orbital velocity of satellite is
(*a*) $\sqrt{\frac{GM}{R}}$ (*b*) $\sqrt{\frac{GM}{R^2}}$
(*c*) $\frac{GM}{R}$ (*d*) gR

56. According to Keplar's law of period, the following is same for all planets
(*a*) $\frac{a^3}{T^3}$ (*b*) $\frac{T^2}{a^2}$
(*c*) $\frac{a^2}{T^3}$ (*d*) $\frac{a^3}{T^2}$

57. Weightlessness is experienced while orbiting the earth in a spaceship as the result of
(*a*) acceleration (*b*) inertia
(*c*) zero gravity (*d*) centre of gravity
[C.P.M.T. 82, 83]

58. The period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius 4 R is
(*a*) T/4 (*b*) T/2
(*c*) 2T (*d*) 4T

59. If V_e and V_0 denote the escape velocity and orbital velocity of a satellite corresponding to a circular orbit of radius R, then

(a) $V_e = V_0$ (b) $V_e = \frac{V_0}{\sqrt{2}}$

(c) $V_e = \sqrt{2}V_0$ (d) V_e and V_o are not related.

[*B.H U. 1983*]

60. Digging a tunnel along the diameter of the earth and dropping a body into the tunnel will result that the body will

(a) stop at the centre of the earth

(b) reach the other end of the tunnel

(c) oscillate about its centre.

(d) will fly away through the other end of the tunnel.

61. Gravitational potential and gravitational intensity are respectively

(a) Scaler and Vector (b) Vector and Scaler

(c) Scaler and Scaler (d) Vector and Vector.

62. A body jumps a distance 6 m at the surface of the earth. What distance can be jump safely at the surface of the moon ?

(a) 36 m (b) 12 m

(c) 6 m (d) 1 m.

63. The earth shrinks to half of its radius. Your weight will

(a) become double (b) become four-times

(c) decrease to one-half (d) decreases to one-fourth.

64. The acceleration due to gravity at a distance 'x' from the centre of the earth ($x >$ radius of the earth), varies as

(a) $g \propto \frac{1}{x}$ (b) $g \propto x$

(c) $g \propto \frac{1}{\sqrt{x}}$ (d) $g \propto \frac{1}{x^2}$

65. If the earth contracts to half of its radius, the duration of the day will be

(a) 24 hrs (b) 12 hrs

(c) 6 hrs (d) 48 hrs.

66. When the speed of rotation of the earth decreases, the weight of the body

(a) increases (b) decreases

(c) remains the same (d) none of them.

☐☐

7

Molecules

IMPORTANT FORMULAE

1. **Law of Definite Proportions (Compositions)** : Whenever a particular compound is formed, the mass of one element required to combine with the fixed mass of the other is always the same.
2. **Law of Multiple Proportions** : When the same two elements combine to form more than one compound, the different quantities of one element that combines with a fixed quantity of the other element are in the ratio of small integers.
3. Avogadro's number,

$$N=\frac{M}{m}\ ;\quad M=\text{Molecular weight}$$
$$m=\text{Absolute mass of a molecule}$$

4. Graham's law

$$\frac{r_1}{r_2}=\sqrt{\frac{\rho_2}{\rho_1}}=\sqrt{\frac{M_2}{M_1}}$$

r_1, r_2=rate of diffusion, ρ_1, ρ_2=density, M_2, M_1=Molecular weight

5. Atomic mass $=\dfrac{6\cdot4}{\text{Specific heat}}$
6. The energy of an α-particle scattered at an angle of 180° by a foil,

$$E=\frac{2kze^2}{r_0}$$

$k=9\times10^9\ Nm^2/\text{coulomb}^2$

z=Atomic No. of atom in the foil

e=Charge on an electron$=1\cdot6\times10^{-19}c$

r_0=radii of nuclei of atoms in the foil or distance of the nearest approach

7. Radius of a nucleus of an element,

$r=r_0A^{\frac{1}{3}}$; r_0=radius of nucleus of H_2 $=1\cdot2\times10^{-15}$ m

A=Mass number of element

8. Thermal velocity (r.m.s. velocity) of a gas molecule,

$$C=\sqrt{\frac{3kT}{m}}$$

k=Boltzmann's Constant
$=1{\cdot}38\times10^{-23}$ J/°k
T=Absolute temp.
m=Absolute mass of a molecule

9. "The amount of any substance that contains $6{\cdot}023\times10^{23}$ number of atoms, molecules, ions, charge or particle is called mole."

mole=gm. mole=gm. molecular mass=$6{\cdot}023\times10^{23}$ molecules
mole=gm. atom=gm atomic mass=$6{\cdot}023\times10^{23}$ atoms.
22·4 litre of every gas contains $6{\cdot}023\times10^{23}$ molecules.

SOLVED EXAMPLES

Example 1. *Calculate the root mean square speed of the suspended molecules in Brown an motion if its mass is 10^{-6} kg and the temperature of the liquid is 27°C. (Boltzmann's constant=$1{\cdot}38\times10^{-23}$ J/°k).*

Solution. $k=1{\cdot}38\times10^{-23}$ J/°k, T=273+27=300 K, $m=10^{-6}$ kg.

So the root mean square (r.m.s.) speed of the molecule,

$$C=\sqrt{\frac{3KT}{m}}$$

$$=\sqrt{\frac{3\times1{\cdot}38\times10^{-23}\times300}{10^{-6}}}$$

$$=1{\cdot}1\times10^{-7}\ \text{ms}^{-1}$$

Example 2. *Calculate the nuclear radius of $8O^{16}$.*

Solution. We know $r_0=1{\cdot}2\times10^{-15}$ m and A=16

$$\therefore\quad r=r_0\,A^{\frac{1}{3}}$$

$$=1{\cdot}2\times10^{-15}\ 16^{\frac{1}{3}}$$

$$=1{\cdot}2\times10^{-15}\times2{\cdot}52$$

$$=3{\cdot}024\times10^{-15}\ \text{m}$$

Example 3. *Calculate the radius of gold nucleus if an α-particle of energy 5 MeV strikes the nucleus and get scattered through an angle of 180°.*

Solution. $k=9\times10^{9}$ Nm²/coulomb²
$z=79$, $e=1{\cdot}6\times10^{-19}$ coulomb
E=5 MeV=$5\times1{\cdot}6\times10^{-13}$ J=8×10^{-13} J

$$\therefore\ \text{By}\quad E=\frac{2kze^2}{r_0},$$

$$r_0=\frac{2kze^2}{E}$$

$$=\frac{2\times9\times10^{9}\times79\times(1{\cdot}6\times10^{-19})^{2}}{8\times10^{-13}}$$

$$=4{\cdot}55\times10^{-14}\text{ m.}$$

Example 4. *Under similar conditions of experiments, the rate of diffusion of hydrogen and the other gas are found to be 28·7 c.c.* s^{-1} *and 7·2 c.c.* s^{-1}. *Identify the gas.*

Solution. $\frac{r_1}{r_2}=\sqrt{\frac{M_2}{M_1}}$

M_1=Molecular wt. of $H_2=1\times2=2$ g

$$\therefore \quad \frac{28{\cdot}7}{7{\cdot}2}=\sqrt{\frac{M_2}{2}}$$

$$M_2=4\times\left(\frac{28{\cdot}7}{7{\cdot}2}\right)^2$$

$$M_2=31{\cdot}78\ g$$

∴ The other gas is oxygen.

Example 5. *16 ml of* H_2 *diffused in 30 s. What will be the volume of* SO_2 *gas diffused in the same time ?*

Solution. M_1=Molecular mass of $H_2=2\times1=2$

M_2= ,, ,, $SO_2=32+2\times16=64$

If volume of SO_2 diffused is V ml,

$$\therefore \text{ By} \quad \frac{r_1}{r_2}=\sqrt{\frac{M_2}{M_1}}$$

$$\frac{16/30}{V/30}=\sqrt{\frac{64}{2}}$$

$$\frac{16}{V}=\frac{8}{\sqrt{2}}$$

$$\therefore \quad V=\frac{16\sqrt{2}}{8}=2\times1{\cdot}414$$

$$=2{\cdot}828\text{ ml}$$

Example 6. *Calculate the total number of electrons present in 1·6 g of methane.* [*J.E.E. 1976*]

Solution. Molecular formula of methane=CH_4

,, mass ,, ,, =16

16 g of methane contains $6{\cdot}023\times10^{23}$ molecular

∴ 1·6 g ,, ,, $=6{\cdot}023\times10^{23}\times\frac{1{\cdot}6}{16}$

$=6{\cdot}023\times10^{22}$ molecules

Now number of electrons in one molecule of methane=10

∴ No. of electrons in 1·6 g of methane

$=10\times6{\cdot}023\times10^{22}$

$=6{\cdot}023\times10^{23}$ electrons

Example 7. *Carbon and oxygen are known to form two compounds. The carbon content in one compound is 42·9% and in other 27·3%. Show that these data are in agreement with law of multiple proportion.*

Solution. Let us fix the mass of carbon=1 g

In first compound, $O_2=100-42{\cdot}9=57{\cdot}1\%$ (say)

∵ 42·9 g carbon combine with 57·1 g O_2

∴ 1 g carbon combine with $=\frac{57{\cdot}1}{42{\cdot}9}=1{\cdot}33$ g O_2

In second compound, $O_2=100-27{\cdot}3=72{\cdot}7\%$

∵ 27·3 g carbon combine with 72·7 g O_2

∴ 1 g carbon combine with $=\frac{72{\cdot}7}{27{\cdot}3}=2{\cdot}66$ g O_2

∴ The ratio of oxygen in two compounds

$=1{\cdot}33:2{\cdot}66=1:2.$

So the given data are in agreement with law of multiple proportions.

Example 8. *Two samples of $ZnCl_2$ are prepared by a scientist*

1st sample contains 7g zinc in 12·7 g of $ZnCl_2$

2nd sample contains 9·5 g zinc in 19·2 g of $ZnCl_2$

Establish law of constant proportion.

Solution. 1st Case :

12·7 g $ZnCl_2$ contains 7 g Zn

100 g $ZnCl_2$ contains $=\frac{7}{12{\cdot}7}\times100=55{\cdot}12\%$ Zn

2nd Case :

19·2 g $ZnCl_2$ contains 9·5 g Zn

∴ 100 g $ZnCl_2$ contains$=\frac{9{\cdot}5}{19{\cdot}2}\times100=55{\cdot}22\%$ Zn.

The fixed percentage of Zn is there in $ZnCl_2$ in each case, proving the law of constant proportions.

Example 9. *1·375 g cuperic oxide was redueed to give 1·098 g Cu. In another experiment 1·179 g of Cu was dissolved in nitric acid and copper nitrate is heated to give 1·476 g CuO. Show these results illustrate law of constant proportions.*

Solution.

Wt. of Cu	Wt. of CuO	% of Cu
1·098	1·375	$=\frac{1{\cdot}098}{1{\cdot}375}\times 100=79{\cdot}85\%$
1·179	1·476	$=\frac{1{\cdot}179}{1{\cdot}476}\times 100=79{\cdot}87\%$

The constant percentage of Cu in these in CuO in both the cases, proving the law of constant proportions.

Example 10. *Calculate mass of carbon atom.*

Solution. We know atomic mass of carbon

$=12{\cdot}00$ amu.

$\therefore$ gm atomic mass$=12{\cdot}00$ g

$\because$ mass of $6{\cdot}023\times 10^{23}$ atoms of carbon

$=12$ g

$\therefore$ mass of 1 atom of carbon

$$=\frac{12}{6{\cdot}023\times 10^{23}}=1{\cdot}99\times 10^{-23}\text{ g}$$

Example 11. *Calculate the number of atoms present in :*
(i) $1{\cdot}792\times 10^{-6}$ *g of silver,* *(ii)* *0·002 mole of sodium*
(*Given* $Ag=107{\cdot}87$, $Na=23$)

Solution.

(*i*) 107·87 g Ag contains$=6{\cdot}023\times 10^{23}$ atom

$\therefore$ 1 g Ag contains $=\frac{6{\cdot}023\times 10^{23}}{107{\cdot}87}$

$\because$ $1{\cdot}792\times 10^{-6}$ g Ag contains$=\frac{6{\cdot}02\times 10^{23}\times 1{\cdot}792\times 10^{-6}}{107{\cdot}87}$

$=10^{16}$ atoms

(*ii*) 1 mole of Sodium contains

$=6{\cdot}023\times 10^{23}$ atoms

$\therefore$ 0·002 mole of sodium contains$=6{\cdot}023\times 0{\cdot}002\times 10^{23}$

$=12046\times 10^{17}$ atoms.

Example 12. *Calculate the mass of 1 molecule of Benzene* (C_6H_6)

Given mass of C $=12{\cdot}01$ *g*

mass of *H* $=1{\cdot}008$ *g*

Solution.

Molecular mass of benzene (C_6H_6)

$=6\times 12{\cdot}01+6\times 1{\cdot}008$

$=78{\cdot}11$ g

Now $6{\cdot}023\times10^{23}$ molecules of Benzene has a mass=78·11 g

$$\therefore \quad 1 \text{ molecule of Benzene has a mass}=\frac{78{\cdot}11}{6{\cdot}023\times10^{23}}$$

$$=12{\cdot}978\times10^{-23}\text{ g}$$

Example 13. *The specific heat at constant volume of a certain metal is approximately 0·33 cal* $g^{-1}\,{}^{\circ}C^{-1}$. *Write the chemical formula of its chloride if it contains 0·390 fraction of the metal.*

Solution. $\text{Atomic mass}=\dfrac{6{\cdot}4}{\text{sp. heat}}$

$$=\frac{6{\cdot}4}{0{\cdot}33}=19{\cdot}4$$

Now $(1-0{\cdot}390)=0{\cdot}710$ chlorine combines with 0·390 metal

∴ 35·5 chlorine will combine the metal

$$=\frac{35{\cdot}5\times0{\cdot}390}{0{\cdot}710}$$

$$=19{\cdot}5$$

$$\therefore \quad \text{Valency of the metal}=\frac{19{\cdot}4}{19{\cdot}5}=1$$

∴ Chemical formula of the chloride=MCl.

Example 14. *Haemocyanin has a density of 1·35 g* cm^{-3}. *For a suspension of haemocyanin in water, the sedimentation constant is formed to be* $17{\cdot}4\times10^{-13}$ *in the useful units. If viscosity of water at the experimental temperature be 1 centipoise, calculate the particle radius and molecular mass of haemocyanin.*

Solution. Sedimentation constant,

$$S=17{\cdot}4\times10^{-13}$$

$\eta=1$ C Poise$=10^{-2}$ Poise, ρ (density of haemocyanin) $=1{\cdot}35\text{ g cm}^{-3}$,

ρ_0 (density of water)$=1\text{ g cm}^{-3}$

$$\text{Now} \qquad S=\frac{2}{9}\,\frac{r^2(\rho-\rho_0)}{\eta}$$

$$\therefore \qquad 17{\cdot}4\times10^{-13}=\frac{2}{9}\,r^2\,\frac{(1{\cdot}35-1)}{10^{-2}}$$

On solving it, $r=4{\cdot}73\times10^{-7}$ cm

gm. molecular mass=mass of one molecule×N

$$=\frac{4}{3}\pi r^3\,\rho\times N$$

$$=\frac{4}{3}\times\frac{22}{7}\times(4{\cdot}73\times10^{-7})^3\times1{\cdot}35\times6{\cdot}023\times10^{23}$$

$$=3{\cdot}606\times10^{5}\text{ g}$$

Example 15. *Polymethyl methacrylate resin is soluble in chloroform. For these solutions the constant k and a are respectively*

$3{\cdot}3\times10^{-6}$ and 0·85. Calculate the molecular mass of the resin if a solution of 2·5 g of it in 1 litre has a specific viscosity of 0·11.

Solution. If the solution is sufficiently dilute,

$$kM^{a}=\frac{\eta}{C}$$

$$3{\cdot}3\times10^{-6}\,M^{0{\cdot}85}=\frac{0{\cdot}11}{2{\cdot}5}$$

$$M^{0{\cdot}85}=\frac{0{\cdot}11}{2{\cdot}5\times3{\cdot}3}\times10^{6}$$

or

$$M^{0{\cdot}85}=13333$$

$$\therefore\quad 0{\cdot}85\log M=\log(13333)$$

$$\therefore\quad \log M=\frac{4{\cdot}1249}{0{\cdot}85}$$

$$\therefore\quad M=\text{Anti}\log(4{\cdot}8528)$$

$$=71250.$$

Example 16. *In an experiment of Brownian motion using a torsion pendulum (a small mirror of area 1 mm^2 mounted on a thin torsion fibre with torsion constant$=1{\cdot}8\times10^{-17}$ J/rad^2) the mean value of angular displacement θ was found to be nearly zero and the fluctuation in θ, that is mean square of θ was found to be $2{\cdot}6\times10^{-4}$ rad^2. Estimate the value of Boltzmann constant from this data and compare it with the correct value. The temperature is 300 K.*

Solution. The mean energy required for angular displacement of torsion fibre required

$$\frac{1}{2}\,\alpha\theta^2=\frac{1}{2}KT$$

$$\therefore\quad K=\frac{\alpha\theta^2}{T}$$

$$=\frac{1{\cdot}8\times10^{-17}\times2{\cdot}6\times10^{-4}}{300}$$

$$=1{\cdot}56\times10^{-23}\text{ J/k}$$

The more accurate value of Boltzmann's constant is $1{\cdot}38\times10^{-23}$ J/k.

Example 17. *The latent heat of vaporization for water is 22·6 $\times10^{5}$ J/kg. Calculate the intermolecular binding energy.*

Solution. Molecular mass of water = 18

Since kg molecular mass has $6{\cdot}023\times10^{26}$ molecules

∴ In 18 kg of water, No. of molecules $=6{\cdot}023\times10^{26}$

∴ In 1 kg of water, No. of molecules, $N=\dfrac{6{\cdot}023}{18}\times10^{26}$

$$=0{\cdot}3346\times10^{26}$$

$\therefore$ The energy required to unbound N molecules in 1 kg of water

$$=22{\cdot}6\times10^5\ \text{J}$$

$$=\frac{22{\cdot}6\times10^5}{1{\cdot}6\times10^{-19}}\text{eV}$$

$$=14{\cdot}125\times10^{24}\ \text{eV}$$

$\therefore$ Intermolecular binding energy

$$=\frac{14{\cdot}125\times10^{24}}{0{\cdot}3346\times10^{26}}$$

$$=0{\cdot}422\ \text{eV}.$$

Example 18. *The ionization energy of sodium is 5·1 eV and the electron affinity for chlorine is 3·6 eV. What is the amount of energy required to form* Na^+ *and* Cl^- *ions ? How close must a* Na^+ *ion and* Cl^- *ion be for the total potential energy to be zero. Calculate also the binding energy of the ionic molecule NaCl when the two ions are 2·4 Å apart.*

Solution. Net energy required to form Na^+ and Cl^- ions

$$=5{\cdot}1-3{\cdot}6$$

$$=1{\cdot}5\ \text{eV}$$

The electric potential energy of ions,

$$E=k\frac{(q)(-q)}{r}$$

If r is such that the $E=-1{\cdot}5$ eV, then the total energy$=E+1{\cdot}5$ $=-1{\cdot}5+1{\cdot}5=0$

$$\therefore \qquad E=-1{\cdot}5\text{eV}=-1{\cdot}5\times1{\cdot}6\times10^{-19}\ \text{J}$$

$$\therefore \quad -1{\cdot}5\times1{\cdot}6\times10^{-19}=\frac{-kq^2}{r}$$

or

$$-2{\cdot}40\times10^{-19}=\frac{-(9\times10^9)(1{\cdot}6\times10^{-19})^2}{r}$$

$$\therefore \qquad r=\frac{9\times1{\cdot}6\times1{\cdot}6}{2{\cdot}4}\times10^{-10}\ \text{m}$$

$$=9{\cdot}6\times10^{-10}\ \text{m}=9{\cdot}6\ \text{Å}$$

The electric potential energy for $r=2{\cdot}4$ Å$=2{\cdot}4\times10^{-10}$ m will be

$$E=-\frac{kq^2}{r}$$

$$=-\frac{(9\times10^9)(1{\cdot}6\times10^{-19})^2}{2{\cdot}4\times10^{-10}}$$

$$=-6\ \text{eV}$$

$\therefore$ The binding energy of the ionic molecule NaCl

$$=(-6+1{\cdot}5)=-4{\cdot}5\ \text{eV}$$

Example 19. *A drop of olive oil of 2 mm diameter when transferred lightly over water dusted with lycopodium powder spreads out into a circular thin film of diameter 79·4 cm. Estimate the size of the molecule of the oil assuming that the film is only one molecule thick.*

Solution. Volume of drop of olive oil, $V=\frac{4}{3}\pi R^3$

Area of circular film, $A=\pi r^2$

∴ Thickness of film,

$$x=\frac{V}{A}=\frac{4/3\ \pi R^3}{\pi r^2}=\frac{4}{3}\frac{R^3}{r^2}$$

Now $R=1\text{ mm}=1\times10^{-3}\text{ m}$

and $r=39{\cdot}7\text{ cm}=0{\cdot}397\text{ m}$

$$\therefore\quad x=\frac{4}{3}\frac{(10^{-3})^3}{(0\ 397)^2}=8{\cdot}4\times10^{-9}\text{ m}$$

Since the film is one molecule thick so the diameter of the molecule is also $8{\cdot}4\times10^{-9}$ m.

EXERCISE 7

1. 100 volume of oxygen take 20 minutes to diffuse out of a vessel. How long will 40 volumes of hydrogen take to diffuse out from the same vessel under the same conditions.
2. 144 ml of a gas X diffuses in 12 minutes. If 120 ml of SO_2 diffuses in 20 minutes. Find molecular mass of gas 'X'.
3. H_2 and gas 'X' have diffusion ratio 6 : 1. Calculate the molecular mass of 'X'.
4. Two gases A and B has molecular mass 16 and 64 respectively. What is the rate of diffusion of gas A if that of B is 50 ml/s ?
5. The ratio of diffusion of H_2 and Cl_2 is 1 : 6. Calculate the density of Cl_2 as compared to H_2.
6. The diffusion rate of O_2 has an average value of $62{\cdot}5\ ccs^{-1}$. The diffusion of another gas under the same conditions is measured to have an average rate of $66\ 8\ ccs^{-1}$. Identify the gas.
7. At what temperature will the root mean square speed of a gas molecule will be half of its value at 0°C.
8. The energy required to dissociate a molecule of H_2 is 4·5 *eV*. Calculate the energy required to dissociate a mole of hydrogen in Joule.
9. When 4·25 g of silver were used to produce silver chloride, it weighed 5·61 g. On the other hand when 1·95 g of silver was extracted from 2·6 g of silver chloride. Verify the law of constant proportions.

10. 2·4 g of Mg on heating in air gave 4g of magnesium oxide. The same oxide obtained by heating magnesium carbonate was found to contain 60% Mg by weight. Verify the law of constant proportions.

11. Silver chloride contains 24·73% of chlorine. Calculate the weight of chlorine that would combine with 12 gm of silver.

12. Two elements X and Y form two different compounds. In the 1st compound, 1 g of X combined with 1·44 g of Y. In the second 0·15 g of X combined, with 0·65 g of Y. Verify the law of multiple proportions.

13. Two sample of H_2S gases was analysed. One contain 0·2 g H_2 and 3·2 g of sulphur whereas other contain 6% H_2 and 94% sulphur. Which law can be illustrated by above data ?

14. In compound 'A' 1g of N_2 unites with 0·87 g of O_2. In compound 'B' 2g N_2 combine with 2·24 g of O_2. In compound C, 3g N_2 combine with 5·11 g of O_2. Show these results obey law of multiple proportions.

15. Three oxides of iron has following composition : (1) Fe=77·78% (2) Fe=70% (3) Fe=72·42%. Show how they illustrate law of multiple proportion.

16. Sodium chloride is derived from three different sources was found to contain Na and Cl_2 only :

    (*i*) 3·2 g NaCl contain 1·940 g chlorine.

    (*ii*) 10 g NaCl contain 6·068 g chlorine.

    (*iii*) 5·3 g NaCl contain 3·216 g chlorine.

    Show that the above data prove the law of constant proportions.

17. Water and H_2O_2 give following % composition :

    (*i*) H_2O contain 11·11% H_2.

    (*ii*) H_2O_2 contain 5·88% H_2.

    Show that these data agree with law of multiple proportions.

18. Two oxides of P gave the following results :

    (*i*) 11 g of oxide 'A' contain 9·8 g of O_2.

    (*ii*) 21·3 g of oxide B contain 12 g of O_2.

    Verify the law of multiple proportions.

19 Find the mass of

    (*a*) 0·15 mole of NH_3

    (*b*) 0·05 mole of MgO

    (*c*) 0·635 mole of copper

    (*d*) 0·107 mole of Ag.

    (Given N=14, Cu=63·5, Mg=24, O=16, Ag=107).

20. An average signature with ac^{12} pencil weigh about 1 mg. Find carbon atoms in signature.

    The mass of a molecule of hydrogen is $3{\cdot}34\times10^{-27}$ kg. Calculate Avogadro's number.

21. Convert into mole :
(*i*) 100 g Sulphur (*ii*) 100 g Fe (*iii*) 100 g Al.

22. How many atoms are present in cube of Iron of side 10 cm.
(density of Fe=7·8 g/cm. and
Atomic mass Fe=55·85).

23. An atom of Hg weight 333×10^{-24} g. Determine atomic mass of Hg.

24. Chlorophyll the green plastid and plant cell responsible for photosynthesis contain 2·68% Mg by weight. Calculate number of Mg atoms in 2g of chlorophyll (Mg=24·3).

25. Calculate number of moles and molecules in 8·516 g NH_3
[N=14·00, H=1·008]

26. Calculate No. of gram of oxygen atom in 0·10 mole of $Na_2CO_3.10\,H_2O$.

27. The mass of C^{12} atom is found to be 2×10^{-26} kg by a particular spectrometer. Calculate the Avogadro's number.

28. The size of the hydrogen molecule may be taken as nearly 1 Å. Calculate the fraction of volume actually occupied in a mole of hydrogen at S.T.P.

29. The specific heat at constant volume of a certain metal is approximately 0·16 cal $g^{-1}\,°C^{-1}$. If it contains 2/7 fraction of the metal, find the chemical formula of its chloride.

30. The chemical formula for a metal is MCl_4. If it contains 0·3 fraction of the metal, calculate C_v (the specific heat at constant volume).

31. Calculate the r.m.s. speed of the molecules of a liquid in Brownian motion if the mass of a molecule is 10^{-5} kg and the temperature of the liquid is 127°C.

32. Polymethyl methacrylate resin is soluble in chloroform. For these solutions the constant K and *a* are respectively $3{\cdot}3 \times 10^{-6}$ and 0·85. Calculate the molecular mass of resin if a solution of 5·0 g of it in 2 litre has a viscosity of 0·22.

33. In an experiment of Brownian motion using a torsion pendulum whose torsion fibre has a torsion constant $1{\cdot}8 \times 10^{-13}$ J/rad^2, the mean value of angular displacement was found to be nearly zero. The fluctuations in angular displacement that is the mean square of its value was found to be $2{\cdot}4 \times 10^{-8}$ rad^2. Calculate the value of Boltzmann's constant if the temperature is 47°C.

34. Calculate the radius and mass density of the following :
(*a*) Lead and (*b*) Oxygen nuclei. (Pb=205)

35. Estimate the radius of a gold nucleus when an α-particle of energy 6 MeV is incident on its centre and gets scattered through 180°.

36. In a head on collision between α-particle and a gold nucleus, the minimum distance of approach is 4×10^{-14} m. Calculate the energy of the α-particle.

37. Nuclear radius is of the order of 10^{-14} m while atomic radius is of the order of 10^{-10} m. Assuming the nucleus and the atom to be spherical, what fraction of atom is occupied by a nucleus ?

38. Calculate the nuclear radius of ${}_{11}Na^{24}$ if nuclear radius of ${}_{82}Pb^{205}$ is 7×10^{-15} m.

39. Calculate the nuclear radius of
(*a*) ${}_{47}Ag^{107}$ (*b*) ${}_{20}Ca^{44}$ (*c*) ${}_{6}C^{12}$

40. Calculate the energy in Joules needed to dissociate 0·04% hydrogen gas occupying volume of 2·8 litres under STP if the binding energy of a hydrogen molecule is 4·75 eV.

41. If the energy required to dissociate 0·2% oxygen gas occupying a volume of 11·2 litre under STP is 424 Joule, calculate the binding energy of a oxygen molecule in eV.

42. The net amount of energy required to form Na^+ and Cl^- ions is 1·5 eV. Calculate the binding energy of the ionic molecule NaCl when the two ions are 2·5 Å apart.

43. The density of water vapour at 100°C and 1 atmosphere is 0·75 kg m^{-3}. Calculate the fraction of molecular volume to the total volume occupied by the water vapour under the above condition of temperature and pressure.

44. The ionization energy of potassium is 4·3 eV and the electron affinity for chlorine is 3·7 eV. Calculate the binding energy of the ionic molecule KCl when the two ions are 3·2 Å apart.

45. Calculate the average intermolecular binding energy per molecule of mercury. Given that its latent heat of vaporization is $2{\cdot}72 \times 10^5$ J kg^{-1} and its molecular mass is 198.

OBJECTIVE TYPE QUESTIONS

46. Which pair diffuse at the same rate
(*a*) CO_2 and N_2O (*b*) CO_2 and NO_2
(*c*) N_2O and NO_2 (*d*) CO and CO_2
[*N.C.E.R.T. 1980*]

47. Equal volume of O_2 and H_2 is filled separately in two balloons. If diffusion rate of O_2 is 65 ml/hour then rate of diffusion of H_2
(*a*) 65 ml/hr (*b*) 130 ml/hr
(*c*) 260 ml/hr (*d*) 325 ml/hr.

48. Relation between diffusion rate and density was given by
(*a*) Dalton (*b*) Graham
(*c*) Boyle (*d*) Charles.

49. Which one is Graham's law ?
(*a*) $\frac{r_1}{r_2}=\sqrt{\frac{M_1}{M_2}}$ (*b*) $\frac{r_1}{r_2}=\frac{M_1}{M_2}$
(*c*) $r_1\times r_2=\sqrt{M_1\times M_2}$ (*d*) $\frac{r_1}{r_2}=\sqrt{\frac{M_2}{M_1}}$

50. The diffusion rate of A is 4 times than that of B. Their density ratio will be
(*a*) 1/4 (*b*) 1/2
(*c*) 1/16 (*d*) 4/1

51. Non-reacting gases have a tendency to mix each other. This property is called
(*a*) diffusion (*b*) fusion
(*c*) mixing (*d*) None.

52. A gas diffuse 4 times as quickly as O_2 the molecular weight of the gas is
(*a*) 2 (*b*) 4
(*c*) 8 (*d*) 16 [*A.M.V. 1986*]

53. The rate of diffusion is given by
(*a*) $r \propto \frac{1}{d}$ (*b*) $r \propto d$
(*c*) $r \propto \frac{1}{\sqrt{d}}$ (*d*) $r \propto \sqrt{d}$

54. The rate of diffusion of SO_2 and O_2 are in the ratio
(*a*) 1 : 32 (*b*) 1 : 4
(*c*) 1 : 2 (*d*) 1 : $\sqrt{2}$

55. The number of molecules of water in a beaker of 250 c.c. full of water is about
(*a*) 6×10^{23} (*b*) $7{\cdot}5\times10^{24}$
(*c*) $1{\cdot}5\times10^{26}$ (*d*) 6×10^{26}.

56. On increasing the temperature of the surrounding medium, the observed Brownian motion will
(*a*) increase (*b*) uneffected
(*c*) decrease (*d*) undecided.

57. The oxides of N_2 like N_2O, NO_2, NO, NO_2, N_2O_5 are example of
(*a*) Law of const. proportionality
(*b*) Law of conservation of mass

(*c*) Law of multiple prop.
(*d*) Law of reciprocal prop.

58. Which one of the following illustrate law of multiple proportion :
(*a*) Na_2O, Na_2O_2
(*b*) HCl, KCl
(*c*) Na_2SO_4, K_2SO_4
(*d*) CaI_2, BaI_2.

59. Two oxides of metal contain 30% and 22·2% oxygen. The weight of metal combined with fixed weight of oxygen are in the ratio of
(*a*) 3 : 5
(*b*) 2 : 1
(*c*) 3 : 4
(*d*) 7 : 10.

60. The positive potential energy of molecules means that the intermolecular force is
(*a*) Attractive
(*b*) Zero
(*c*) Repulsive
(*d*) Either attractive or repulsive.

61. The nature of interatomic or intermolecular forces of attraction or repulsion is always
(*a*) Gravitational
(*b*) Electrical
(*c*) Magnetic
(*d*) None of the above.

62. If m is the absolute mass of a molecule and M is the molecular mass of the same substance then Avogadro's number is
(*a*) $M \times m$
(*b*) $M + m$
(*c*) m/M
(*d*) M/m.

63. Brownian motion of molecules increases with
(*a*) increase in size of Brownian particle
(*b*) decrease in density of medium
(*c*) decrease in temperature of medium
(*d*) decrease in viscosity of medium.

□ □

8

Properties of Solids

IMPORTANT FORMULAE

1. Normal stress and tangential stress,

$$T=\frac{F}{A}$$

2. Hydrostatic stress, $T=\Delta p$;

Δp=Excess (difference) pressure

3. Linear or longitudinal strain,

$$e=\frac{\Delta l}{l}$$

4. Bulk or volume strain,

$$e=\frac{\Delta V}{V}$$

5. Young's modulus of elasticity,

$$Y=\frac{F.l}{A.\Delta l}$$

$$=\frac{Mg\,l}{\pi r^2\,\Delta l};$$

M=Load on wire

r=radius of wire.

6. Bulk modulus of elasticity,

$$K=\frac{\Delta p.\,V}{\Delta V}$$

Reciprocal of K *i.e.*, $\frac{1}{K}$ is called compressibility.

7. Shear modulus of elasticity or coefficient of rigidity,

$$\eta=\frac{F}{A.\,\theta};$$

θ=Shearing angle.

8. Pull on a clamped rod when it is cooled,

$$F=YA\,\alpha\,\Delta T$$

SOLVED EXAMPLES

Example 1. *A 4m long aluminium wire with cross-sectional area 1.0×10^{-6} m^2 is used to support a weight of 50 N. If the elongation of the wire is 2.5 mm., calculate Young's modulus for aluminium.*

Solution. F=50N ; $A=1.0\times10^{-6}$ m^2 ; l=4m and Δl=2.5 mm.

$=2.5\times10^{-3}$ m.

$$Y=\frac{F.l}{A.\Delta l}$$

$$=\frac{50\times4}{1\times10^{-6}\times2.5\times10^{-3}}=8\times10^{10}\ \mathrm{Nm^{-2}}$$

Example 2. *A 2 m long wire with diameter 2 mm is stretched by 3.5 mm by a load of 50 kg. Calculate (a) the stress, (b) the strain, and (c) Young's modulus.*

Solution. $l=2\mathrm{m}$; $A=\pi\frac{D^2}{4}=\frac{\pi(2\times10^{-3})^2}{4}=\pi\times10^{-6}\ \mathrm{m^2}$

$\Delta l=3.5\times10^{-3}$ m ; F=Mg=50×9.8=490 N.

(*a*) Stress $=\frac{F}{A}$

$$=\frac{490}{\pi\times10^{-6}}=15.6\times10^{7}\ \mathrm{Nm^{-2}}.$$

(*b*) Strain $=\frac{\Delta l}{l}$

$$=\frac{3.5\times10^{-3}}{2}=1\ 75\times10^{-3}$$

(*c*) $Y=\frac{\text{Stress}}{\text{Strain}}$

$$=\frac{15.6\times10^{7}}{1.75\times10^{-3}}=8.91\times10^{10}\ \mathrm{Nm^{-2}}.$$

Example 3. *Calculate the force required to double the length of a steel wire of diameter 4 mm. Y for steel is 20×10^{10} Nm^{-2}.*

Solution. $Y=20\times10^{10}\ \mathrm{Nm^{-2}}$, $A=\pi\frac{D^2}{4}=3.14\times\frac{(4\times10^{-3})^2}{4}$

$=12.56\times10^{-6}\ \mathrm{m^2}$

if $l=x$ metre, $\Delta l=2x-x=x$ metre

Now $Y=\frac{F.l}{\Delta l.A}$

$\therefore$ $F=\frac{Y.\Delta l.A}{l}$

$$=\frac{20\times10^{10}\times x\times12.56\times10^{-6}}{x}$$

$$=2.512\times10^{6}\ \mathrm{N}.$$

Example 4. *When a wire is strained longitudinally by 0·15%, the stress developed in the wire is 3·0 × 10⁸ Nm⁻². Calculate Young's modulus of wire.*

Solution.

$$\text{Strain} = \frac{\Delta l}{l}$$

$$= \frac{0{\cdot}15}{100} = 0{\cdot}0015$$

$$\therefore \quad Y = \frac{\text{Stress}}{\text{Strain}}$$

$$= \frac{3{\cdot}0 \times 10^8}{0{\cdot}0015} = 20 \times 10^{10}\ \text{Nm}^{-2}.$$

Example 5. *A wire increases by 10^{-3} of its length when the force acting per unit area on the wire normally is $1{\cdot}2 \times 10^8$ Nm⁻². What is the Young's modulus of the material of the wire ?*

Solution.

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$= \frac{1{\cdot}2 \times 10^8}{10^{-3}} = 12 \times 10^{10}\ \text{Nm}^{-2}.$$

Example 6. *What increase in pressure will be needed to decrease the volume of 1·0 m³ of water by 10 c.c. ? The bulk modulus of water is $0{\cdot}21 \times 10^{10}$ Nm⁻².*

Solution. $V = 1{\cdot}0\ \text{m}^3;\ \Delta V = 10\ \text{c.c.} = 10 \times 10^{-6}\ \text{m}^3$

$K = 0{\cdot}21 \times 10^{10}\ \text{Nm}^{-2}$

Now

$$K = \frac{\Delta p}{\frac{\Delta V}{V}}$$

$$\therefore \quad \Delta p = K\frac{\Delta V}{V}$$

$$= 0{\cdot}21 \times 10^{10} \times \frac{10^{-5}}{1{\cdot}0} = 2{\cdot}1 \times 10^4\ \text{Nm}^{-2}.$$

Example 7. *A steel wire of diameter 3·5 mm is heated uniformly to 300°C and then rigidly clamped at its two ends. If it is cooled to 20°C, calculate the pull exerted on clamps.*

(Y for steel = 20×10^{10} Nm⁻² and α for steel = 11×10^{-6} K⁻¹)

Solution. $A = \pi\frac{D^2}{4} = \frac{22}{7}\ \frac{(3{\cdot}5 \times 10^{-3})^2}{4} = 9{\cdot}625 \times 10^{-6}\ \text{m}^2$

Now $F = YA\alpha\Delta T$

$= 20 \times 10^{10} \times 9{\cdot}625 \times 10^{-6} \times 11 \times 10^{-6} \times (300-20)$

$= 5929$ N.

EXERCISE 8

1. A steel wire 50 cm long is stretched to a length of 50·50 cm. What is the strain ?

2. A copper wire 2 m long and 0·063 cm^2 in cross-section is stretched by ·03 mm with a load of 105 N. Calculate (*a*) Strain, (*b*) Stress, (*c*) Young's modulus of elasticity.

3. A telephone wire between the two poles is 50 m long and 1 mm in radius. When it is stretched by a load of 65 kg, its length becomes 50·12 m. Calculate Young's modulus of elasticity.

4. A 6 m long steel wire whose diameter is 7 mm is used to support a mass of 40 kg. What will be elongation of the wire ? (Y for steel$=20\times10^{10}$ Nm^{-2}).

5. An aluminium wire increases by 2×10^{-2} of its length when a stress of $1{\cdot}4\times10^{9}$ Nm^{-2} is applied to it. Calculate Young's modulus for aluminium.

6. Young modulus for steel is 20×10^{10} Nm^{-2}. Calculate the stress developed in a steel wire when it is strained by 0·30%.

7. A brass wire is having radius 0·35 mm. Calculate the force required to make its length 1·5 times the initial one. (Y for brass$=10\times10^{10}$ Nm^{-2}).

8. Find the length of the wire that will break under its own weight when suspended vertically. (Density of wire$=7{\cdot}8\times10^{3}$ Kg m^{-3} and breaking stress$=7{\cdot}8\times10^{8}$ Nm^{-2}).

9. A piece of lead having volume 500 c.c. is dropped into the sea. What is its new volume when it is on the sea bed were the pressure is $8{\cdot}0\times10^{7}$ Nm^{-2} greater than that at the surface ? (K for lead$=4{\cdot}3\times10^{10}$ Nm^{-2}).

10. The volume of 1000 litres of water decreases by 100 c.c. by increase of pressure by $2{\cdot}0\times10^{5}$ Nm^{-2}. Calculate the value of Bulk modulus and compressibility of water.

11. A rubber cube of 4 cm side has one face fixed rigidly while its opposite face is sheared by 2 mm parallel to the fixed face by a tangential force 10 N. Calculate modulus of rigidity.

12. An ocean is 10^4 metres deep. If a certain quantity of water is taken from the surface to its bottom, calculate the percentage change in its volume. The compressibility of water is 1/22000 per atmosphere.

13. A brass rod 50 cm long and 4 mm in diameter is heated from 20°C to 90°C. Calculate the force needed to compress the rod back to its original length. (α for brass$=18\times10^{-6}\,°C^{-1}$ and Y for brass$=9\times10^{10}$ Nm^{-2}).

14. How much will be the increase in length of a 3 m long copper wire if a weight of 10·0 kg is suspended from one end and the other end is fixed. The diameter of wire is 0·4 mm (Y for copper$=10\times10^{10}$ Nm^{-2}). [*A.I.S.S.E. 1979*]

15. Taking the Young's modulus of brass to be 10^{11} Nm^{-2}, calculate the increase in energy of a bar 0·1 m long and of cross-section area 1·0 cm^2, when compressed with a load of 10 kg wt. along its length ($g=10$ ms^{-2}). [*A.I.S.S.E. 1982*]

16. Calculate the force required to double the length of a wire of diameter 2 mm. Young's modulus of elasticity$=12\times10^{11}$ Nm^{-2}. [*D.S.S.E. 1980*]

17. The length of a wire increases 8 mm when a weight of 5 kg. is hung. If all conditions are the same, but the radius of the wire is doubled, what will be the increase in the length. [*A.I.S.S.* (*Compt.*) *E. 1982*]

18. A steel wire 3 metres long, when subjected to a tensile load of 40 kg. extends through 6 mm. If Young's modulus of steel is 2×10^{11} Nm^{-2}, calculate the stress and strain in the wire. Also determine the area of cross-section of wire ($g=10$ ms^{-2}). [*D.S.S.E. 1983*]

19. A longitudinal stress of 8×10^7 Nm^{-2} is applied to a wire, which increases its length 10^{-3} times. Calculate Young's modulus of the material of the wire. [*D.S.S.E. 1985*]

20. A steel wire of length 1 m, mass 0·1 kg and uniform cross-sectional area 10^{-6} m^2 is rigidly fixed at both ends. The temperature of the wire is lowered by 20°C. If transverse waves are set up by plucking the string in the middle, calculate the frequency of the fundamental mode of vibration.
(Y for steel$=2\times10^{11}$ Nm^{-2}; α for steel$=12{\cdot}1\times10^{-6}$ K^{-1}) [*I.I.T. J.E.E. 1984*]

21. Write the dimensions of rigidity modulus in terms of mass, length and time. [*I.I.T. J.E.E. 1983*]

22. Which of the two wires of the same material having the length 100 cm ; diameter 1 mm and length 50 cm ; diameter 0·5 mm has the larger extension when the same tension is applied. [*I.I.T. J.E.E. 1981*]

23. When a stress of 10^7 Nm^{-2} is applied to a suspended wire, its length increases by 10^{-4} of its original length. Calculate Young's modulus of elasticity of the material of the wire. [*D.S.S E. 1986*]

24. Two wires, one of copper and the other of the nickel, each 2·5 m long, 2·0 mm in diameter, are joined end to end to form a composite wire 4 mm thick. What tension in the wire will produce a total extension of 0·50 mm ?
(Y for Cu$=1\times10^{11}$ N/m^2 and Y for Ni$=2{\cdot}2\times10^{11}$ N/m^2)

25. What load when suspended from an aluminium wire 2 mm in diameter and 5 m long will stretch it by 1 mm ?
($Y=7\times10^{10}$ Nm^{-2}) *[D.S.S.E. 1987]*

26. A wire having a length of 150 cm and area of cross-section equal to 1 mm^2 stretched by a weight of 3 kg. Determine the increase in length. Young's modulus of the material of the wire$=2\times10^{11}$ N/m^2. *[A.I.S.S.E. 1987]*

27. Two wires of the same material have same diameter but length l and $2l$ respectively. Both are subjected to the same load. Will the strain in the two wires be the same or different ?
[A.I.S.S.E. 1988]

28. A 4 m long copper wire with diameter 0·5 mm is used to support a 5 kg weight. Determine the elongation of the wire [Y for copper is 10^{11} Nm^{-2}] *[D.S.S.E. 1988]*

29. The ends of a cylindrical bar are kept at 100°C and 0°C respectively. The diameter of the bar is 2 cm and its length is 20 cm. How much heat in joules flows (under steady state conditions) through the bar in 10 minutes ? (Given coefficient of thermal conductivity=0·025 watt m^{-1} k^{-1}.)
[D S S.E. 1989]

30. An iron sphere has a diameter of 20 cm at 20°C. Calculate the change in its volume if it is heated to 200°C. Coefficient of linear expansion of iron is 12×10^{-6} °C^{-1} and iron can be assumed to be isotropic material.
[D.S.S.C.E. 1989]

OBJECTIVE TYPE QUESTIONS

31. Out of rubber, glass, copper and steel which has maximum and minimum value of Young's modulus of elasticity respectively
(*a*) copper and rubber (*b*) glass and copper
(*c*) steel and rubber (*d*) rubber and steel.

32. Dimensions of modulus of elasticity are :
(*a*) ML^2T^{-2} (*b*) MLT^{-2}
(c) $ML^{-1}T^{-1}$ (*d*) $ML^{-1}T^{-2}$.

33. S.I. units of modulus of elasticity are :
(*a*) Nm^{-2} (*b*) dyne cm^{-2}
(*c*) Nm^{-1} (*d*) dyne cm^{-1}.

34. Bulk modulus of elasticity (k) and compressibility (c) of a material are related with each other as

(*a*) $k.c=2$ (*b*) $k.c=1$

(*c*) $k=c$ (*d*) $\frac{k}{c}=a$ constant.

35. A wire is replaced by another one of the same length and material but of twice the diameter. How will this effect the elongation under a given load ?

(*a*) Four rimes (*b*) Two times

(*c*) One-fourth (*d*) One-half.

36. The two wires of same diameter and length are loaded with the same weight. If the ratio of extension of the two wires is 1 : 2, what will be the ratio of their elasticity :

(*a*) 1 : 2 (*b*) 1 : 4

(*c*) 4 : 1 (*d*) 2 : 1.

37. Which is most elastic ?

(*a*) Steel (*b*) Iron

(*c*) Rubber (*d*) Putty. [*C.P.M.T. 1988*]

38. 4 metre wire of cross-section area $1{\cdot}2\times10^{-4}$ m^2 is expanded by $4{\cdot}8\times10^3$ N force, the stress is given by

(*a*) 4×10^7 N/m^2 (*b*) 2×10^7 N/m^2

(*c*) 6×10^7 N/m^2 (*d*) None. [*C.P.M.T. 1988*]

□□

9

Properties of Liquids

IMPORTANT FORMULAE

1. Pressure, $P=\frac{F}{A}$; F=thrust, A=area

2. Pressure of a fluid column of height h,

$P=h\rho g$; ρ=Density of fluid
g=Acc. due to gravity.

3. In a hydraulic lift,

(*a*) $\frac{W_2}{W_1}=\frac{R_2^2}{R_1^2}$; where W_2=weight on press plunger
W_1=weight on pump plunger
R_2=Radius of press plunger
R_1=Radius of pump plunger

(*b*) Mechanical advantage of a lever

$=\frac{\text{power arm}}{\text{weight arm}}$

4. Relative density has no unit and it is numerically equal to density in *c.g.s.* system.

5. Archimedes' principle :

(*a*) Relative density of a solid heavier than and insoluble in water,

$\text{R.D.}=\frac{W_1}{W_1-W_2}$; W_1=weight of solid in air
W_2=weight of solid in water

(*b*) Relative density of solids soluble in water,

$\text{R.D.}=\frac{W_1}{W_1-W_2}\times d$; W_1=wt. of solid in air
W_2=wt. of solid in liquid
d=R.D. of liquid

(*c*) Relative density of a solid lighter than water as cor
glass etc.

$$\text{R.D} = \frac{W_1}{W_2 - W_3};$$

W_1=wt. of solid in air

W_2=wt. of solid in air and sinker in water

W_3=wt. of solid and sinker both in water

6. Surface tension of a liquid,

$$\sigma = \frac{F}{l};$$

F=Force

l=length

and also

$$\sigma = \frac{W}{A};$$

W=Work or surface energy

A=Surface area

7. Excess pressure :

(*i*) on the curved surface of a liquid,

$$p = \frac{2\sigma}{r};$$

r=radius of curvature of the curved surfaces

(*ii*) inside a liquid drop, $p = \frac{2\sigma}{r}$,

r=radius of the drop.

(*iii*) inside a soap bubble,

$$p = \frac{4\sigma}{r},$$ since it has two spherical surfaces one external and another internal.

8. Capillary rise,

$$h = \frac{2\sigma \cos \theta}{\rho g r};$$

ϑ=angle of contact

ρ=density of liquid

9. Coefficient of viscosity 'η' is defined as

$$F = -\eta A \frac{dv}{dx};$$

F=Tangential force of viscosity

$\frac{dv}{dx}$=Velocity gradient (change in velocity per unit change in height)

10. Poiseuille's formula :

Volume of liquid issuing out per sec. through *a* pipe of length '*l*' and radius '*r*' is given by

$$V=\frac{V'}{t}=\frac{\pi}{8}\frac{r^4}{\eta}\left(\frac{p}{l}\right)$$

p=Diff. in pressure at the two ends of the pipe.

11. Reynold number,

$$N_R=\frac{\rho vD}{\eta}$$

12. Stoke's law, $F=6\pi\ \eta r\ v_0$;

r=radius of sphere moving through the liquid.

v_0=terminal velocity

and $$v_0=\frac{2}{9}\ \frac{r^2g\ (\rho-\rho_0)}{\eta}\ ;$$

ρ=density of sphere

ρ_0=density of liquid

13. Equation of continuity,

$$A_1,\ v_1=A_2\ v_2$$

A_1, A_2=Area of cross-section

v_1, v_2=Velocity of flow of liquid.

14. Bernoulli's theorem,

$$p+\rho gh+\tfrac{1}{2}\rho v^2=a \text{ constant}$$

p=pressure energy per unit volume

ρgh=potential energy per unit volume

$\frac{1}{2}\rho\ v^2$=kinetic energy per unit volume.

15. Torricelli's theorem,

$$v=\sqrt{2\ gh}$$

16. In venturi meter, the volume of liquid flowing per sec. is given by

$$V=\frac{V'}{t}=A_1\ A_2\sqrt{\frac{2gh}{(A_1^2-A_2^2)}};$$

A_1, A_2=Area of cross-section

h=Diff. of pressure heads at A_1 and A_2.

SOLVED EXAMPLES

Example 1. *Calculate the atmospheric pressure in N/m² if it be equal to the height of mercury column (in the barometer) of 76 cm. and what will be the height of water column that has the same pressure. (The density ρ of mercury=13,600 kg/m³ and g=9·8 m/s²).*

Solution. Here we have $h=76\text{ cm}=0{\cdot}76\text{ m}$

$g=9{\cdot}8\text{ m/s}^2$, $\rho=13{,}600\text{ kg/m}^3$

Hence pressure, $p=hg\rho$

$=0{\cdot}76\text{ m}\times 9{\cdot}8\text{ m/s}^2\times 13{,}600\text{ kg/m}^3$

$=1{\cdot}013\times 10^5\text{ N/m}^2$

Height of water column for the same pressure,

$\rho'=0{\cdot}76\text{ m}\times 13{\cdot}6$

$=10{\cdot}336\text{ m}$

Alternative method : Height of water

$$\text{Column}=\frac{1{\cdot}013\times 10^5}{9{\cdot}8\times 10^3}\text{ m}$$

$$=10{\cdot}336\text{ m}$$

Example 2. *At what depth below the surface of water will the pressure be equal to 2 atmospheres if the atmospheric pressure be equal to 10^6 dynes/sq. cm. Given $g=981$ cms./sec.^{2}*

Solution. The pressure on the surface of water is one atmosphere. Hence the question is now to find the height of water column which will exert a pressure of one atmosphere. Let h be the height of this water column. Then

$$h\rho g=\text{one atmosphere}$$

$$=10^6\text{ dynes}$$

or

$$h=\frac{10^6}{\rho\times g}=\frac{10^6}{981}$$

$$=1019{\cdot}4\text{ cms.}$$

Example 3. *The neck and bottom of a bottle are ·5 cm and 4 cm in diameter respectively. If the cork in the neck is pressed with a force of 1 kg wt. when the bottle is full of oil, what is the force exerted on the bottom of the bottle ?*

Solution. Area of cross-section of neck$=\frac{\pi\times(\cdot 5)^2}{4}\text{cm}^2$

Area of the bottom $=\pi\times 2^2\text{ cm}^2$

The cork is pressed with a force of 1 kg wt.

$$\therefore\quad \text{Pressure}=\frac{1\times 4}{\pi\times(\cdot 5)^2}$$

$$\therefore\quad \text{Thrust at the bottom}=\frac{4}{\pi\times(\cdot 5)^2}\times(\pi\times 2^2)$$

$$=64\text{ kg wt.}$$

Example 4. *The area of a small piston of an hydraulic press is 1 sq. cm and of the large piston is 22 sq. cm. How much weight can be raised on the large piston by a force of 200 kg. acting on the small piston.*

Solution. Area of a small piston=1 sq. cm.

Force acting on this piston=200 kg. wt.

Pressure on this piston=200 kg wt./sq. cm.

Area of the bigger piston=20 sq. cm.

Thrust$=200\times20$

=4000 kg. wt.

Example 5. *The radii of press and pump plungers of a hydraulic press are 60 cm and 20 cm respectively. A force of 60 kg is applied to the end of a lever whose power arm is 60 cm. and weight arm is 15 cm attached to the press plunger. Calculate the load raised by the plunger.*

Solution. The force is applied to the end of a lever whose power arm is 60 cm. and weight arm is 15 cm.

∴ The mechanical advantage of lever$=\frac{60}{15}=4$

Hence force exerted on the pump plungers$=60\times4$

=240 kg. wt.

Area of the pump plunger$=\pi\times20^2=400\pi$

Area of the press plunger$=\pi\times60^2=3600\pi$

Pressure on the pump plunger$=\frac{240}{400\pi}$ kg wt /cm^2.

Force transmitted to the press plunger$=\frac{240}{400\pi}\times3600\pi$

=2160 kg.

∴ The load raised=2 60 kg.

Example 6. *How much will a body of 70 N weigh in water if it displaces 200 ml of water ?*

Solution. Weight of the body=70 N

Volume of water displaced=200 ml

=0·2 litre

∴ Mass of water displaced=0·2 kg.

Weight of water displaced$=0\cdot2\times9\cdot8$ N

=1·96 N

By Archimedes' principle the loss in weight of the body is equal to the weight of the water displaced by the body.

Therefore, weight of the body in water$=70-1\cdot96$

=68·04 N

Example 7. *A solid weighs 3 N in air, 2·5 N in water and 2·6 N in a liquid. Calculate the R.D. of the liquid.*

Solution. Wt. of solid in air $X = 3$ N

Wt. of solid in liquid $X' = 2.6$ N

Wt. of solid in water $X'' = 2.5$ N

Loss of wt. of the solid in liquid $= 3 - 2.6 = 0.4$ N

Loss of wt. of the solid in water $= 3 - 2.5 = 0.5$ N

$$\therefore \text{R.D. of the liquid} = \frac{0.4}{0.5}$$

$$= 0.8. \textbf{ Ans.}$$

Example 8. *A copper cube of mass 0.50 kg is weighed in water (density 10^3 kg/m³). The mass comes out to be 0.40 kg. Is the cube hollow or solid ?*

(Given, density of copper $= 8.96 \times 10^3$ kg/m³).

Solution. Let V be the volume of the cube Then by Archimedes' Principle :

Loss of weight in water = weight of water displaced

or $$(0.50 - 0.40) = V \text{ (density of water)}$$

or $$(0.10) = V \times 10^3$$

or $$V = \frac{0.1}{10^3} \text{ m}^3 = 10^{-4} \text{ m}^3.$$

$$\therefore \quad \text{Density of cube} = \frac{m}{V} = \frac{0.50}{10^{-4}} = 5 \times 10^3 \text{ kg/m}^3.$$

which is less than that of copper. Hence the cube must be hollow.

Example 9. *A piece of ice floats on water. What fraction of its volume will be above the surface of water ? You are given that the density of ice is 0.90×10^3 kg/m³.*

Solution. Let V be the total volume of the piece of ice and let V′ be the volume of the part inside water. By Archimedes' Principle (as applied to a floating body) :

Weight of body in air = weight of the water displaced

or $$V\ 0.90 \times 10^3 = V'\ 10^3\ g,$$

or $$\frac{V'}{V} = \frac{0.90 \times 10^3}{10^3} = 0.90$$

or $$\left(1 - \frac{V'}{V}\right) = (1 - 0.90)$$

$$\therefore \quad \frac{(V - V')}{V} = 0.10 = \frac{1}{10}$$

$\therefore$ $\frac{1}{10}$ of the ice block is above water.

Example 10. *When a solid is immersed wholly in water the loss of weight is 0.4 kg. What is the volume of the solid ?*

Solution. By Archimedes' Principle,

Wt. of water displaced = Loss in weight of solid

$\therefore$ Wt. of water displaced = 0·4 kg.

But wt. of water displaced = Volume of water displaced × density of water

$$\therefore \quad 0{\cdot}4 = V \times 10^3$$

$$V = 0{\cdot}4 \times 10^{-3}\ \text{m}^3$$

The same will be the volume of the solid.

Example 11. *A solid weighs 10 N in air. Its weight decreases by 2 N when weighed in water. What is the density of solid ?*

Solution. Wt. in air = 10 N

Loss of wt. in water = 2 N

$$\text{R. Density} = \frac{\text{Wt. in air}}{\text{Loss of wt. in water}} = \frac{10}{2} = 5$$

$$\text{Now} \quad \text{R.D.} = \frac{\text{Density of substance}}{\text{Density of water at } 4^\circ\text{C}}$$

$$\therefore \quad 5 = \frac{\text{Density of solid}}{1000\ \text{kg/m}^3}$$

$\therefore$ Density of solid = 5000 kg/m³. **Ans.**

Example 12. *A solid weighs 6 kg in air. If its density is 2000 kg/m³. What will be its apparent weight in water ?*

[*A.I.S.S.E., 1981 Comptt.*]

Solution. Mass of the solid $m = 6$ kg

Density of the solid $d = 2000\ \text{kg/m}^3$

$$\therefore \quad \text{Volume of the solid } V = \frac{m}{d} = \frac{6}{2000}\ \text{m}^3$$

$$\text{Volume of water displaced} = \frac{6}{2000}\ \text{m}^3$$

$$\text{Density of water} = 1000\ \text{kg/m}^3$$

$$\therefore \quad \text{Mass of water displaced} = V \times d = \frac{6}{2000} \times 1000 = 3\ \text{kg.}$$

By Archimedes' Principle : Loss of weight in water is equal to the weight of water displaced.

$\therefore$ Apparent weight in water = 6 kg − 3 kg = 3 kg.

Example 13. *A liquid drop of diameter 'D' breaks up into 64 tiny drops. Find the resulting change in the energy.*

Solution. Let the radius of each tiny drop be 'r'. Volume of 64 tiny drops=Volume of one big drop.

$$64\times\frac{4}{3}\pi r^3=\frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$

$$r^3=\frac{1}{4^3}\left(\frac{D}{2}\right)^3$$

$$\therefore \quad r=\frac{D}{8}$$

$\therefore$ Change in surface area,

$$\Delta S=64\times4\pi r^2-4\pi\left(\frac{D}{2}\right)^2$$

$$=64\times4\pi\left(\frac{D}{8}\right)^2-4\pi\left(\frac{D}{2}\right)^2$$

$$=4\pi\left[D^2-\frac{D^2}{4}\right]$$

$$=3\pi D^2.$$

$\therefore$ Resulting change in the energy,

$$\Delta E=\sigma\Delta S$$

$$=\sigma\,(3\pi\,D^2)$$

$$=3\pi\,\sigma\,D^2.$$

Example 14. *Calculate the work done in blowing a soap bubble of diameter 4 cm. Surface tension of soap solution=3×10^{-2} Nm^{-1}.*

Solution. Surface area of bubble,

$$A=2\times4\pi r^2$$

$$=2\times4\times\frac{22}{7}\times(0{\cdot}02)^2\ \text{m}^2$$

$$=0{\cdot}01\ \text{m}^2$$

$\therefore$ Work done $W=\sigma\,A$

$$=3\times10^{-2}\times{\cdot}01=0{\cdot}0003\ \text{J}.$$

Example 15. *If a number of little drops of mercury all of the same radius 'r' collapse to form a single big drop of radius 'R'. Show that rise in temperature is given by*

$$\Delta T=\frac{2\sigma}{SJ\rho}\left(\frac{1}{r}-\frac{1}{R}\right)$$

where σ, S and ρ are surface tension, specific heat and density of liquid respectively.

Solution. $n\times\frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3,$

$$\therefore \quad n=\left(\frac{R}{r}\right)^3$$

∴ Decrease in surface area,

$$\Delta S=[n\times 4\pi r^2-4\pi R^2]$$

$$=4\pi\left[\left(\frac{R}{r}\right)^3 r^2-R^2\right]$$

$$=4\pi R^2\left[\frac{R}{r}-1\right]$$

∴ Energy released,

$$\Delta E=\sigma\Delta S$$

$$=\sigma 4\pi R^3\left[\frac{1}{r}-\frac{1}{R}\right]$$

∴ Heat produced,

$$\Delta Q=\frac{\Delta E}{J}$$

$$\Delta Q=\frac{4\pi\ \sigma R^3}{J}\left(\frac{1}{r}-\frac{1}{R}\right) \quad \ldots(1)$$

Also

$$\Delta Q=mS\ \Delta T=\left(\frac{4}{3}\pi\ R^3\rho\right)S\ \Delta T \quad \ldots(2)$$

$$\therefore \quad \frac{4}{3}\pi R^3\ \rho\ S\ \ \Delta T=\frac{4\pi\ \sigma\ R^3}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$$

$$\boxed{\Delta T=\frac{3\sigma}{\rho\ S\ J}\left(\frac{1}{r}-\frac{1}{R}\right)}$$

Example 16. *Three capillaries of lengths l_1, l_2 and l_3 and radii r_1, r_2 and r_3 respectively are joined in series. Show that the volume of liquid flowing per second in given by*

$$v=\frac{\pi p}{8\eta\left(\frac{l_1}{r_1^4}+\frac{l_2}{r_2^4}+\frac{l_3}{r_3^4}\right)}$$

where p is the pressure difference across the combination of capillaries.

Solution. Let p_1, p_2 and p_3 be the pressure differences across the three capillaries as shown in the figure.

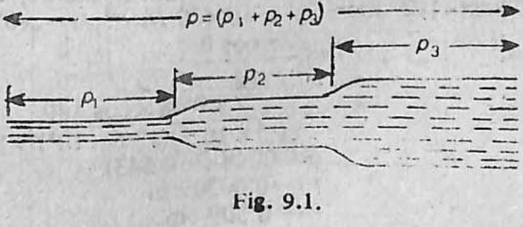

Fig. 9.1.

As the same volume of liquid is flowing through each capillary, we have according to Poiseuille's formula,

$$v=\frac{\pi p_1 r_1^4}{8\eta l_1}=\frac{\pi p_2 r_2^4}{8\eta l_2}=\frac{\pi p_3 r_3^4}{8\eta l_3}$$

$$\therefore \qquad p_1=\frac{8\eta l_1}{\pi r_1^4}v\ ;$$

$$p_2=\frac{8\eta l_2}{\pi r_2^4}v\ ;$$

and

$$p_3=\frac{8\eta l_3}{\pi r_3^4}\ v$$

∴ The effective pressure difference '*p*' cross the combination will be

$$p=(p_1+p_2+p_3)$$

$$p=\frac{8\eta l_1}{\pi r_1^4}\ v+\frac{8\eta l_2}{\pi r_2^4}\ v+\frac{8\eta l_3}{\pi r_3^4}\ v$$

or

$$p=\frac{8\eta}{\pi}\ v\ \left(\frac{l_1}{r_1^4}+\frac{l_2}{r_2^4}+\frac{l_3}{r_3^4}\right)$$

$$\therefore \qquad v=\frac{\pi p}{8\eta\left(\frac{l_1}{r_1^4}+\frac{l_2}{r_2^4}+\frac{l_3}{r_3^4}\right)}.$$

Example 17. *What would be the pressure inside an air bubble of 0·2 mm in diameter situated just below the surface of water? (Surface tension of water is 0·07 Nm^{-1}, atmospheric pressure= 1·013×10^5 N/m^2).*

Solution. The pressure,

P=Atmospheric pressure+hydrostatic pressure+excess pressure

$$=P_0+h\rho g+\frac{2\sigma}{r}$$

$$=1{\cdot}013\times10^5+(0{\cdot}1\times10^{-3})\times10^3\times9{\cdot}8+\frac{2\times0{\cdot}07}{0{\cdot}1\times10^{-3}}$$

$$=1{\cdot}0272\times10^5\ \text{Nm}^{-2}.$$

Example 18. *The internal diameter of the tube of a mercury barometer is 3·00 mm. Find the corrected reading of the barometer after allowing for the error due to surface tension, if the observed reading is 76·00 cm. (For mercury σ=0·480 Nm^{-1}, θ=140°).*

Solution,

$$h=\frac{2\sigma\cos\theta}{r\rho g}$$

$$=\frac{2\times0{\cdot}48\times\cos 140}{(1{\cdot}5\times10^{-3})(13{\cdot}6\times10^3)\ 9{\cdot}8}$$

$$=0{\cdot}0048\ (-0{\cdot}643)$$

$$=-0{\cdot}00309\ \text{m}$$

$$=-0{\cdot}309\ \text{cm}$$

The negative sign indicates that the reading of the barometer is lower than the correct value by 0·309 cm.

$\therefore$ Correct reading$=76\ 00+0·309$

$=76·309$ cm.

Example 19. *Two mercury drops of radii a and b coalesce to form a single drop of radius c. If external pressure is P, prove that the surface tension of mercury is given by*

$$\sigma=\frac{P(c^3-a^3-b^3)}{2(a^2+b^2-c^2)}.$$

Solution. Since the two drops coalesce under isothermal conditions and total mass remains constant,

$$(P_1V_1+P_2V_2)=P'V'$$

$$\left(P+\frac{2\sigma}{a}\right)\frac{4}{3}\pi a^3+\left(P+\frac{2\sigma}{b}\right)\frac{4}{3}\pi b^3=\left(P+\frac{2\sigma}{c}\right)\frac{4}{3}\pi c^3$$

$$2\sigma(a^2+b^2-c^2)=P(c^3-a^3-b^3)$$

$$\therefore\qquad \sigma=\frac{P(c^3-a^3-b^3)}{2(a^2+b^2-c^2)}.$$

Example 20. *A U-tube is such that the diameter of one limb is 2 mm that of the other is 5 mm. Find the difference in the level of water in the limbs. [Surface tension of water=0·07 Nm^{-1} and angle of contact=0].*

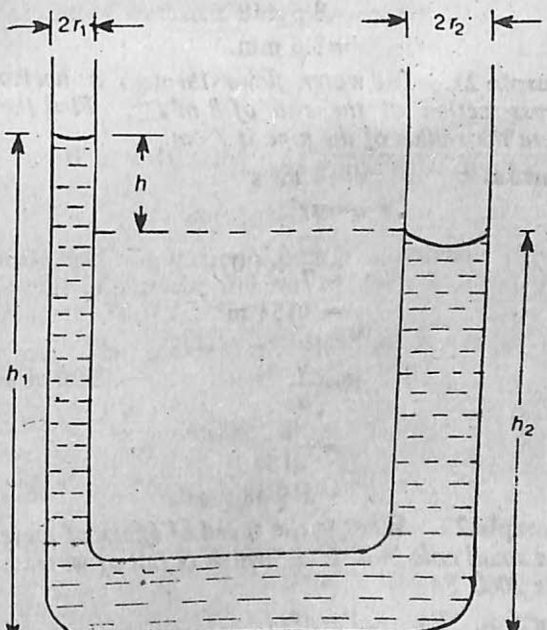

Fig. 9.2.

Solution. By $h=\dfrac{2\sigma\cos 0}{r\rho g}$

$$h_1=\frac{2\sigma}{r_1\rho g}$$

and $$h_2=\frac{2\sigma}{r_2\rho g} \qquad [\because \cos\theta=\cos 0=1]$$

$\therefore$ Difference in level of water in two limbs,

$$\therefore \quad h=h_1-h_2$$
$$=\frac{2\sigma}{r_1\rho g}-\frac{2\sigma}{r_2\rho g}$$
$$=\frac{2\sigma}{\rho g}\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$

Now $$r_1=1\times10^{-3}\text{ m}$$
$$r_2=2\cdot5\times10^{-3}\text{ m}$$
$$\sigma=0\cdot07\text{ Nm}^{-1}$$
$$\rho=10^3\text{ kg m}^{-3}$$

and $$g=9\cdot8\text{ ms}^{-2}$$

$$\therefore \quad h=\frac{2\times0\cdot07}{10^3\times9\cdot8}\left[\frac{1}{1\times10^{-3}}-\frac{1}{2\cdot5\times10^{-3}}\right]$$
$$=8\cdot6\times10^{-3}\text{ m}$$
$$=8\cdot6\text{ mm.}$$

Example 21. *The water flows through a horizontal pipe of varying cross-section at the rate of 8 m^3s^{-1}. Find the velocity at a section where the radius of the pipe is 7 cm.*

Solution. $$V=8\text{ m}^3\text{ s}^{-1}$$
$$a=\pi r^2$$
$$=\frac{22}{7}\times(\cdot07)^2$$
$$=\cdot0154\text{ m}^2$$

By $$V=a\,.\,v$$
$$v=\frac{V}{a}$$
$$=\frac{8}{\cdot0154}$$
$$=519\cdot48\text{ ms}^{-1}.$$

Example 22. *What is the speed of efflux of water which flows out from a small hole in a tank which is full of water. The pressure of water is 2000 Pa.*

Solution. By $P=h\rho g,$

$$h=\frac{P}{\rho g}$$

$$=\frac{2000}{1000\times 9{\cdot}8}=\frac{2}{9{\cdot}8}\text{ m}$$

∴ Speed of efflux of water

$$=\sqrt{2gh}$$

$$=\sqrt{2\times 9{\cdot}8\times\frac{2}{9{\cdot}8}}$$

$$=2\text{ ms}^{-1}.$$

Example 23. *A pipe is running full of water. At a certain place tapers from 20 cm radius to 8 cm. The difference between the pressure on the two sides of the taper is 75 cm of water column. Find the volume of liquid flowing per second.*

Solution. Here $A_1=\pi r_1^2$

$$=\pi(0{\cdot}2)^2$$

$$=0{\cdot}04\,\pi\text{ m}^2$$

$$A_2=\pi r_2^2$$

$$=\pi(0{\cdot}08)^2$$

$$=0{\cdot}0064\,\pi\text{ m}^2$$

$$h=0{\cdot}75\text{ m}$$

and $g=9{\cdot}8\text{ ms}^{-1}$

∴ Volume of water flowing per second,

$$V=A_1A_2\sqrt{\frac{2gh}{({}_1A^2-A_2^2)}}$$

$$=0{\cdot}04\,\pi\times 0{\cdot}0064\,\pi\sqrt{\frac{2\times 9{\cdot}8\times 0{\cdot}75}{[(0{\cdot}04\,\pi)^2-(0{\cdot}0064\,\pi)^2]}}$$

$$=0{\cdot}03868\text{ m}^3\text{ s}^{-1}.$$

Example 24. *What should be the maximum average velocity of water in a pipe of diameter 5·0 cm so that the flow is laminar [The viscosity of water $=0{\cdot}001\ Nm^{-2}\ s$].*

Solution. The maximum value of Reynold number for laminar flow is 2000.

$$\therefore\quad \frac{\rho v D}{\eta}=2000$$

$$\therefore\quad v=\frac{2000\,\eta}{\rho D}$$

$$=\frac{2000\times 0{\cdot}001}{10^3\times 0{\cdot}05}$$

$$=0{\cdot}04\text{ ms}^{-1}$$

$$=4\text{ cms}^{-1}.$$

Example 25. *At what speed will the velocity head of a stream of water be equal to* (*a*) *10 cm of* H_2O (*b*) *40 cm of Hg.*

Solution. (*a*) $g=9{\cdot}8\ \text{ms}^{-2}$,

$$h=10\ \text{cm}=0{\cdot}10\ \text{m}$$

The velocity head of stream of water,

$$\begin{aligned} v&=\sqrt{2gh}\\ &=\sqrt{2\times9{\cdot}8\times0{\cdot}1}\\ &=1{\cdot}4\ \text{ms}^{-1}. \end{aligned}$$

(*b*)

$$\begin{aligned} g&=9{\cdot}8\ \text{ms}^{-2}\\ h&=40\ \text{cm of Hg}\\ &=0{\cdot}40\times13{\cdot}6\ \text{m of } H_2O \end{aligned}$$

$$\therefore\quad \begin{aligned} v&=\sqrt{2gh}\\ &=\sqrt{2\times9{\cdot}8\times0{\cdot}4\times13{\cdot}6}\\ &=10{\cdot}326\ \text{ms}^{-1}. \end{aligned}$$

Example 26. *Water is conveyed through a horizontal pipe 15 cm in diameter and 5 km in length at the rate of 10 litre per second. Assuming only viscous resistance, calculate the pressure required.* [*For water,* $\eta=0{\cdot}001$ *PaS*]

Solution. Here

$$\begin{aligned} r&=7{\cdot}5\ \text{cm}=7{\cdot}5\times10^{-2}\ \text{m}\\ l&=5\ \text{km}=5\times10^{3}\ \text{m}\\ V&=20\ \text{litre s}^{-1}\\ &=20\times10^{-3}\ \text{m}^3\ \text{s}^{-1}\\ \eta&=0{\cdot}001\ \text{PaS} \end{aligned}$$

$\therefore$ By

$$V=\frac{\pi}{8}\frac{pr^4}{\eta l}$$

$$\begin{aligned} p&=\frac{8\eta l\,V}{\pi r^4}\\ &=\frac{8\times0{\cdot}001\times5\times10^{3}\times20\times10^{-3}}{3{\cdot}14\times(7{\cdot}5\times10^{-2})^4}\\ &=8{\cdot}05\times10^{3}\ P_a. \end{aligned}$$

Example 27. *A pilot tube is mounted on an aeroplane wing to measure the speed of the plane. The Tube contains alcohol and shows a level difference of 50 cm. What is the speed of the plane relative to air?* (*Density of alcohol=700 kg* m^{-3} *and density of air* $=1{\cdot}4$ *kg* m^{-3}).

Solution. Applying Bernoulli's theorem for the air at the points A and B respectively of pilot tube, we have

$$p_1+\tfrac{1}{2}\rho v_1^2=p_2+\tfrac{1}{2}\rho v_2^2$$

$$\therefore\quad (p_1-p_2)=\tfrac{1}{2}\rho(v_2^2-v_1^2)$$

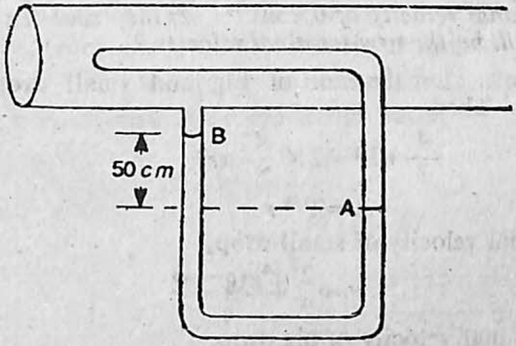

Fig. 9.3.

But $p_1 - p_2 = h\rho_0 g$

where $h = 0{\cdot}50 \text{ m}, \ \rho_0 = 700 \text{ kg m}^{-3}$

$g = 9{\cdot}8 \text{ ms}^{-2}$

and $v_1 = 0$ [Air at A is stand still]

$\therefore$ $h\rho_0 g = \frac{1}{2} \rho v_2^2$

or
$$v_2 = \sqrt{\frac{2h\rho_0 g}{\rho}}$$
$$= \sqrt{\frac{2 \times 0{\cdot}5 \times 700 \times 9{\cdot}8}{1{\cdot}4}}$$
$$= 70 \text{ ms}^{-1}$$
$$= 70 \times \frac{18}{5} \text{ kmh}^{-1}$$
$$= 252 \text{ kmh}^{-1}.$$

Example 28. *With what terminal velocity will an air bubble 0·8 mm in diameter rise in a liquid of viscosity 0·15 Nm⁻² S and specific gravity 0·9 ?* [*D.S.S.E. 1984*]

Solution. Here $r = 0{\cdot}4 \text{ mm} = 0{\cdot}4 \times 10^{-3} \text{ m}$

$\eta = 0{\cdot}15 \text{ Nm}^{-2} \text{ S}$

$\sigma = 0{\cdot}9 \times 10^3 \text{ kg m}^{-3}$

$= 900 \text{ kg m}^{-3}$

$\rho =$ density of air

$= 1{\cdot}3 \text{ kg m}^{-3}$

$\therefore$ Terminal velocity of rising air bubble,
$$v = \frac{2}{9} \frac{r^2 g(\sigma - \rho)}{\eta}$$
$$= \frac{2}{9} \frac{(\cdot 4 \times 10^{-3})^2 \, 9{\cdot}8 \, (900 - 1{\cdot}3)}{0{\cdot}15}$$
$$= 209 \times 10^{-5} \text{ ms}^{-1}.$$

Example 29. *Two equal drops of water are falling through air with terminal velocity of 0·2 ms^{-1}. If they coalesce to form a big drop, what will be the new terminal velocity ?*

Solution. Let the radii of big and small drops be R and r respectively. Then

$$\frac{4}{3}\pi R^3 = 2\times\frac{4}{3}\pi r^3$$

$$\therefore \quad R = 2^{1/3}\, r$$

Terminal velocity of small drop,

$$v = \frac{2}{9}\,\frac{r^2 g(\rho-\sigma)}{\eta}$$

and the terminal velocity of big drop,

$$V = \frac{2}{9}\,\frac{R^2 g(\rho-\sigma)}{\eta}$$

$$\therefore \quad \frac{V}{v} = \frac{R^2}{r^2}$$

$$= \frac{(2^{1/3}\, r)^2}{r^2}$$

$$\therefore \quad V = 2^{2/3}\times v$$

$$= 1{\cdot}59\times 0{\cdot}2$$

$$= 0{\cdot}318 \text{ ms}^{-1}.$$

Example 30. *Water stands at a depth 'H' in a tank whose side walls are vertical (Fig. 9.4). A hole is made on one of the wall at a depth 'h' below the water surface. Calculate :*

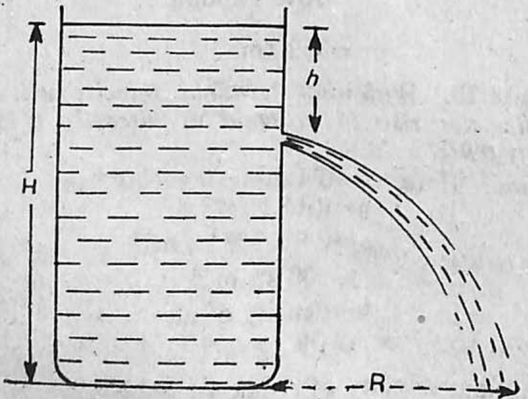

Fig. 9.4.

(*a*) *At what distance 'R' from the foot of the wall does the stream of water strike the floor ?*

(*b*) *For what value of h is the range maximum ?*

Solution. (*a*) Applying Bernoulli's theorem for water at the free surface and at the hole, we have

$$H\rho g+p_1+\tfrac{1}{2}\rho(0)^2=(H-h)\rho g+p_2+\tfrac{1}{2}\rho V^2$$

But $p_1=p_2$ (each equal to atmospheric pressure)

$\therefore$ $V=\sqrt{2gh}$, along horizontal direction.

$\therefore$ For vertical downward motion of water,

by
$$s=v_it+\tfrac{1}{2}at^2$$
$$(H-h)=0+\tfrac{1}{2}\times gt^2$$

$$\therefore \quad t=\sqrt{\frac{2(H-h)}{g}}$$

$$\therefore \quad R=V\times t$$
$$=\sqrt{2gh}\sqrt{\frac{2(H-h)}{g}}$$
$$R=2\sqrt{h(H-h)}$$

(*b*)
$$R=2\sqrt{h(H-h)}$$
$$=2\sqrt{hH-h^2}$$
$$=2\sqrt{\left(\frac{H}{2}\right)^2-\left(\frac{H}{2}\right)^2+hH-h^2}$$

or
$$R=2\sqrt{\left(\frac{H}{2}\right)^2-\left(\frac{H}{2}-h\right)^2}$$

$\therefore$ For R to be maximum,
$$\left(\frac{H}{2}-h\right)=0$$

$$\therefore \quad \boxed{h=\frac{H}{2}}$$

Example 31. *A glass tube of 1 mm bore is dipped vertically into a container of mercury, with its lower end 2 cm below the mercury surface. What must be the gauge pressure of air in the tube to blow a hemispherical bubble at its lower end ? (σ for mercury$=0{\cdot}49$ Nm^{-1}).*

Solution. The gauge pressure of air in the tube,

$$p=\frac{2\sigma}{r}+h\rho g$$
$$=\frac{2\times0{\cdot}49}{(0{\cdot}5\times10^{-3})}+0{\cdot}02\times(13{\cdot}6\times10^3)\times9{\cdot}8$$
$$p=4625{\cdot}6\ \text{Nm}^{-2}$$

Example 32. *A sphere is dropped under gravity through a fluid of viscosity η. Taking the average acceleration as half of initial* *acceleration, show that the time to attain the terminal velocity is independent of fluid density.*

Solution. Initial acceleration of the sphere

$$= g\,\frac{(\rho-\sigma)}{\rho}$$

∴ Average acceleration,

$$a = \tfrac{1}{2}\,g\,\frac{(\rho-\sigma)}{\rho}$$

We know terminal velocity,

$$V_f = \frac{2}{9}\,r^2 g\,\frac{(\rho-\sigma)}{\eta}$$

∴ By

$$V_f = V_i + at$$

$$t = \frac{V_f - V_i}{a} = \frac{V_f}{a} \qquad [\because\ V_i = 0]$$

∴ Time to attain the terminal Velocity,

$$t = \frac{\frac{2}{9}\,r^2 g\,\frac{(\rho-\sigma)}{\eta}}{\frac{1}{2}\,g\,\frac{(\rho-\sigma)}{\rho}}$$

$$= \frac{4}{9}\,\frac{r^2\rho}{\eta}$$

which is independent of fluid density (σ).

EXERCISE 9

1. Calculate the atmospheric pressure in N/m² when the barometric height is 750 mm.

2. Calculate the height of an oil barometer when the mercury barometer stands at 76 cms. Density of mercury is 13·6 and that of oil is 0·85 g/c.c.

3. A lift pump is used to pump kerosene of relative density 0·8 from a lower tank into the upper tank. What is the maximum possible height of the upper tank above the lower tank, if the atmospheric pressure is 76 cms. of mercury column.

4. A force of 50 kilograms is applied to the smaller piston of a hydraulic press. Neglecting friction, find the force exerted on the large piston ; the diameters of the pistons being 2 and 20 cms respectively. [*P.U. 1925*]

5. The diameters of two pistons of a hydraulic press are 1 and 6 cms respectively. What multiplication of force will be produced ? [*Delhi H.S. 1949*]

6. The two plungers of a hydraulic press have diameters of 2·5 cm and 30 cm. respectively. The downward force on the smaller plunger is 56 kg wt. What is the force exerted by press plunger. If the stroke of the smaller piston is 1 cm ? Through what distance will the press plunger rise after 36 strokes. [*Pat. U. 1946 S*]

7. The cross-section of the two pistons in a hydraulic press are 2 and 150 sq. cms. respectively. Calculate the minimum force necessary to support a weight of 3 metric ton placed on the broader base of the press.

8. The power arm of the lever of a Bramah press is 1 m and its weight arm 10 cm. Find the force applied at the end of the lever to support a load of 1 metric ton placed on the press plunger. The diameter of the press plunger is 10 times that of pump plunger.

9. What is the weight of a lead sphere of density 11·4 gms./c.c. having a radius of 2·1 cms ?

10. A lump of rock salt having a volume of 2 c.c. is dissolved in 90 c.c. of water without increasing the volume. If the density of the rock salt is 2·16 gms./c.c. what is the density of the solution ?

11. A piece of glass weighs 8·6 grams in air, 5·17 gms. in water and 6·4 grams in alcohol. Find the specific gravity of alcohol and glass.

12. A body weighs 60 gms. in vacuum and 42 grams in water. FInd the volume and specific gravity.

13. A substance weighs 6·02 grams in air and 0·14 gram in oil. A glass stopper weighs 23·84 grams in air, 13·34 grams in water and 15·02 grams in the same oil. Find the density of the substance.

14. A sinker which weighs 25·23 grams in water is fastened to a piece of wax and the two together weigh 24·106 grams in water. If the wax weighs 11·375 grams in air, find its specific gravity.

15. Two bodies are in equilibrium when suspended in water from the arms of a balance. The mass of one body is 28 grams and its density is 5·6 gms/c.c. If the mass of the other body is 36 grams, what is its density ?

16. A silver ornament is suspected to be hollow. Its weight is 288·75 grams and can displace 30 c.c. of water. If the specific gravity of silver is 10·5, what is the volume of the cavity ?

17. The crown of Hiero weighed 10 kg. Archimedes found that it lost 0·625 kg. when immersed in water. The crown was made of gold and silver. Find the weights of these metals

present in the crown. Relative density of gold is 19·8 and that of silver is 10·5.

18. A test tube 30 cms. long and 0·5 sq. cm. in cross-section weighs 4 grams. 10 grams of mercury are poured into it. What will be the specific gravity of a liquid in which it floats with 2 cms. length of its stem above the surface.

19. A block of ice of specific gravity 0·9 floats in a vessel containing water and filled up to the brim. What fraction of volume of the block will be above the surface of water ?

20. Specific gravity of ice is 0·9 and that of sea water is 1·1. What fraction of the whole volume of the floating ice berg appears above the surface of sea ?

21. A cube of 5 centimetres edge is made of wood of specific gravity 0·8. Find the weight which when placed on it will just submerge it in liquid of specific gravity 0 9.

22. An iceberg of specific gravity 0·940 floats in sea water of sp. gravity 1·02. What fraction of the total volume is above the surface of water ?

23. When two mercury drops each of radius r are brought in contact, they form one drop. Calculate the energy released.

24. A water drop of radius 1 mm is sprayed into a million droplets of the same size. Find the work done. [Surface tension of water $=0\ 072\ \text{Nm}^{-1}$].

25. Calculate the work done in blowing an air bubble of radius 0·5 cm. in water. [Surface tension of water $=0{\cdot}07\ \text{Nm}^{-1}$].

26. Calculate the energy needed to expand a soap bubble from radius 10 cm to 15 cm. [Surface tension of soap solution $=0\ 035\ \text{Nm}^{-1}$].

27. A mercury drop of radius R breaks up into one million tiny drops of the same size. Calculate the change in energy of the surface of the drop if σ for mercury is $0{\cdot}49\ \text{Nm}^{-1}$.

[*Roorkee Entrance Exam. 1984*]

28. A liquid drop of diameter D is broken up into 27 tiny drops of equal size. Find the energy required.

29. What is the work done in breaking a mercury drop of radius 4 mm into 125 tiny drops of equal size. [Surface tension of mercury $=0{\cdot}49\ \text{Nm}^{-1}$]

30. Find the amount of work done in breaking a drop of petrol of volume of 1 c.c. into a thousand million drops of equal size. [Surface tension of petrol is $0{\cdot}026\ \text{Nm}^{-1}$].

31. Calculate the energy released when 64 droplets of water, each of radius 0·07 mm, coalesce to form one large drop. [Surface tension of water $=0{\cdot}07\ \text{Nm}^{-1}$]

32. Find the difference of air pressure between that inside and outside a soap bubble 5 mm in diameter. Assume surface tension to be 1·6 Nm^{-1}. *[A.I.S.S.E. 1979]*

33. A liquid drop of diameter 4 mm breaks into 1000 drops of equal size. Calculate the resultant change in the surface energy. (Surface tension of liquid=0·07 Nm^{-1}) *[D.S.S.E. 1979 Compt.]*

34. Calculate the work done required to break a spherical drop of liquid into eight drops of equal size. Express the result in terms of the radius of the large drop R and the suface tension of the liquid σ. *[D.S.S.E. 1980 Compt.]*

35. Calculate the work done in blowing a soap bubble of radius 5 cm. Surface tension of soap solution=3×10^{-2} Nm^{-1}. *[D.S.S.E. 1982]*

36. What should be the maximum average velocity in a tube of diameter 2·0 cm so that the flow is laminar ? The viscosity of water is 0·001 Nm^{-2}. *[D.S.S.E. 1985, A.I.S.S.E. 1981]*

37. U-tube with limbs of diameter 5·00 mm and 2·00 mm contains water of surface tension $7{\cdot}0\times10^{-2}$ Nm^{-1}, angle of contact is zero and density is $1{\cdot}0\times10^{3}$ kg m^{-3}. Find the difference in levels (g=10 ms^{-2}). *[D.S.S.E. 1984]*

38. A U-tube is such that the diameter of one limb is 0·4 mm and that of the other is 0·8 mm. Find the difference in the level of water in the limbs (σ=0·07 Nm^{-1}).

39. A capillary tube of radius 2·5 mm stands vertically in a cup containing a liquid of surface tension 0·035 Nm^{-1}. If density of liquid is 8 g/cc. and angle of contact is 0, find the rise of the liquid in the capillary.

40. What should be the radius of the capillary tube so that water rises to a height of 4 cm in it (σ=0·07 Nm^{-1}) ?

41. A liquid in a capillary tube of radius 2 mm. rises 6 mm. If the surface tension of liquid is 0·042 Nm^{-1}, find out the density of the liquid.

42. At what speed will the velocity head of a stream of water be equal to 40 cm ?

43. Water rises to height of 5 cm in a certain capillary tube. In the same tube the level of mercury surface is depressed by 1·54 cm. Compare the surface tension of water and mercury. (Density of mercury=13·6 g/cm^3 and the angle of contact for water is 0 and for mercury is 130°).

44. A mercury barometer's tube is 4 mm in diameter reads 76·2 cm. Calculate the error in reading due to surface tension of mercury. (Density of mercury is 13·6 g cm^{-3}, angle of contact is 140° and surface tension is 0·490 Nm^{-1}).

45. Water stands at a depth '*h*' in a tank whose side walls are vertical. A hole is made on one of the walls near the bottom of the tank. Find the velocity of the emerging stream. (Assume that the speed of water at the top is zero).
[*D.S.S.E. 1980 Compt.*]

46. A what distance from the foot of the tank does the emerging stream of water from a hole in the tank strike the floor if height of tank, full of water, is 5 m and the hole is 1 m below its top.

47. What is the speed with which water flows from a small hole 4·9 m below the water level in a full tank which is 5 m high ?

48. A glass tube of 1·4 mm bore is dipped vertically into a container of mercury 1 cm below the mercury surface. What must be the gauge pressure of air in the tube to blow a hemispherical bubble at its lower end (σ for mercury=0·49 Nm^{-1}).

49. A steel ball of diameter 4 mm falls through a liquid of density 1·2 g/c.c. Find the terminal velocity of the ball. (Given density of steel=8·2 g/c.c., coefficient of viscosity for liquid= 8·4 poise and g=980 cm s^{-2}). [*D.S.S E. 1980 Compt.*]

50. Calculate the terminal velocity of an air bubble of radius 0·5 mm rising in a liquid of viscosity 0·15 Nm^{-2} s. Density of the liquid=9 g/c.c.

51. Determine the radius of a drop of water falling in air if the terminal velocity of the drop is 1·2 cm s^{-1}. (Coefficient of viscosity for air=$1{\cdot}8\times10^{-3}$ PaS, density of air=1·3 kg m^{-3}).

52. A pilot tube is mounted on an aeroplane wing to measure the speed of the plane. The tube contains alcohol and shows a level difference of 40 cm. What is the speed of the plane relative to air (Assume specific gravity of alcohol=0·8 and density of air=1 kg m^{-3}).

53. Calculate the speed at which the velocity head of a stream of water is equal to (*a*) 50 cm of H_2O, (*b*) 50 cm of Hg.

54. Prove that the initial acceleration of a sphere of density 'ρ' dropped from rest into a liquid of density σ is given by

$$a=g\ \frac{(\rho-\sigma)}{\rho}$$

55. Eight spherical rain drops of equal size are falling vertically through air with terminal velocity of 0·01 ms^{-1}. What would be the velocity if these 8 drops were to combine to form one large spherical drop ? [*D.S.S.E., 1980*]

56. An air bubble of diameter 2 cm is allowed to rise through a long cylindrical column of a viscous liquid and travels at the rate of 0·21 cm s^{-1}. If density of liquid is 1·47 g/c.c., find the coefficient of viscosity (Neglect the density of air).
[*D.S.S.E., 1979*]

57. A water drop is observed to fall through a fluid of density of 1 kgm^{-3} with a constant velocity of $11{\cdot}1 \text{ ms}^{-1}$. What is the radius of the drop ? [Viscosity of fluid$=2\times10^{-5}$ PaS].

58. Two drops of water of same size are falling through air with terminal velocity of $0{\cdot}5 \text{ ms}^{-1}$. If they coelesce to form a big drop, what will be the new terminal viscosity ?

59. If the maximum speed of flow of blood through a capillary of radius $3{\cdot}0\times10^{-6}$ m and length 2 mm is $5\times10^{-4} \text{ ms}^{-1}$, calculate the pressure difference at its two ends. [The viscosity of blood is 4×10^{-3} PaS].

60. Blood flows in a capillary of radius 1×10^{-3} m and length 1·5 mm at a speed of $0{\cdot}5 \text{ mm s}^{-1}$. The pressure difference between the ends of the capillary is 450 Nm^{-2}. Calculate

(*a*) Volume of blood flowing per second; and

(*b*) Work done per second to maintain this flow.

61. The water entering a house flows with a speed of $0{\cdot}20 \text{ ms}^{-1}$ through a pipe of 2·4 cm inside diameter. If the pipe tapers to a diameter of 6 mm, what is the speed of water at this point ?

62. Water enters the filter pump at a pressure of $1{\cdot}5\times10^5 \text{ Nm}^{-2}$ at a rate $0{\cdot}18 \text{ m}^3 \text{ min}^{-1}$. The cross-sectional area of the pipe is $4{\cdot}00 \text{ cm}^2$. What should be the cross-sectional area of the constriction to achieve pressure as low as $3\times10^4 \text{ Nm}^{-2}$?

63. Water is escaping from a cistern by way of horizontal capillary tube 110 cm long and 0·2 mm in diameter at a distance 40 cm below the free surface of in the cistern. If coefficient of viscosity of water is 0·001 PaS, calculate the rate of flow of water.

64. Find the velocity of efflux of water from a tank in which pressure of water is 500 P*a*.

65. Find the velocity of efflux of a liquid of density 800 kgm^{-3} from a tank, in which the pressure of the liquid at the orifice is 480 Nm^{-2}.

66. A metal plate 5 cm square is placed on a 0·5 mm thick castor oil layer. If a force of 22500 dyne is needed to move the plate with a velocity of 3 cms^{-1}, calculate coefficient of viscosity of castor oil. [*Punjab, 1987*]

67. Calculate the terminal velocity of a rain drop of radius 0·3 mm. Take coefficient of viscosity of air$=1{\cdot}83\times10^{-5}$ P*a*S, density of air$=1{\cdot}3 \text{ kgm}^{-3}$, density of water$=1000 \text{ kgm}^{-3}$ and $g=9{\cdot}8 \text{ ms}^{-2}$. [*D.S.S.E., 1981, 1986*]

68. Water flows in a pipe of diameter 2·00 cm located in the basement of a building. The pipe rises to a height of 11 m to the second floor with the diameter of the pipe tapering to 1·00 cm.

Find the water pressure and the velocity in the pipe at the second floor, when the water pressure and velocity at the basement are 2×10^5 Nm^{-2} and 10 cms^{-1} respectively.

OBJECTIVE TYPE QUESTIONS

69. 20 water drops merge with each other to form a larger drop. In this process energy is
    (*a*) absorbed
    (*b*) liberated
    (*c*) neither absorbed nor liberated
    (*d*) released as a conversion of mass. [*D.P.M.T., 1982*]

70. Soap helps in cleaning of the clothes because
    (*a*) Chemicals of soap changes
    (*b*) It increases the surface tension of solution
    (*c*) It absorbs dirt
    (*d*) It lowers the surface tension of solution. [*D.P.M.T., 1983*]

71. Surface tension is defined as
    (*a*) work done in increasing the area of the surface by unity
    (*b*) force on unit length of an imaginary line on the surface of a liquid towards interior of the liquid
    (*c*) the free energy of the surface
    (*d*) none of them. [*D.P.M.T., 1983*]

72. When the temperature of a liquid increases, the angle of contact of the liquid
    (*a*) increases
    (*b*) decreases
    (*c*) remains the same
    (*d*) first decreases and then increases. [*D.P.M.T., 1983*]

73. Small liquid drops assume spherical shape because
    (*a*) of gravitational force
    (*b*) atmospheric pressure from all sides
    (*c*) adhesion
    (*d*) liquid surface tends to have minimum surface. [*D.P.M.T., 1983*]

74. An iron needle placed gently over the surface of water floats because of
    (*a*) iron is lighter than water
    (*b*) viscosity of water
    (*c*) surface tension of water
    (*d*) Buoyant force equals to the weight of the needle.

75. A liquid drop breaks into a large number of drops. The temperature of the liquid drop
(*a*) increases (*b*) decreases
(*c*) remains the same (*d*) none of them.

76. Terminal velocity is achieved when
(*a*) viscous force becomes equal to force causing the motion
(*b*) viscous force becomes zero
(*c*) viscous force becomes equal to the weight of the body
(*d*) none of them.

77. When Yamuna flows from open area through the space between the pillars of Yamuna bridge, its velocity
(*a*) decreases (*b*) increases
(*c*) remains the same (*d*) none of them.

78. The drying of hands by a towel is due to
(*a*) capillarity (*b*) viscosity
(*c*) evaporation (*d*) surface tension.

79. When some detergent is added to water, its surface tension
(*a*) increases (*b*) decreases
(*c*) remains the same (*d*) none of them.

80. Choose the one which does not involve capillarity
(*a*) absorption of ink by blotting paper
(*b*) rising of oil in a wick
(*c*) spray painting
(*d*) bunsen burner.

81. Choose the one which does not involve Bernoulli's theorem
(*a*) spray painting (*b*) oil rising in a wick
(*c*) lifting of aeroplane (*d*) bunsen burner.

82. The excess pressure inside a soap bubble is
(*a*) $\frac{r}{4\sigma}$ (*b*) $\frac{2\sigma}{r}$
(*c*) $\frac{3\sigma}{r}$ (*d*) $\frac{4\sigma}{r}$

83. Angle of contact for pure water and clean glass is
(*a*) 0 (*b*) 4°
(*c*) 8° (*d*) 18°.

84. Angle of contact for silver and pure water is
(*a*) 145° (*b*) 90°
(*c*) 45° (*d*) 18°.

85. When a capillary tube is dipped in a trough of mercury, the mercury......in capillary due to......

(*a*) rises, surface tension (*b*) falls, viscosity

(*c*) falls, surface tension (*d*) rises, viscosity.

86. It is easier to swim in sea water than that in river water because of

(*a*) sea water has more density

(*b*) sea water has less density

(*c*) sea water has salt

(*d*) sea water is more displaced.

87. Heavy iron ships can float on water while a piece of iron sinks down. It is because of

(*a*) ship has less density than piece of iron

(*b*) ship displaces less amount of water than that by piece of iron

(*c*) ship displaces much more amount of water than that by piece of iron.

(*d*) the weight of water displaced by ship is equal to its weight.

88. If weight of a body of 50 g is equal to the buoyant force acting on it. The apparent weight of the body is

(*a*) 49000 dyne (*b*) 100 g

(*c*) 50 g (*d*) 0.

89. The density of pure water at 4°C is

(*a*) 1000 kg m^{-3} (*b*) 100 kg m^{-3}

(*c*) 10 kg m^{-3} (*d*) 1 kg m^{-3}.

90. A body of mass 2 kg weighs zero when fully immersed in water. Its density is

(*a*) 1000 g cm^{-3} (*b*) 100 g cm^{-3}

(*c*) 10 g cm^{-3} (*d*) 1 g cm^{-3}.

91. If a person is standing close to a fast moving train. He is likely to fall

(*a*) away from the train

(*b*) towards the train

(*c*) in the direction of the train

(*d*) uneffected.

□□

10

Properties of Gases

IMPORTANT FORMULAE

1. Root mean square velocity of the molecules of the gas,

$$C=\sqrt{\frac{C_1^2+C_2^2+\dots C_n^2}{n}}$$

$$=\sqrt{\frac{3P}{\rho}}$$

where P=pressure of the gas
ρ=density of the gas.

2. Pressure of a gas,

$$P=\frac{1}{3}\,\frac{M}{V}\,\overline{C^2}$$

$$=\frac{1}{3}\,\rho\,\overline{C^2}$$

M=Mass of the gas
V=Volume of the gas
$\overline{C^2}$=Mean square velocity of molecules of the gas.

3. Boyle's law, $P_1V_1=P_2V_2$;
P_1 and P_2=Initial and final pressure
V_1 and V_2=Initial and final volume.

4. Charle's law,

$$\frac{V_1}{T_1}=\frac{V_2}{T_2}\,;$$

T_1 and T_2=Initial and final temp. in Kelvin.

5. Pressure law,

$$\frac{P_1}{T_1}=\frac{P_2}{T_2}$$

6. Gas equation, $PV=nRT$;
P=Pressure of the gas
R=Gas constant=8·31 J mol^{-1} K^{-1}
T=Temp. in Kelvin.

7. Thermal velocity (*r.m.s.* velocity) of gas molecules,

$$C=\sqrt{\frac{3RT}{M}};$$

M=Molecular weight of the gas.

8. Energy per gram mole of a gas,

$$E=\frac{3}{2}RT$$

9. Boltzmann's constant,

$$K=\frac{R}{N};$$

N=Avogadro's Number

$=6{\cdot}023\times10^{23}$ mole^{-1}

10. Graham's law ;

$$\frac{C_1}{C_2}=\sqrt{\frac{\rho_2}{\rho_1}}$$

C_1, C_2=rate of diffusion

ρ_1, ρ_2=density.

11. $C_p-C_v=R$:

C_p=Molar specific heat at constant pressure

C_v=Molar specific heat at constant volume.

12. $C\propto\sqrt{T}$

SOLVED EXAMPLES

Example 1. *If three molecules have velocities 0·5, 1·1 and 2 km/s respectively. Calculate the relation of the root mean square speed and the average speed.*

Solution. Average speed,

$$C=\frac{0{\cdot}5+1{\cdot}1+2}{3}$$

$$=1{\cdot}2\text{ km s}^{-1}$$

Root mean square speed,

$$C_1=\sqrt{\frac{(0{\cdot}5)^2+(1{\cdot}1)^2+2^2}{3}}$$

$$=1{\cdot}35\text{ km s}^{-1}$$

$$\therefore\quad \frac{C}{C_1}=\frac{1{\cdot}2}{1{\cdot}35}=8:9.$$

Example 2. *Using the ideal gas equation. determine the value of R. (one gram molecule of a gas at S.T.P. occupies 22·4 litres).*

[*DSSE 1978*]

Solution.

$$P = 76 \text{ cm of Hg (at S.T.P.)}$$
$$= 0{\cdot}76 \text{ m of Hg}$$
$$= 0{\cdot}76\,(13{\cdot}6\times10^3)\times9{\cdot}8 \qquad (\because P = hdg)$$
$$= 1{\cdot}013\times10^5 \text{ N/m}^2$$
$$V = 22{\cdot}4 \text{ litres} = 22{\cdot}4\times10^{-3} \text{ m}^3$$
$$n = 1$$

and $T = 273$ K

By $PV = nRT$,

$$R = \frac{PV}{nT}$$
$$= \frac{1{\cdot}013\times10^5\times22{\cdot}4\times10^{-3}}{1\times273}$$
$$= 8{\cdot}31 \text{ J mole}^{-1}\text{ K}^{-1}.$$

Example 3. *Calculate the root mean square velocity of the molecules of air at 27°C.*

Solution. Air pressure at N.T.P.

$$P = hdg = {\cdot}76\times(13{\cdot}6\times10^3)\times9{\cdot}8$$
$$= 1{\cdot}013\times10^5 \text{ N/m}^2$$

and density, $\rho = 1{\cdot}293$ kg m^{-3}

∴ Root mean square speed at N.T.P.,

$$C = \sqrt{\frac{3P}{\rho}}$$
$$= \sqrt{\frac{3\times1{\cdot}013\times10^5}{1{\cdot}293}}$$
$$= 4{\cdot}848\times10^2 \text{ ms}^{-1}$$
$$= 0{\cdot}485 \text{ km s}^{-1}$$

∴ $C = 0{\cdot}485$ km s^{-1} at $T = 273$ K

Let the root mean square speed at 27°C be C', then,

$$T' = 27 + 273 = 300 \text{ K}$$

∴ By

$$\frac{C'}{C} = \sqrt{\frac{T'}{T}}$$
$$C' = \sqrt{\frac{300}{273}}\times0{\cdot}485$$
$$= 0{\cdot}508 \text{ km s}^{-1}$$

Example 4. (*a*) *Calculate the root mean square speed and mean K.E. of one gram molecule of hydrogen at S.T.P., given that the density of hydrogen is 0·09 kg m^{-3}.*

(*b*) *Calculate Avogadro's number given that the mass of a molecule of hydrogen is 3·34×10^{-27} kg.*

(*c*) *Calculate Boltzmann's constant.*

Solution. (*a*) At S.T.P.,

$$P=0{\cdot}76\times(13{\cdot}6\times10^3)\times9{\cdot}8=1{\cdot}013\times10^5\ \text{Nm}^{-2}$$

$$T=273\ \text{K}$$

and $$\rho=0{\cdot}09\ \text{kg m}^{-3}$$

∴ By $$C=\sqrt{\frac{3P}{\rho}}$$

$$C=\sqrt{\frac{3\times1{\cdot}013\times10^5}{0{\cdot}09}}=1{\cdot}837\times10^3\ \text{ms}^{-1}$$

$$=1{\cdot}837\ \text{km s}^{-1}$$

Now we know for H_2 molecule,

$$m=2{\cdot}016\ g=2{\cdot}016\times10^{-3}\ \text{kg}$$

∴ $$\text{K.E.}=\tfrac{1}{2}\,mc^2$$

$$=\tfrac{1}{2}\times2{\cdot}016\times10^{-3}\,(1{\cdot}837\times10^3)^2$$

$$=3387\ \text{J.}$$

(*b*) Avogadro's number,

$$N=\frac{0{\cdot}002016}{3{\cdot}34\times10^{-27}}$$

$$=6{\cdot}036\times10^{23}\ \text{molecules.}$$

(*c*) $$R=8{\cdot}31\ \text{J mole}^{-1}\ \text{K}^{-1}$$

and $$N=6{\cdot}036\times10^{23}\ \text{molecules}$$

$$K=\frac{R}{N}$$

$$=\frac{8{\cdot}31}{6{\cdot}036\times10^{23}}$$

$$=1{\cdot}377\times10^{-23}\ \text{J mole}^{-1}\ \text{K}^{-1}.$$

Example 5. *Calculate in calories the specific heat at constant volume for a monoatomic gas and also its specific heat at constant pressure. (Given $R=8{\cdot}31$ J mole^{-1} k^{-1} and $J=4{\cdot}18$ J/cal).*

Solution. For a monoatomic gas, sp. heat at constant volume,

$$C_v=\frac{3}{2}R$$

∴ $$C_v\ \text{(in cal.)}=\frac{3}{2}\,\frac{R}{J}$$

$$=\frac{3\times8{\cdot}13}{2\times4{\cdot}18}$$

$$=2{\cdot}982\ \text{cal mole}^{-1}\ \text{k}^{-1}$$

∴ $$C_p=(C_v+R/J) \qquad (\therefore\ C_p-C_v=R/J)$$

$$=\left(\frac{3}{2}\,\frac{R}{J}+\frac{R}{J}\right)$$

$$=\frac{5}{2}\;\frac{R}{J}=\frac{5\times 8{\cdot}31}{2\times 4{\cdot}18}$$
$$=4{\cdot}97 \text{ cal mole}^{-1}\text{ k}^{-1}.$$

Example 6. *Calculate the temperature at which the nitrogen molecules will have the same r.m.s. velocity as the hydrogen molecule at 127°C. (Molecular weight of nitrogen is 28 and that of hydrogen is 2).*

Solution. We know,

$$C=\sqrt{\frac{3RT}{M}}$$

$$\therefore \quad C_N=\sqrt{\frac{3R\,T_N}{M_N}}$$

and
$$C_H=\sqrt{\frac{3R\,T_H}{M_H}}$$

But
$$C_N=C_H$$

$$\therefore \quad \frac{T_N}{M_N}=\frac{T_H}{M_H}$$

$$T_N=\frac{M_N}{M_H}T_H$$
$$=\frac{28}{2}(273+127)$$
$$T_N=5600\text{ K}$$
$$=(5600-273)°\text{C}$$
$$=5327°\text{C}.$$

Example 7. *One gram molecule of oxygen at 17°C and one atmospheric pressure is enclosed in a vessel (a) Assuming molecules to be moving with C_{rms}, find the number of collisions per second which the molecule make per unit area of the vessel wall. (b) The vessel is thermally insulated and moved with a constant speed V_0. It then stopped, the process result in the rise of the temperature by 1°C Calculate the speed V_0.* [*I.I.T. J.E.E.* 1983]

Solution. (*a*) Pressure of the gas in vessel,

$$P=n\,(2\,mC_{rms})$$

where n = No. of collisions per unit area per sec. of the vessel wall.

$(2m\,C_{rms})$ = change in momentum of one molecule in one collision, m being mass of a molecule.

Now
$$m=\frac{M}{N}$$

and $$C_{rms}=\sqrt{\frac{3RT}{M}}$$

$\therefore$ $$n=\frac{PN}{2\,(3RTM)^{1/2}}$$

Now $$P=1\cdot013\times10^{5}\ N/m^2,$$
$$N=6\cdot023\times10^{26}\ \text{per kg mole}$$
$$R=8\cdot31\ J\ mole^{-1}\ k^{-1},$$
$$T=(273+17)=290\ K$$

and $$M=32\ g=\cdot032\ kg.$$

$\therefore$ $$n=\frac{(1\cdot013\times10^{5})\,(6\cdot023\times10^{26})}{2\,(3\times8\cdot31\times290\times\cdot032)^{1/2}}$$
$$=2\times10^{30}\ \text{collision per m}^2\ \text{per s}$$

(*b*) $$V_0=C_{rms}\sqrt{\frac{2}{T}}$$
$$=\sqrt{\frac{3RT}{M}\times\frac{2}{T}}$$
$$V_0=\sqrt{\frac{6R}{M}}$$
$$=\sqrt{\frac{6\times8\cdot31}{\cdot032}}$$
$$=39\cdot47\ ms^{-1}.$$

EXERCISE 10

1. Four molecules have velocities 0·5, 1·2, 1·5 and 2·5 km s^{-1}. Calculate the average speed and root mean square speed of the molecules.
2. If 5 molecules of a gas have velocities 0·6, 0·9, 1·0, 2·0, 2·5 km s^{-1} Calculate the relation of the average speed and the root mean square speed.
3. If three molecules have speed v_1, v_2 and v_3 respectively, then what will be their average speed and the root mean square speed ? [*A.I.S.S.E. 1981*]
4. Four molecules of a gas have speeds 2, 4, 6 and 8 km/s respectively. Calculate their (*a*) average speed, (*b*) root mean square speed. [*A.I.S.S.E. 1982*]
5. Calculate the root mean square velocity of hydrogen gas at N.T.P. (density of hydrogen=0·089 kgm^{-3}).
6. At what temperature will the *r.m.s.* velocity of hydrogen molecules be double of its value at N.T.P. ?
7. Calculate the mean square speed of the oxygen molecules at (*a*) N.T.P. (*b*) 37°C (density of oxygen=1·424 kg m^{-3}).

8. Calculate C_p and C_v for Argon. (Given $R=8{\cdot}3$ J $mole^{-1}$ k^{-1} and $J=4{\cdot}18$ J/cal.)

9. Calculate C_p and C_v for Hydrogen. (Given $R=8{\cdot}3$ J $mole^{-1}$ k^{-1} and $J=4{\cdot}2$ J/cal.)

10. If *r.m.s.* velocity of hydrogen molecules at S.T.P. is 1·84 kms^{-1}, what is the *r m.s.* velocity of oxygen at S.T.P. ?

11. Calculate the temperature at which the oxygen molecule will have the same *r.m s.* velocity as the hydrogen molecules at 27°C. (Molecular weight of oxygen is 32 and that of hydrogen is 2).

12. If the Avogadro's number is $6{\cdot}06\times10^{23}$ per gram mole, what is the mass of a molecule of hydrogen in S.I. unit ? Calculate Boltzmann's constant if $R=8{\cdot}3$ J $mole^{-1}$ K^{-1} ?

[*A.I S.S.E 1980*]

13. Calculate the *r.m.s.* velocity of hydrogen at N.T P. Density of hydrogen is 0·089 kg/m^3 and normal pressure is $1{\cdot}01\times10^5$ N/m^2.

14. Find the root mean square speed of argon molecules at 60°C, given that the *r.m.s.* speed of oxygen molecules is 0·420 kms^{-1} at 1092 k. Molecular weight of oxygen and argon are 32 and 40 respectively. [*A.I.S.S.E. 1986*]

15. At what temperature will the root mean square speed of a gas molecules will be half of its value at 0°C ?

16. Calculate the volume occupied by 6·4 g of oxygen at 76 cm of Hg at 27°C. [$R=8{\cdot}31$ J $mole^{-1}$ k^{-1}].

17. Calculate the ratio of specific heats for nitrogen, given that the specific heat at constant pressure is 0·235 and density at N.T.P. is 1·234 g/litre.

18. Calculate the root mean square speed of hydrogen at 127°C. Mass of a hydrogen molecule is $3{\cdot}34\times10^{-24}$ g and Boltzmann's constant is $1{\cdot}38\times10^{-23}$ J/mole/k.

19. Calculate the difference in specific heats of a gram of helium gas. The molecular weight of helium = 4 g $mole^{-1}$.

20. Calculate the K.E. of the molecules in 10 g of ammonia gas at 20°C. Molecular weight of ammonia is 17·03 g

OBJECTIVE TYPE QUESTIONS

21. Four molecules have speed, 1 kms^{-1}, 2 kms^{-1}, 3 kms^{-1} and 4 kms^{-1}. Their average speed is
(*a*) 1·5 kms^{-1} (*b*) 2 kms^{-1}
(*c*) 2·5 kms^{-1} (*d*) 2·7 kms^{-1}.

22. Four molecules have speed 1 kms^{-1}, 2 kms^{-1}, 3 kms^{-1} and 4 kms^{-1}. Their root mean square speed will be

(*a*) 1·5 kms^{-1} (*b*) 2 kms^{-1}
(*c*) 2·5 kms^{-1} (*d*) 2·7 kms^{-1}.

23. The postulates of kinetic theory of gases were given by
(*a*) Planck (*b*) Clausius and Maxwell
(*c*) Bohr (*d*) Avogadro.

24. For one gram mole of a triatomic gas, the value of C_v is
(*a*) 9/2 R (*b*) 7/2 R
(*c*) 5/2 R (*d*) 3/2 R.

25. The number of degree of freedom of oxygen is
(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 5.

26. The unit of Boltzmann's constant is
(*a*) Joule $\text{mole}^{-1}\ \text{k}^{-1}$ (*b*) Joule k^{-1}
(*c*) Joule mole^{-1} (*d*) Joule.

27. When the absolute temperature of a gas becomes four times, the *r.m.s.* speed of the gas molecules will become
(*a*) one-fourth (*b*) one-half
(*c*) double (*d*) four times.

28. If pressure in a closed vessel is reduced by drawing out some gas, mean free path of the molecules
(*a*) is decreased (*b*) is increased
(*c*) remain unchanged
(*d*) increase or decrease depending on the nature of gas.
[*B.H.U. 1983*]

29. Pressure exerted by a perfect gas is proportional to the
(*a*) mean square velocity of the gas molecules.
(*b*) root mean square velocity of gas molecules.
(*c*) total kinetic energy of all the molecules in a unit volume of the gas. [*A.I.I.M.S. 1983*]

30. At absolute zero of temperature
(*a*) The molecular energy is minimum and not zero.
(*b*) All the molecules come to rest and have zero energy.
(*c*) P.E. is zero.
(*d*) Molecules travel faster. [*D.P.M.T. 1982*]

31. Moon has no atmosphere because
(*a*) R.M.S. velocity of all gases is more than the escape velocity from moon's surface.
(*b*) Its surface is not smooth.
(*c*) It is quite far away from the earth.
(*d*) It does not have population of plants. [*D.P.M.T. 1983*]

32. The thermal capacity of 40 g of aluminium having specific heat 0·2 is
(*a*) 0·4 cal (*b*) 8 cal
(*c*) 160 cal (*d*) 4000 cal. [*D.P.M.T. 1982*]

33. Specific heat at constant volume for one gram mole of a diatomic gas is equal to
(*a*) R (*b*) 3/2 R
(*c*) 5/2 R (*d*) 7/2 R. [*D.P.M.T. 1984*]

34. Which of the following obey both Charles' and Boyles' law ?
(*a*) Saturated vapour (*b*) Unsaturated vapour
(*c*) Both (*d*) None. [*D.P.M.T 1984*]

35. A gas in a container A is in thermal equilibrium with another gas of the same mass in container B. If we denote the corresponding pressures and volumes by the suffixes A and B then which of the following statements is most likely to be true ?
(*a*) $P_A = P_B$, $V_A \neq V_B$ (*b*) $P_A \neq P_B$, $V_A = V_B$
(*c*) $P_A/V_A = P_B/V_B$ (*d*) $P_A V_A = P_B V_B$.
[*A.I.I.M.S. 1982*]

36. We have a jar A filled with a gas characterised by the parameters P, V, T and another as B, filled with parameters 2P, 4V and 2T, where the symbols have their usual meanings. The ratio of the number of molecules of jar A to those of B is
(*a*) 4 : 1 (*b*) 2 : 1
(*c*) 1 : 2 (*d*) 1 : 1 [*A.I.I.M.S 1982*]

37. A gas is heated from 27° to 327°C. As a result of the increase in temperature, the average K.E. of the molecules is
(*a*) halved (*b*) doubled
(*c*) increased by a factor of $\sqrt{2}$
(*d*) quadrupled. [*B.H.U. 1983*]

38. The temperature at which the kinetic energy of a gas will be half of that at room temperature 27°C is given by
(*a*) 123°C (*b*) −123°C
(*c*) 27°C (*d*) 13·5°C [*C.P.M.T 1988*]

39. At which temperature velocity of O_2 molecule will be equal to velocity of N_2 molecule at 0°C.
(*a*) 40°C (*b*) 93°C
(*c*) 39°C (*d*) cannot be calculated.
[*C.P.M.T. 1988*]

40. In an ideal gas the collision between the molecules is taken to be
(*a*) perfectly elastic (*b*) imaginary
(*c*) perfectly inelastic (*d*) none of the above.
[*C.P.M.T. 1988*]

□ □

11

Heat and Thermodynamics

IMPORTANT FORMULAE

1. Amount of heat gained or lost by a substance,
$\Delta Q = mS\ \Delta T$; m=mass of substance
S=specific heat of substance
ΔT=Rise or fall in temperature.

2. Specific heat at constant volume,

$$C_v = \frac{\Delta Q}{\Delta T}$$

3. Coefficient of linear expansion,

$$\alpha = \frac{\Delta L}{L \Delta T}$$

4. Length at t°C, $L_t = L_0 (1+\alpha t)$ L_0=Length at 0°C

5. Coefficient of cubical or volume expansion,

$$\beta = \frac{\Delta V}{V\ \Delta T}$$

6. Volume at t°C, $V_t = V_0 (1+\beta t)$ V_0=Volume at 0°C

7. Relation between α and β,

$$\beta = 3\alpha$$

8. Coefficient of thermal conductivity (K) is given by

$$\frac{\Delta Q}{\Delta t} = KA \frac{\Delta T}{\Delta x}$$

9. Pull on a clamped rod when it is cooled,

$$F = YA\alpha . \Delta T$$

10. Temperature in Kelvin,

$T = (t+273)$; t=temperature in °C

11. Joule's law, $W = JH$;
W=Work done=mgh or $\frac{1}{2}$ mv^2
J=Mechanical equivalent of heat=4·18 J/cal.
H=Heat produced
$= ms\Delta T$ or mL or sum of the two

12. First law of thermodynamics.

$$\Delta Q = \Delta U + \Delta W \;;$$

ΔQ = Heat absorbed
ΔU = Increase in internal energy
ΔW = Work done

13. In isothermal expansion of a gas (T = constant) heat absorbed by the gas is the same as the work done by the gas. There is no increase in the internal energy of the gas in this process.

$$\therefore \quad \Delta Q = \Delta W = nRT \log_e \frac{V_f}{V_i} \;;$$

n = No of moles
R = gas constant = 8·31 J $\text{mole}^{-1}K^{-1}$
T = constant temp of gas in Kelvin
V_f = Final volume of gas
V_i = Initial volume of gas

14. In an isobaric process (P = constant), work done by an expanding gas is given by

$$W = P(V_f - V_i)$$

15. In an adiabatic expansion of a gas (Q = constant),

$$PV^{\gamma} = \text{constant}$$

and $$TV^{\gamma-1} = \text{constant}$$

The work done is given by

$$W = \frac{(P_2V_2 - P_1V_1)}{(\gamma - 1)}.$$

where $$\gamma = \frac{C_p}{C_v}.$$

16. The efficiency of a heat engine is given by

$$\eta = \frac{W}{Q_1} \;;$$

W = useful work ;

Q_1 = heat extracted from source at constant temp. T_1

or $$\eta = \left(1 - \frac{Q_2}{Q_1}\right);$$

Q_2 = heat rejected to sink at constant temperature T_2 $(T_2 < T_1)$

or $$\eta = \left(1 - \frac{T_2}{T_1}\right)$$

(**Note.** It is supposed here heat engine is working in a reversible cycle for which

$$\left.\frac{Q_2}{Q_1}=\frac{T_2}{T_1}\right)$$

17. The coefficient of performance of a refrigerator,

(*a*) $$\text{C.O.P.}=\frac{Q_2}{W};$$

Q_2 = Heat removed from inside of refrigerator at constant temperature T_2

W = Work done

$$=\frac{Q_2}{(Q_1-Q_2)}$$

Q_1 = heat rejected to outside atmosphere at const. temp. T_1

$$=\frac{T_2}{(T_1-T_2)}$$

SOLVED EXAMPLES

Example 1. *Consider a steel bridge 300 m long in a locality where temperature varies from 263 K to 303 K. Find the change in the length in the bridge for the above variation in temperature.* (α *for steel* $=11\times10^{-6}\ K^{-1}$).

Solution. Change in length,

$$\Delta l=\alpha l\ \Delta T$$

$$=11\times10^{-6}\times300\times(303-263)$$

$$=132\times10^{-3}\text{ m}=13.2\text{ cm.}$$

Example 2. *A circular hole of diameter 12 cm, is drilled in a brass sheet at 290 K. What will be the diameter of this hole when the sheet is heated to 410 K.* (α *for brass is* $18.7\times10^{-6}\ K^{-1}$).

Solution. Final diameter $=l\,(1+\alpha\,\Delta T)$

$$=12\,[1+18.7\times10^{-6}\,(410-290)]$$

$$=12\,(1+0.002244)$$

$$=12.027\text{ cm.}$$

Example 3. *What should be the ratio of the lengths of brass rod and iron rod at 0°C so that the difference between their lengths may remain the same at any other temperature.* (α *for brass and iron* $=18\times10^{-6}$ *and* $12\times10^{-6}\ K^{-1}$ *respectively*).

Solution. Here $\Delta l=\Delta l'$

$\therefore$ $$l_0\alpha\Delta T=l'_0\alpha'.\Delta T$$

$\therefore$ $$\frac{l_0}{l_0'}=\frac{\alpha'}{\alpha}$$

$$=\frac{12\times10^{-6}}{18\times10^{-6}}=2:3.$$

Example 4. *An iron ball whose diameter is 10 cm rests upon a brass ring whose internal diameter is 9·9 cm, both being at 0°C. To what temperature must both be heated in order that the ball may just slide through the ring.* (α *for brass* $=18\times10^{-6}$ *and* α *for iron* $=12\times10^{-6}\ K^{-1}$ *respectively*).

Solution. Let t°C be the required temperature

For iron ball,

$$\begin{aligned} l_t &= l_0\,(1+\alpha.\Delta T)\\ &=10\,[1+12\times10^{-6}\,(t-0)]\\ &=10+12\times10^{-5}t \qquad ..(i)\end{aligned}$$

For brass ring

$$\begin{aligned} l'_t &= l'_0\,(1+\alpha'\,\Delta T)\\ &=9{\cdot}9\,[1+18\times10^{-6}\,(t-0)]\\ &=9{\cdot}9+17{\cdot}82\times10^{-5}t \qquad ...(ii)\end{aligned}$$

But $\quad l'_t=l_t$

$$\therefore\ 9{\cdot}9+17{\cdot}82\times10^{-5}t=10+12\times10^{-5}t$$

$$\therefore\qquad t=\frac{0{\cdot}1}{5{\cdot}82\times10^{-5}}=1718{\cdot}2\ ^\circ\mathrm{C}.$$

Example 5. *A rectangular block of height 40 cm has its top 60 cm×80 cm. When its base is fixed rigidly, its top is sheared by 5 cm parallel to the fixed base by a tangential force of 15 kg wt. Calculate* (*a*) *Stress,* (*b*) *Strain,* (*c*) *Shearing modulus.*

Solution. $\quad A=60\ \mathrm{cm}\times80\ \mathrm{cm}=0{\cdot}6\ \mathrm{m}\times0{\cdot}8\ \mathrm{m}=0{\cdot}48\ \mathrm{m}^2$

$$F=Mg=15\times9{\cdot}8=147{\cdot}0\ \mathrm{N}$$

$$\therefore\qquad \text{Stress}=\frac{F}{A}=\frac{147}{0{\cdot}48}=306{\cdot}25\ \mathrm{Nm}^{-2}$$

$$\text{Strain}=\frac{5}{40}=0{\cdot}125$$

$\therefore$ **Shearing modulus,**

$$\eta=\frac{\text{Stress}}{\text{Strain}}=\frac{306{\cdot}25}{0{\cdot}125}=2{\cdot}45\times10^{3}\ \mathrm{Nm}^{-2}.$$

Example 6. *A hollow brass sphere with internal radius 2·1 cm at 0°C is heated to 100°C. Calculate increase in its volume.* (α *for brass* $=18\times10^{-6}\ ^\circ\mathrm{C}^{-1}$).

Solution. $\quad V_0=\frac{4}{3}\pi r^3$

$$=\frac{4}{3}\times\frac{22}{7}\,(2{\cdot}1)^3=38{\cdot}808\ \mathrm{cm}^3.$$

$$\beta=3\,\alpha=3\times18\times10^{-6}=54\times10^{-6}\ ^\circ C^{-1}.$$

$$\Delta T=100-0=100\ ^\circ C$$

∴ Increase in Volume,

$$\Delta V=\beta V\Delta T$$

$$=54\times10^{-6}\times38{\cdot}808\times100=0{\cdot}209\ cm^3.$$

Example 7. *On heating a glass block of 10,000 cm³, from 25°C to 40°C, its volume increases by 4 cm³. Calculate coefficient of linear expansion of glass.*

Solution.

$$V=10{,}000\ cm^3$$

$$\Delta V=4\ cm^3$$

$$\Delta T=40-25=15^\circ C$$

∴

$$\beta=\frac{\Delta V}{V.\Delta T}$$

∴

$$\beta=\frac{4}{10000\times15}=26{\cdot}67\times10^{-6}\ ^\circ C^{-1}$$

Now

$$\alpha=\frac{\beta}{3}$$

$$=\frac{26{\cdot}67\times10^{-6}}{3}$$

$$=8{\cdot}89\times10^{-6}\ ^\circ C^{-1}.$$

Example 8. *An aluminium plate of area 20 cm × 15 cm and thickness 1 cm has one face kept at 100°C and the other face at 20°C. If flow of heat through this plate is 10 Kcal s⁻¹, calculate coefficient of thermal conductivity of aluminium.*

Solution.

$$K=\frac{\Delta Q\ \Delta x}{\Delta t A \Delta T}$$

$$=\frac{10{,}000\times1}{1\times20\times15\times(100-20)}$$

$$=0{\cdot}4167\ cal.\ cm^{-1}\ s^{-1}\ ^\circ C^{-1}.$$

Example 9. *A layer of ice 10 cm. thick is formed on a pond. The temperature of air is −10°C. Calculate how long it will take for the thickness of ice to increase by 1 mm. (Take density of ice=1, K for ice=0·005 cal. cm⁻¹ s⁻¹ °C⁻¹, Latent heat of ice=80 cal g⁻¹).*

Solution. If A cm² be the area of 1 mm thick ice layer, its mass,

$$m=(A\times0{\cdot}1)\times1\ g=0{\cdot}1A\ g$$

∴

$$\Delta Q=mL=(A\times0{\cdot}1)\times80=8A\ cal.$$

Now we know,

$$\frac{\Delta Q}{\Delta t}=K.A.\ \frac{\Delta T}{\Delta x}$$

∴

$$\Delta t=\frac{\Delta Q.\ \Delta x}{KA\ \Delta T}$$

$$=\frac{8A\times 10}{0{\cdot}005\times A\times[0-(-10)]}=1600\ S$$

$$=26\ mt\ 40\ S.$$

Example 10. *Heat is conducted through a slab of two layers of different materials of thermal conductivity K_1 and K_2 and thickness x_1 and x_2 respectively. A is the area of cross-section of each plate and the temperatures of outer faces of the slabs are θ_1 and θ_2. Drive an expression for (a) the rate of flow of heat and (b) the temperature of common face between two layers.*

Solution. The rate of flow of heat $\frac{\Delta Q}{\Delta t}$ will be same throughout the two layers of slab. Let θ_3 be the temperature of the common face between the two layers.

For first layer,

$$\frac{\Delta Q}{\Delta t}=K_1A\ \frac{(\theta_1-\theta_3)}{x_1}$$

$$\therefore (\theta_1-\theta_3)=\frac{\Delta Q}{\Delta t A}\ \frac{x_1}{K_1} \qquad \ldots(1)$$

For second layer,

$$\frac{\Delta Q}{\Delta t}=K_2A\ \frac{(\theta_3-\theta_2)}{x_2}$$

$$\therefore\ (\theta_3-\theta_2)=\frac{\Delta Q}{\Delta t.A}\ \frac{x^2}{K_2} \qquad \ldots(2)$$

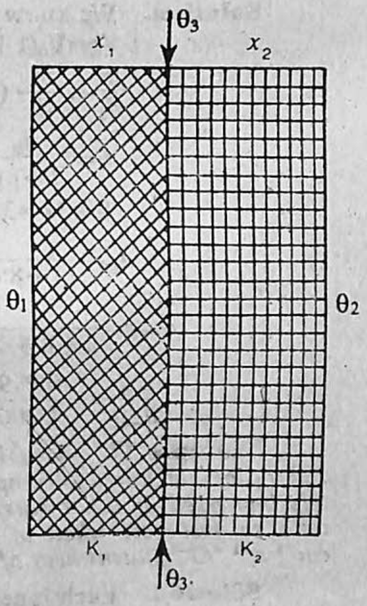

Fig. 11·1

(*a*) Adding the two expressions for θ, we have

$$(\theta_1-\theta_3)+(\theta_3-\theta_2)$$

$$=\frac{\Delta Q}{\Delta t.A}\frac{x_1}{K_1}+\frac{\Delta Q}{\Delta A.t}\ \frac{x_2}{K_2}$$

$$\therefore\ (\theta_1-\theta_2)$$

$$=\frac{\Delta Q}{\Delta t.A}\left(\frac{x_1}{K_1}+\frac{x_2}{K_2}\right)$$

∴ Rate of flow of heat,

$$\boxed{\frac{\Delta Q}{\Delta t}=\frac{A(\theta_1-\theta_2)}{(x_1/K_1+x_2/K_2)}} \qquad \ldots(3)$$

(*b*) From expression (2), we have

$$\theta_3=\frac{\Delta Q}{\Delta t}\ \frac{1}{A}\ \frac{x_2}{K_2}+\theta_2$$

Now putting the value of $\frac{\Delta Q}{\Delta t}$ from expression (3), we have

$$\theta_3=\frac{A\,(\theta_1-\theta_2)}{\left(\frac{x_1}{K_1}+\frac{x}{K_2}\right)}\times\frac{1}{A}\;\frac{x_2}{K_2}+\theta_2$$

$$\therefore\quad \boxed{\theta_3=\frac{\left(\theta_1\,\frac{x_2}{K_2}+\theta_2\,\frac{x_1}{K_1}\right)}{\left(\frac{x_1}{K_1}+\frac{x_2}{K_2}\right)}}$$

Example 11. *Compare the density of lead at 80°C. and −80°C (α for lead$=2{\cdot}8\times10^{-6}\,°C^{-1}$).*

Solution. We know volume of a material at t°C is given by

$$V_t=V_0(1+\beta t)$$

or

$$\frac{m}{d_t}=\frac{m}{d_0}\,(1+\beta t)$$

$$\therefore\quad d_t=\frac{d_0}{1+\beta t}$$

$$\beta=3\alpha=3\times2{\cdot}8\times10^{-6}=8{\cdot}4\times10^{-6}\,°C^{-1}$$

$$\therefore\quad d_{80}=\frac{d_0}{1+8{\cdot}4\times10^{-6}\times(80)}=\frac{d_0}{1{\cdot}000672}$$

$$d_{-80}=\frac{d_0}{1+8{\cdot}4\times10^{-6}(-80)}=\frac{d_0}{0{\cdot}999328}$$

$$\therefore\quad \frac{d_{80}}{d_{-80}}=\frac{0{\cdot}999328}{1{\cdot}000672}=0{\cdot}9986$$

Example 12. *A cubical hollow copper box of 11 cm outer side and 0·5 cm thickness is completely filled with ice at 0°C. It is completely immersed in a hot water bath at 100°C. How much time will it take to just melt whole of the ice in the box? (K for Cu=0·9 cal cm^{-1} s^{-1} °C^{-1}, latent heat of ice=80 cal g^{-1}, take density of ice=0·9).*

Solution. Each inner side of cube$=11-(0{\cdot}5+0{\cdot}5)=10$ cm.

$$\therefore\quad A=6 \text{ times area of one face}$$

$$=6\times10\times10=600\text{ cm}^2$$

$$\Delta x=0{\cdot}5\text{ cm}$$

$$K=0{\cdot}9$$

$$\Delta T=(100-0)=100°C$$

Inner volume of cube,

$$V=10\times10\times10=1000\text{ c.c.}$$

Mass of ice in the cube$=V\times\rho$

$$=1000\times0{\cdot}9=900\text{ g}$$

$$\therefore \quad \Delta Q = mL$$
$$= 900 \times 80 = 7{\cdot}2 \times 10^4 \text{ cal.}$$

Now $$\frac{\Delta Q}{\Delta t} = KA \; \frac{\Delta T}{\Delta x}$$

$$\therefore \quad \Delta t = \frac{\Delta Q . \Delta x}{KA \; \Delta T}$$
$$= \frac{7{\cdot}2 \times 10^4 \times 0{\cdot}5}{0{\cdot}9 \times 600 \times 100} = 0{\cdot}67 \text{ s.}$$

Example 13. *What volume of iron will have the same thermal capacity as a litre of water ? The specific heat of iron is 0·12 and its specific gravity is 7·5.* [*A.I.S.S.* (*Comp.*) *E 1982*]

Solution. Mass of 1 litre of water = 1000 g.,
sp. heat of water = 1.

$\therefore$ Thermal capacity of water, $c = $ m s
$= 1000 \times 1 = 1000$ cal.

$\therefore$ The mass of iron,

$$m = \frac{c}{S}$$
$$= \frac{1000}{0{\cdot}12} \text{ g.}$$

Now $$V = \frac{m}{\rho}$$
$$= \frac{1000}{0{\cdot}12 \times 7{\cdot}5} \text{ c.c.}$$
$$= 1111{\cdot}111 \text{ c.c.}$$

Example 14. *The volume of a metal sphere is 10^{-3} m^3 at 273 K. What would be its volume at 373 K ? (Coefficient of linear expansion of the metal = 0·000018 per K).* [*A.I.S.S.* (*Comp.*) *E. 1985*]

Solution. $\Delta T = 373 - 273 = 100$ K
$\beta = 3\alpha = 3 \times 0{\cdot}000018$
$= 0{\cdot}000054$ per K

Now final volume $V' = V(1 + \beta \Delta T)$
$= 10^{-3}(1 + 0{\cdot}000054 \times 100)$
$= 1{\cdot}0054 \times 10^{-3} \text{m}^3$.

Example 15. *In a joule experiment two weights of 5 kg each fall through a height of 3 m and rotate a paddle wheel which stirs 0·1 kg water. What is change in the temperature of water for 200 falls of weight ? (Water equivalent of calori meter = 0·2 kg).*

Solution. $W = 2Mgh\,n$
$= 2 \times 5 \times 9{\cdot}8 \times 3 \times 200$
$W = 58800$ Joule

$$H = ms\Delta T + w\Delta T$$
$$= (0{\cdot}1 \times 1 + 0{\cdot}2)\Delta T$$
$$= 0{\cdot}3\Delta T \text{ kcal}$$

Now $W = JH$

$\therefore$ $58800 = 4200 \times 0{\cdot}3\Delta T$ [$\because$ J=4200 J/kcal]

$\therefore$ $\Delta T = 46{\cdot}67°C$.

Example 16. *A stream of water is 500 m high. What will be the rise in temperature of water on striking the ground after falling freely? Assume all the heat produced is used in heating water.*

($J = 4{\cdot}2$ J/col and $g = 9{\cdot}8$ ms^{-2}) *[D.S.S.E. 1983, 84 C]*

Solution. Let the mass of falling water be m kg. Then

$$W = mgh$$
$$= m \times 9{\cdot}8 \times 500$$

or $W = 4900\, m$ Joule

and $H = ms\Delta T$

$= m \times 1 \times \Delta T$ ($\because$ For water, $s = 1$)

or $H = m\Delta T$ kcal

Now $J = 4200$ Joule/kcal

$\therefore$ By $W = JH$

$$4900\, m = 4200 \times m\Delta T$$

$\therefore$ $\Delta T = 1{\cdot}167°C$.

Example 17. *From what height a piece of ice should be dropped so that it may just melt completely on striking the ground? Assume that half of the heat produced is absorbed by ice.*

($L = 80$ kcal kg^{-1} and $J = 4200$ J kcal^{-1})

Solution. $W = mgh = m \times 9{\cdot}8 \times h$

$$H = \frac{W}{J} = \frac{m \times 9{\cdot}8 \times h}{4200}$$
$$= \frac{7\,mh}{3000}$$

But Heat absorbed $= \frac{1}{2}H = mL$

$\therefore$ $\frac{1}{2} \times \frac{7\,mh}{3000} = m \times 80$

$\therefore$ $h = \frac{80 \times 3000 \times 2}{7}$

$= 68571$ m

$= 68{\cdot}571$ km

Example 18. *A lead bullet weighing 20 g and moving with velocity of 100 ms^{-1} comes to rest in a fixed block of wood. Calculate* *the heat developed and rise in temperature of the bullet assuming that 60% heat is absorbed by the bullet. The specific heat of lead is 0·03.*

Solution.
$$m = 20\text{ g} = 0{\cdot}02\text{ kg}$$
$$W = \tfrac{1}{2}mv^2$$
$$= \tfrac{1}{2} \times 0{\cdot}02 \times 100^2$$
$$= 100\text{ J}$$
$$H = \frac{W}{J} = \frac{100}{4{\cdot}2} = 23{\cdot}8\text{ cal}$$
$$\text{Heat absorbed} = \frac{60}{100}H = \frac{3}{5} \times 23{\cdot}8\text{ cal}$$

Also Heat absorbed $= ms\Delta T$
$$\frac{3}{5} \times 23{\cdot}8 = 20 \times {\cdot}03 \times \Delta T$$
$$\therefore \quad \Delta T = 23{\cdot}8°\text{C}.$$

Example 19. *An electrical drilling machine of 200 watt drill a hole in a metal sheet of 1 kg for 20 s. Calculate the increase in temperature of the metal. (sp. heat of metal=0·11)*

Solution.
$$W = P \times t$$
$$= 200 \times 20$$
$$= 4000\text{ J}$$
$$H = ms\Delta T$$
$$= 1 \times {\cdot}11 \times \Delta T$$
$$= {\cdot}11\Delta T\text{ kcal}$$

Now
$$H = \frac{W}{J}$$
$${\cdot}11\Delta T = \frac{4000}{4200}$$
$$\therefore \quad \Delta T = 8{\cdot}658°\text{C}.$$

Example 20. *Calculate the change in internal energy of a block of copper of mass 200 g when it is heated from 25°C to 75°C. Assume the change in volume as negligible. (Sp. heat of copper=0·1).*

Solution.
$$\Delta W = PdV = P \times 0 = 0$$
$$\Delta Q = ms\Delta T$$
$$= 200 \times {\cdot}1 \times (75 - 25)$$
$$= 1000\text{ cal.}$$

By $\Delta Q = \Delta W + \Delta U$
$$\Delta U = \Delta Q = 1000\text{ cal} = 1000 \times 4{\cdot}2$$
$$= 4200\text{ J}.$$

Example 21. *2 g of ice at −10°C is made to melt to form water at 50°C. If the change in volume is negligible, calculate the* *increase in the internal energy of ice. (Sp. heat of ice=0·5 and latent heat of ice=80 cal* g^{-1}).

Solution. $\Delta W = P dV = P \times 0 = 0$

Heat will be gained by ice in 3 stages—(1) to increase its temp. from −10°C to 0°C, (2) to convert into water at 0°C, (3) to raise the temperature of water from 0°C to 50°C.

$$\therefore \quad \Delta Q = ms\Delta T + mL + ms\Delta T$$
$$= 2 \times \cdot 5 \times [0-(-10)] + 2 \times 80 + 2 \times 1 \times (50-0)$$
$$= 10 + 160 + 100$$
$$= 270 \text{ cal.}$$

$\therefore$ By $\quad \Delta Q = \Delta W + \Delta U$

$$\therefore \quad \Delta U = \Delta Q$$
$$= 270 \text{ cal}$$
$$= 270 \times 4{\cdot}2 \text{ J}$$
$$= 1134 \text{ J.}$$

Example 22. *1 g of water at 373 K is converted into steam at the same temperature. The volume of 1 c.c. of water becomes 1671 c.c. on boiling. Calculate the change in the internal energy of the system if the heat of vapourisation is 540 cal/g.* [*A.I.S.S.E. 1979*]

Solution. Water is heated at constant atmospheric pressure and we know

$$1 \text{ atmosphere} = 1{\cdot}013 \times 10^5 \text{ Nm}^{-2}$$
$$\therefore \quad P = 1{\cdot}013 \times 10^5 \text{ Nm}^{-2}$$
$$V_i = 1 \ c.c. = 10^{-6} \text{m}^3$$
$$V_f = 1671 \ c.c. = 1671 \times 10^{-6} \text{m}^3$$
$$\therefore \quad \Delta W = P(V_f - V_i)$$
$$= 1{\cdot}013 \times 10^5 (1671 - 1) 10^{-6}$$
$$= 169{\cdot}17 \text{ J}$$
$$= \frac{169{\cdot}17}{4{\cdot}2} \text{ cal.}$$
$$= 40{\cdot}3 \text{ cal.}$$

$\therefore$ By $\quad \Delta Q = \Delta W + \Delta U$

$$\therefore \quad \Delta U = \Delta Q - \Delta W$$
$$= 540 - 40{\cdot}3$$
$$= 499{\cdot}7 \text{ cal.}$$

Example 23. *A steam engine takes steam from the boiler at 500 K and rejects it to the air at 373 K. What is its efficiency?*

Solution. Efficiency,

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

$$=\left(1-\frac{373}{500}\right)\times 100\%$$
$$=\frac{127}{500}\times 100\%$$
$$=25{\cdot}4\%.$$

Example 24. *In a refrigerator heat from inside at 277 K is transferred to a room at 300 K. How many joules of heat will be delivered to the room for each one joule of electric energy consumed ideally.*

Solution. We know

$$\frac{Q_2}{Q_1}=\frac{T_2}{T_1}$$
$$\frac{(Q_1-W)}{Q_1}=\frac{T_2}{T_1}$$
$$\therefore \quad \frac{(Q_1-1)}{Q_1}=\frac{277}{300}$$
$$300\ Q_1-300=277\ Q_1$$
$$\therefore \quad Q_1=\frac{300}{23}$$

$\therefore$ Heat delivered to the room $=13{\cdot}04$ J.

Example 25. *2 moles of an ideal gas kept at a constant temperature at 27°C are expanded from a volume of 2 litres to a volume of 5 litres. Calculate the work done in the process.*

($R=8{\cdot}31\ Jmole^{-1}\ k^{-1}$)

Solution. $T=(273+27)=300$ k

Work done,

$$W=nRT\log_e\left(\frac{V_f}{V_i}\right)$$
$$=(2\times 8{\cdot}31\times 300)\log_e\left(\frac{5}{2}\right)$$
$$=(4986)\times 2{\cdot}3\log_{10}(2{\cdot}5)$$
$$=4986\times 2{\cdot}3\times 0{\cdot}3979$$
$$=4563\ \text{J}.$$

Example 26. *What is the minimum power that must be supplied to a refrigerator that freezes 1·0 kg of water at 25°C into ice at 0°C in a time interval of 4 minutes. The room temperature is 25°C.*

Solution. $Q_2=ms\Delta T+mL$

$$=1\times 1\times(25-0)+1\times 80$$
$$Q_2=105\ \text{kcal}$$
$$T_2=0+273=273\ \text{K}$$
$$T_1=25+273=298\ \text{K}$$

$$\frac{Q_1}{Q_2}=\frac{T_1}{T_2}$$

$$Q_1=\frac{298}{277}\times 105$$

$$=112{\cdot}96 \text{ kcal}$$

$$\therefore \quad W=(Q_1-Q_2)$$

$$=(112{\cdot}96-105)$$

$$=7{\cdot}96 \text{ kcal}$$

$$=7{\cdot}96\times 4200 \text{ J}$$

$$=33432 \text{ J}$$

$$P=\frac{W}{t}=\frac{33432}{4\times 60} \text{ J/s}$$

$$P=139{\cdot}3 \text{ watt.}$$

Example 27 *One gram mole of an ideal gas at S.T.P. expands adiabatically to double its volume. Find the change in the internal energy in the process.*

Solution. For an adiabatic process,

$$T_2V_2^{\gamma-1}=T_1V_1^{\gamma-1}$$

$$T_2=\left(\frac{V_1}{V_2}\right)^{\gamma-1}T_1$$

$$=\left(\frac{V_1}{2V_1}\right)^{(1\cdot 4-1)}\times 273$$

$$=(0{\cdot}5)^{0\cdot 4}\times 273$$

$$T_2=204{\cdot}7 \text{ K} \qquad \text{(Solved by log method)}$$

$$\therefore \quad \triangle W=\frac{(RT_2-RT_1)}{1-r}$$

$$=\frac{8{\cdot}31(204{\cdot}7-273)}{(1-1{\cdot}4)}$$

$$=1419 \text{ J}$$

Since for adiabatic process, $\triangle Q=0$,

By

$$\triangle Q=\triangle W+\triangle U,$$

$$0=\triangle W+\triangle U$$

$$\triangle U=-\triangle W$$

$$=-1419 \text{ J.}$$

So there is a decrease in the internal energy as the work is done by gas.

Example 28 *In a refrigerator the temperature of deep freezer is – 3°C. If the room temperature is 37°C, calculate the coefficient of performance of the refrigerator. How much heat will be removed per second from its site for a supply of 200 watt.*

Solution.

$$T_1=273+37=310 \text{ K}$$

and

$$T_2=273-3=270 \text{ K}$$

∴ The coefficient of performance of refrigerator,

$$c.o.p. = \frac{T_2}{(T_1 - T_2)}$$

$$= \frac{270}{(310-270)}$$

$$= \frac{270}{40} = 6{\cdot}75$$

Now $$W = P \times t$$

$$= 200 \times 1 = 200 \text{ J}$$

Also $$c.o.p. = \frac{Q_2}{W}$$

∴ $$Q_2 = c.o.p. \times W$$

$$= 6{\cdot}75 \times 200$$

$$= 1350 \text{ cal.}$$

Example 29. *A reversible engine converts one-fifth of heat which it absorbs from source into work. When the temperature of the sink is reduced by 70°, its efficiency is doubled. Calculate the temperature of the source and the sink.*

Solution.

$$W = \frac{1}{5} Q_1$$

$$Q_2 = Q_1 - W = Q_1 - \frac{1}{5} Q_1 = \frac{4}{5} Q_1$$

or $$\frac{Q_1}{Q_2} = \frac{5}{4}$$

∴ By $$\frac{T_1}{T_2} = \frac{Q_1}{Q_2}$$

∴ $$\frac{T_1}{T_2} = \frac{5}{4} \qquad \text{...(1)}$$

Let efficiency $\left(1 - \frac{T_2}{T_1}\right) = x$ ...(2)

In second case,

$$\left(1 - \frac{T_2 - 70}{T_1}\right) = 2x \qquad \text{...(3)}$$

On dividing eqn. (3) by (2)

$$\frac{T_1 - T_2 + 70}{T_1} = 2\left(\frac{T_1 - T_2}{T_1}\right)$$

∴ $$(T_1 - T_2) = 70$$

∴ From eqn. (1)

$$\left(\frac{5}{4} T_2 - T_2\right) = 70$$

$$\therefore \qquad T_2=280\text{ K}$$

$$\therefore \qquad T_1=\frac{5}{4}\,T_2$$

$$=\frac{5}{4}\times 280$$

$$T_1=350\text{ K.}$$

Example 30. *Calculate the work done when one mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 10^5 Nm^{-2} and 6l respectively. The final volume of the gas is 2 l. Molar specific heat at constant volume is $\frac{3}{2}$ R.*

[I.I.T. J.E.E. 1982]

Solution.

$$C_v=\frac{3}{2}R$$

$$\therefore \qquad C_p=C_v+R=\frac{3}{2}R+R=\frac{5}{2}R$$

$$\therefore \qquad \gamma=\frac{C_p}{C_v}=\frac{5/2R}{3/2R}=\frac{5}{3}$$

For adiabatic process,

$$P_2V_2^{\gamma}=P_1V_1^{\gamma}$$

$$P_2 2^{\gamma}=10^5\, 6^{\gamma}$$

$$P_2=10^5\left(\frac{6}{2}\right)^{\gamma}$$

$$P_2=10^5\,3^{5/3}$$

Now let $\qquad x=3^{5/3}$

$$\log x=\frac{5}{3}\log 3=\frac{5}{3}\times 0{\cdot}4771$$

or $\qquad \log x=0{\cdot}7952$

$$\therefore \qquad x=\text{Antilog }(0{\cdot}7952)$$

$$=6{\cdot}24$$

$$\therefore \qquad P_2=6{\cdot}24\times 10^5\text{ Nm}^{-2}$$

Now $\qquad P_1=10^5\text{ Nm}^{-2},$

$$V_1=6\,l=6\times 10^{-3}\text{ m}^3$$

$$V_2=2\,l=2\times 10^{-3}\text{ m}^3$$

and $\qquad \gamma=\frac{5}{3}$

∴ Work done,

$$W=\frac{(P\,V_2-P_1V_1)}{(\gamma-1)}$$

$$=\frac{(6{\cdot}24\times10^{5}\times2\times10^{-3}-10^{5}\times6\times10^{-3})}{\left(\frac{5}{3}-1\right)}$$

$$=\frac{6{\cdot}48\times10^{2}}{2/3}\,\mathrm{J}$$

$$=972\ \mathrm{J}.$$

Example 31. *Calculate the work done by an expanding gas. If its indicator diagram is the one as shown in the Fig. 11.2 below.*

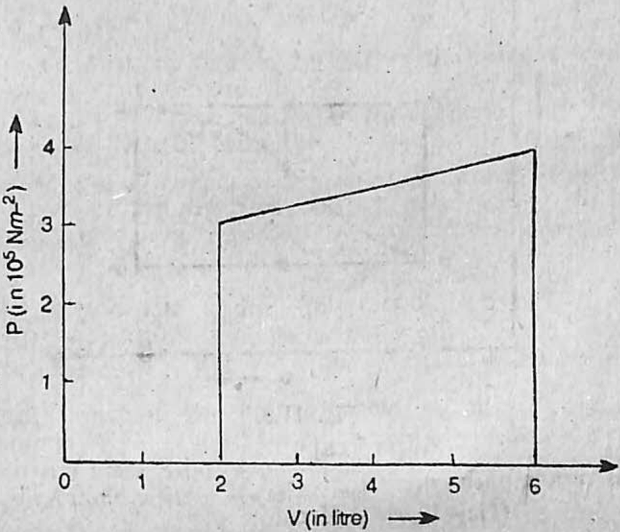

Fig. 11.2.

Solution. Area of the indicator diagram is the measure of work done by the expanding gas.

$\therefore$ W = Area of Trapezium

= ½ (sum of ∥ sides) × perp. distance between them

$$=\tfrac{1}{2}(3+4)10^{5}\times(6-2)10^{-3}$$

[∵ 1 litre = 10^{-3} m³]

$$=1400\ \mathrm{J}.$$

Example 32. *When a system is taken from a to c along the path abc, it is found that Q=80 cal. and W=35 cal. If it is taken from a to c along the path ab′c, it is found Q=50 calories. What is W along this path? If W=—15 cal. for the curved return path, what is Q for this path?*

Solution. According to the first law of thermodynamics, let along the path *abc*,

$$Q_1=W_1+(U_c-U_a)$$

$$\therefore \quad 80=35+(U_c-U_a)$$

$$\therefore \quad (U_c-U_a)=45 \text{ cal,}$$

Let along the path *ab′c*,

$$Q_2=W_2+(U_c-U_a)$$

$$\therefore \quad 50=W_2+45$$

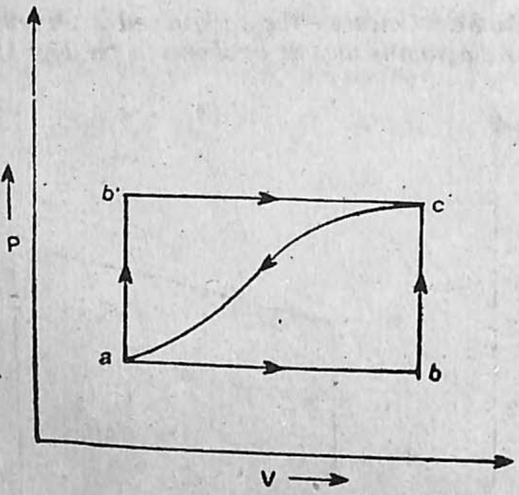

Fig. 11.3.

$$\therefore \quad W_2=5 \text{ cal.}$$

For return path *ca*,

$$(U_c-U_a)=-45 \text{ cal.}$$

$$\therefore \text{ By} \quad Q=W+(U_c-U_a)$$

$$Q=-15-45$$

$$=-60 \text{ cal.}$$

EXERCISE 11

1. The Yamuna bridge of Delhi is 800 m long. If the temperature in Delhi varies from 10°C to 45°C, calculate the change in the length of the bridge. (α for steel$=11\times10^{-6}$ K^{-1}).
2. What gap between steel rails 66 m long each should be left for the maximum increase of temperature of 50°C. (α for steel$=11\times10^{-6}$ °C^{-1}).
3. An iron rim 0·64 m in radius at 15°C is to be put on a wooden wheel with circumference 4·072 m. Calculate the temperature upto which the rim should be heated. (α for Iron$=12\times10^{-6}$ °C^{-1}). Take $\pi=3·14$.
4. A copper disc of 1 m diameter has a circular hole of radius 10 cm at its centre at 0°C. Calculate the radius of the hole if

the disc is heated to a temperature of 100°C. (α for copper $= 16\times 10^{-6}\ {}^\circ\mathrm{C}^{-1}$).

5. A brass cube has its side as 10 cm long at 20°C. Calculate its volume if it is heated to a temperature of 60°C. (α for brass $= 18\times 10^{-6}\ {}^\circ\mathrm{C}^{-1}$).

6. An iron ball with diameter as 3 cm rest upon a brass ring whose internal diameter is 2·95 cm, both being at 0°C. To what temperature must both be heated in order that the ball may just slide through the ring. (α for iron $= 12\times 10^{-6}\ {}^\circ\mathrm{C}^{-1}$ and α for brass $= 18\times 10^{-6}\ {}^\circ\mathrm{C}^{-1}$).

7. A brass rod 50 cm long and 4 mm in diameter is heated from 20°C to 90°C. Calculate the force needed to compress the rod back to its original length. (α for brass $= 18\times 10^{-6}\ {}^\circ\mathrm{C}^{-1}$ and Y for brass $= 9\times 10^{10}\ \mathrm{Nm}^{-2}$).

8. What lengths of copper and steel at 0°C must be used so that the difference between their lengths may remain 10 cm at all temperatures. (α for Cu $= 16\times 10^{-6}\ {}^\circ\mathrm{C}^{-1}$ and α for steel $= 12\times 10^{-6}\ {}^\circ\mathrm{C}^{-1}$).

9. A rod of brass and another of steel differ by 12 cm in length at all temperatures Calculate the length of each rod at 0°C (α for brass $= 18\times 10^{-6}\ {}^\circ\mathrm{C}^{-1}$ and α for steel $= 12\times 10^{-6}\ {}^\circ\mathrm{C}^{-1}$).

10. The difference of temperatures between outside and inside of a room is 10°C. Find how much heat is conducted per minute through a glass window of 0·8 m × 1·0 m and thickness 2 mm. (thermal conductivity for glass $= 1{\cdot}3\times 10^{-5}$ cal $\mathrm{s}^{-1}\ \mathrm{cm}^{-1}\ {}^\circ\mathrm{C}^{-1}$).

11. A steel pipe is 20 m long at 0°C. Calculate its length when steam at 100°C is passed through it. (β for steel $= 36\times 10^{-6}\ {}^\circ\mathrm{C}^{-1}$).

12. An aluminium plate of 15 cm × 10 cm and thickness 2·5 cm has one side maintained at 100°C by steam and other side at 25°C by running water. If the rate of flow of heat through this plate is 2250 cal s^{-1}, calculate the coefficient of thermal conductivity of aluminium.

13. A layer of ice 15 cm thick is formed on a pond in Kashmir. The temperature of the air over the pond is −8°C. Calculate how long time it will take for the thickness of ice to increase by 1 mm. (For ice L $= 80$ cal g^{-1}, K $= 0{\cdot}005$ cal $\mathrm{s}^{-1}\ \mathrm{cm}^{-1}\ {}^\circ\mathrm{C}^{-1}$ and take density of ice as 1).

14. Water is boiling in a rectangular aluminium tank the bottom of which is 3 mm. thick. If water level falls steadily at 1 cm in 3 minutes, calculate the temperature of the lower surface of the tank. (K for aluminium $= 0{\cdot}5$ cal $\mathrm{s}^{-1}\ \mathrm{cm}^{-1}\ {}^\circ\mathrm{C}^{-1}$; Latent heat of steam $= 540$ cal g^{-1}).

15. The inner volume of a hollow brass sphere is 400 c.c at 20°C. Calculate its volume at 400°C. (α for brass $= 18\times 10^{-6}\ {}^\circ\mathrm{C}^{-1}$).

16. The volume of an iron sphere is 100 c.c. at 0°C. When its temperature is raised to 200°C, its volume becomes 100·72 c.c. Calculate coefficient of linear expansion of iron.

17. A hollow spherical iron ball of internal radius 7 cm and thickness 2 mm. is completely filled with ice at 0°C and is immersed completely in a hot water bath at 100°C. Calculate (*a*) how much ice will melt in 1 s, (*b*) the time in which all the ice will melt. (for ice $L=80$ cal g^{-1}, for iron $K=0·14$ cal s^{-1} cm^{-1} $°C^{1}$, take density of ice as 1 g cm^{-3}). Take $\pi=22/7$.

18. Heat is conducted through a wall 20 cm × 10 cm consisting of a 4 mm thickness of copper and 8 mm thickness of aluminium. The temperature of the outer faces of copper and aluminium are 100°C and 70°C respectively. Calculate the rate of flow of heat and the temperature of the joint between copper and aluminium. (K for copper=1 cal s^{-1} cm^{-1} $°C^{-1}$ and K for aluminium=0·5 cal s^{-1} cm^{-1} $°C^{-1}$).

19. Compare the heat loss for an equal time interval through equal areas of a 2 cm thick wooden wall and 5 mm thick glass in window. (K for wood=0·00025 cal s^{-1} cm^{-1} $°C^{-1}$ and K for glass=0·00125). [*D.S.S.E., 1986*]

20. A metallic ball has a radius 9 cm at 0°C. Calculate the change in volume when it is heated to 90°C. (Coefficient of linear expansion of the metal=12×10^{-6} S.I. unit). [*A.I.S.S.E., 1986*]

21. In a Joule experiment two weights of 5 kg each fall through a height of 3 m and rotate a paddle wheel which stirs 0·1 kg of water. What is the change in temperature of the water ?

22. In a Joule experiment two weights each of 14 kg falls through a height of 2·1 m and rotate a paddle wheel which stirs the water in calorimeter. If the weights are allowed to fall 50 times, calculate the rise in temperature of the water. (Water equivalent of calorimeter=7 kg).

23. In a Joule experiment two weights each of 2·5 kg fall through a height of 4 m and rotate a paddle wheel which stirs 0·15 kg of water in a calorimeter whose water equivalent is 2·5 kg. If the weights are allowed to fall 250 times, what is the rise in temperature. Assume that the velocity acquired by the weights is zero.

24. In a Joule experiment two weights each of 3 kg fall through a height of 2 m. If the weights fall 100 times and rise in temperature is 5·6°C, calculate the value of J. Water equivalent of calorimeter is 0·3 kg and water in it is 0·2 kg.

25. The height of Niagra fall is 50 m. Calculate the difference of temperature at the top and bottom of the fall. Assume all the heat produced is absorbed by water.

26. The height of Bhakra fall is 210 m. Calculate the difference of temperature at the top and the bottom of the fall and assuming that all the energy is used in heating of water.

27. From what height must a piece of ice must be dropped so that it may just melt completely on striking the ground. (L=80 k cal kg^{-1}, J=4200 J Kcal^{-1}).

28. A 1·5 g bullet moving with a speed of 500 ms^{-1} becomes dead stop on striking with a target. How much heat is produced ?

29. A piece of metal (specific heat=0·03) is dropped from a certain height so that rise in temperature of the piece is 20°C. Calculate the height. (J=4·2 J cal^{-1}).

30. From what height must a body be dropped so that its temperature increases by 0·2°C. Assume that only 30% heat produced is absorbed by the body. (Sp. heat of body=0·07).

31. Calculate the velocity with which a hailstorm at 0°C must strike the ground in order that if $\frac{5}{8}$ of its kinetic energy were converted into heat in the hailstorm, the whole just melted into water.

32. An electric kettle of 500 watt is used to heat the 200 g of water for 1·4 minute. Calculate the rise in temperature of water.

33. An electric kettle increases the temperature of 1 kg of water by 30°C in 4 minutes. Calculate the power of electric kettle.

34. 3 moles of an ideal gas kept at a constant temperature of 300 K are compressed from a volume of 4 litres to a volume of 1 litre. Calculate the work done in the process [R=8·31 J mole^{-1} k^{-1}].

35. A lead bullet just melts when stopped by an obstacle. Assume that 25% of the heat is absorbed by the obstacle, find the velocity of the bullet if its initial temperature is 27°C. (Melting point of lead=327°C and heat of fusion of lead=6 cal/g and sp. heat of lead=0·03 cal g^{-1} °C^{-1}).

[I.I.T. J.E.E. 1981]

36. Calculate the efficiency of a Carnot's engine working between 927°C and 27°C.

37. A perfect Carnot engine utilizes an ideal gas. The source temperature is 500 k and that of sink is 375 k. If the engine takes 600 k cal. per cycle from the source, calculate
    (*a*) Efficiency of the engine.
    (*b*) Work done per cycle.
    (*c*) The heat rejected to the sink per cycle.

[A.I.S.S.E. 1982, D.S.S.E. 1982]

38. Calculate the work done if one mole of an ideal gas in compressed isothermally at a temperature 27°C from a volume of 5 litres to 1 litre. ($R=8{\cdot}31$ J $mole^{-1}$ k^{-1})

[*D.S.S.E. 1978 Comp.*]

39. Calculate the change in internal energy of a block of copper of mass 400 g when it is heated from 25°C to 275°C. Assume change in volume is negligible (sp. heat of copper=0·1).

40. By how many Joules does the internal energy increase when 50 g of ice at 0°C melts to form water at 0°C ? Latent heat of fusion of ice 80 cal g^{-1}. Neglect the small change in volume.

41. 1 g of ice at −5°C is made to melt to form water at 80°C. If the change in volume is negligible, calculate the increase in the internal energy of ice. Latent heat of ice=80 cal g^{-1} and sp. heat of ice=0 5).

42. How much work is done by a gas in expanding from a volume of 5 c c. to 25 cc. against a constant pressure of 2 atmosphere.

43. Two kg of water having volume of 2 litres become 3342 litres of steam when boiled at a pressure of 1 atmosphere. Calculate the work done.

44. Two moles of an ideal gas at 102°C are compressed isothermally from a volume of 7·5 litres to a volume of 1·5 litre. Calculate the work done by the system.

45. 3 kg of water at 100°C is converted into steam at the same temperature. The volume of 1 c.c. of water becomes 1671 c.c. of steam of boiling at 1 atm. Calculate the change in the internal energy of the system if latent heat of vapourisation of water is 536 cal g^{-1}.

46. Find the efficiency of a Carnot's engine working between 227°C and 27°C.

47. A steam engine takes heat from a boiler at 177°C and rejects it to sink at 27°C. Calculate its efficiency.

48. A steam engine takes 250 k cal of heat from a boiler and rejects 150 k cal of heat to air. Calculate its efficiency.

49. A Carnot's engine whose temperature of the source is 480 k takes 600 k cal. of heat from it and rejects 400 k cal. to sink. What is the temperature of the sink ? Calculate also the efficiency of the engine.

50. In a refrigerator heat from inside at 280 k is transferred to a room at 300 k. (*a*) What is the coefficient of performance of refrigerator ? (*b*) How many joules of heat will be delivered to a room for each two joule of electric energy consumed ideally ?

51. A Carnot's engine working as a refrigerator between 250 k and 300 k receives 1 k cal. of heat from reservoir at lower temperature. Calculate (*a*) the C.O.P. of refrigerator, (*b*) the

amount of heat rejected to the reservoir at the higher temperature, and (*c*) the amount of work done in each cycle to operate the refrigerator.

52. Find the change in the internal energy in calories during an adiabatic compression if 168 J of work is done on the gas.

53. A Carnot's refrigerator takes 60 k cal. of heat from the freezing chamber at 250 k and rejects heat at 300 k. (*a*) How much work is done ? (*b*) How many calories are rejected ? (*c*) Calculate C.O.P. of refrigerator.

54. In a diesel engine the rate of production of heat due to the combustion of the fuel is 5×10^6. calories per hour. The efficiency of the engine is 40%. Calculate the horse power of the engine. (Given J=4·2 J/cal.).

55. What is the minimum power that must be supplied to a refrigerator that freeze 2 kg of water at 0°C into ice at 0°C in a time interval of 5 minutes. The room temperature is 20°C.

56. The motor in a certain refrigerator has a power input of 200 watt. The freezer is maintained at −3°C and the outside air at 27°C. Calculate (*a*) the shortest time in which 1·8 kg of water at 0°C can convert into ice, (*b*) the minimum amount of heat which can be extracted from the freezer in 10 minutes.

57. A domestic freeze which is working between inside temperature of −3°C and outside atmosphere temperature of 17°C, can be regarded as a reversible engine. Calculate, (*a*) its C.O.P., (*b*) the energy required to freeze 1 kg of water at 20°C, (*c*) what is the power of the freeze if the said water freezes in 4 minutes.

58. When a system is taken from *a* to *c* along the path *abc*, it is found that Q=75 cal and W=30 cal. If it is taken from *a* to *c* along the path *ab′c*, it is found that Q=60 cal. Find the value of 'W' along this path (Fig. 11·4).

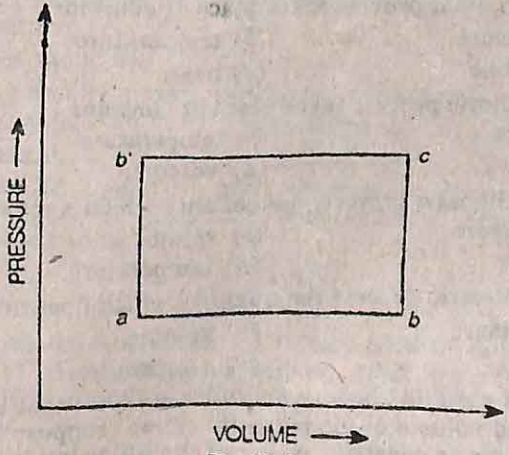

Fig. 11.4.

59. If in the Problem 58, Q=100 cal and W=30 cal along the path *abc*, what is the value of Q along the path *ab′c* if W along this path is 15 cal.

60. A cyclic process ABCA shown is V-T diagram (Fig. 11·5) is performed with constant mass of an ideal gas. Show the same process on a P-V diagram.

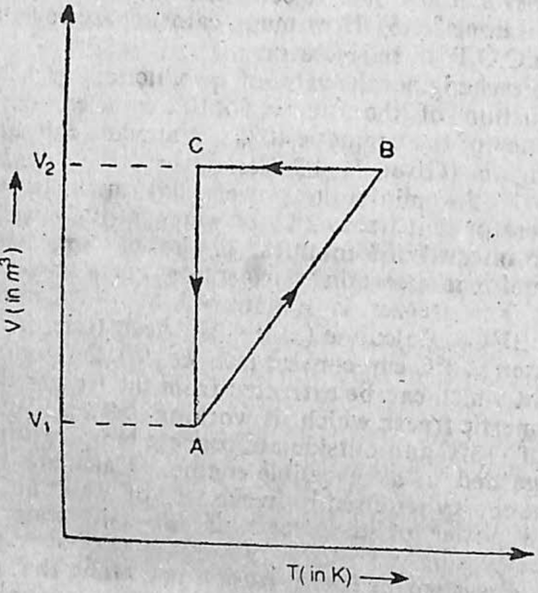

Fig. 11.5.

OBJECTIVE TYPE QUESTIONS

61. An isothermal process takes place at constant
(*a*) pressure
(*b*) temperature
(*c*) volume
(*d*) heat.

62. An isochoric process takes place at constant
(*a*) pressure
(*b*) temperature
(*c*) heat
(*d*) volume.

63. In an adiabatic process, the quantity which remains constant is
(*a*) pressure
(*b*) volume
(*c*) heat
(*d*) temperature.

64. In an isobaric process the quantity which does not change is
(*a*) pressure
(*b*) volume
(*c*) heat
(*d*) temperature.

65. When air is filled in a cycle tube, we find that both the pressure and volume of air increase. If we suppose the temperature of air as constant, which of the following is correct.

(*a*) Boyle's law can be applied.
Boyle's law cannot be applied because
(*b*) temperature is constant
(*c*) mass of the air is constant
(*d*) mass of the air is not constant.

66. During an adiabatic compression the gas warms up because
(*a*) internal energy remains constant
(*b*) internal energy increases
(*c*) internal energy decreases
(*d*) nothing can be decided.

67. The volume of a gas becomes twice on expanding isothermally. What will its pressure become ?
(*a*) One-fourth (*b*) Twice
(*c*) Half (*d*) Unchanged.

68. If the door of a freeze in a room is kept opened. The temperature of the room
(*a*) rises (*b*) falls
(*c*) falls very slowly (*d*) unchanged.

69. The work done by a gas during isochoric process is
(*a*) zero (*b*) $P(V_2-V_1)$
(*c*) $nRT \log_e \left(\frac{V_2}{V_1}\right)$ (*d*) none of them.

70. The work done by a gas depends upon
(*a*) initial state
(*b*) final state
(*c*) path along which the gas is taken from initial state to final state
(*d*) none of the above.

71. The work done by a gas is maximum if it expands
(*a*) isothermally (*b*) isochorically
(*c*) adiabatically (*d*) isobarically.

72. A gas is heated under constant pressure so that its volume becomes double. If the initial temperature of gas was 27°C, its final temperature will be
(*a*) 54°C (*b*) 108°C
(*c*) 327°C (*d*) 927°C.

73. If a gas is heated such that its volume remains constant. If the pressure of gas becomes 4 times, what will be the final temperature of the gas if its initial temperature was 27°C.
(*a*) 54°C (*b*) 108°C
(*c*) 327°C (*d*) 927°C.

74. What is the efficiency of a Carnot's heat engine if it operates between 500 k and 300 k ?

(*a*) 60% (*b*) 40%
(*c*) 20% (*d*) None of these.

75. The efficiency of an engine which has a work output 2100 J per k cal of heat input is

(*a*) 50% (*b*) 42%
(*c*) 40% (*d*) 21%.

76. The amount of work done to convert 1 g of ice at 0°C into steam at 100°C is approximately

(*a*) $720 \times 4{\cdot}2$ J (*b*) $180 \times 4{\cdot}2 \times 10^7$ J
(*c*) $640 \times 4{\cdot}2$ J (*d*) $620 \times 4{\cdot}2 \times 10$ J.

[*B.H.U. 1983*]

77. The internal energy is a unique function of any state because change in it

(*a*) depends upon path
(*b*) does not depend upon path
(*c*) corresponds to an adiabatic process
(*d*) corresponds to an isothermal process. [*B.H.U. 1983*]

78. There are two Carnot's engines working between the same temperatures of the reservoirs and the sink. If Q_1 and Q_2 be the amounts of heat extracted from the reservoir and Q_1' and Q_2' be the heats rejected to the sink, then which of the following statements is likely to be true ?

(*a*) $Q_1 = Q_1'$ (*b*) $Q_2 = Q'_2$
(*c*) $Q_1 - Q_2 = Q_1' - Q'_2$ (*b*) $\dfrac{Q_1}{Q_2} = \dfrac{Q_1'}{Q'_2}$.

[*A.I.I.M.S. 1983*]

79. Water is heated by

(*a*) conduction (*b*) convection
(*c*) radiation (*d*) none of them.

80. For same length and rise in temp. which will expand maximum

(*a*) Iron (*b*) Steel
(*c*) Copper (*d*) Brass.

☐ ☐

12

Harmonic Oscillations

IMPORTANT FORMULAE

1. The displacement (ξ) of a particle executing harmonic oscillations, at an instant 't' is given by,

$$\boxed{\xi = a\cos(\omega t + \phi_0)}$$

; a = Displacement amplitude
ω = Angular frequency
ϕ_0 = Initial phase

2. Frequency,

$$\boxed{\nu = \frac{1}{T}}$$

; T = Time period

3. Angular frequency,

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi\nu}$$

4. Velocity amplitude *i.e.*, maximum velocity,

$$\boxed{V_0 = \omega a = 2\pi\nu a}$$

5. Acceleration amplitude *i.e.*, maximum acceleration,

$$\boxed{g_0 = \omega^2 a = 4\pi^2\nu^2 a}$$

6. Velocity at any instant,

$$\boxed{V = \omega\sqrt{a^2 - \xi^2}}$$

7. Restoring force,

$$\boxed{F = -K\xi}$$

; K = Force constant

8. Frequency of harmonic oscillation,

$$\boxed{\nu=\frac{1}{2\pi}\sqrt{\frac{\text{Spring factor}}{\text{Inertia factor}}}=\frac{1}{2\pi}\sqrt{\frac{K}{m}}}\;;$$

m=mass of oscillating body

9. Frequency of angular harmonic oscillation,

$$\boxed{\nu=\frac{1}{2\pi}\sqrt{\frac{C}{I}}}$$

C=Restoring torque per unit radian twist
I=Moment of Inertia of oscillating body

10. Frequency of simple pendulum,

$$\boxed{\nu=\frac{1}{2\pi}\sqrt{\frac{g}{l}}}$$

g=Acceleration due to gravity
l=Length of the pendulum

11. Frequency of Liquid column in a U-tube

$$\boxed{\nu=\frac{1}{2\pi}\sqrt{\frac{2g}{L}}}$$

L=Length of liquid column

12. Frequency of a L-C electric oscillator,

$$\boxed{\nu=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}}$$

L=Inductance
C=Capacitance

13. Energy of oscillation,

$$\boxed{\begin{aligned}E&=E^0_k=\tfrac{1}{2}mV_0^2\\&=E^0_p=\tfrac{1}{2}ka^2\end{aligned}}$$

E^0_k=Maximum K.E.
E^0_p=Maximum P.E.

SOLVED EXAMPLES

Example 1. *The body of mass 16 kg is oscillating on a spring of force constant 256 Nm^{-1}. Deduce angular frequency and frequency.*

Solution. $m=16$ kg, $K=256\ Nm^{-1}$

$$\omega=\sqrt{\frac{K}{m}}=\sqrt{\frac{256}{16}}=4\text{ Hz}$$

$$\nu=\omega/2\pi=\frac{4}{2\pi}=\frac{2}{\pi}\text{ Hz.}$$

Example 2. *A spring is stretched by 0·02 m with a load of 0·3 kg wt. If a body of mass of 0·4 kg is attached to the spring and it* *is set into vibrations with an amplitude 0·10 m, what will be (a) spring constant, (b) frequency, (c) velocity amplitude, (d) acceleration amplitude.*

Solution. $\xi=0{\cdot}02$ m, $F=0{\cdot}3\times9{\cdot}8$ N

(*a*) $\therefore \quad K=\dfrac{F}{\xi}=\dfrac{0{\cdot}3\times9{\cdot}8}{{\cdot}02}=3\times49=147\ \text{Nm}^{-1}$

(*b*) $m=0{\cdot}4$ kg, $K=147\ \text{Nm}^{-1}$.

$\therefore \quad \nu=\dfrac{1}{2\pi}\sqrt{\dfrac{K}{m}}=\dfrac{1}{2\pi}\sqrt{\dfrac{147}{0{\cdot}4}}=\dfrac{19{\cdot}17}{6{\cdot}28}=3$ Hz

(*c*) $V_0=2\pi\nu a=2\times3{\cdot}14\times3\times0{\cdot}1=1{\cdot}884\ \text{ms}^{-1}$.

(*d*) $g_0=(2\pi\nu)^2.\ a=2\pi\nu.\ V_0=2\times3{\cdot}14\times3\times1{\cdot}884=35{\cdot}494\ \text{ms}^{-2}$.

Example 3. *A body of mass 0·2 kg is executing simple harmonic motion with a periodic time of 1 sec. and amplitude 10 cm. Calculate (a) maximum velocity (b) maximum acceleration (c) maximum restoring force (d) velocity at a displacement of 6 m.*

Solution. $m=0{\cdot}2$ kg, $T=1$ s, $a=10$ cm$=0{\cdot}1$ m

$$\omega=\frac{2\pi}{T}=\frac{2\times3{\cdot}14}{1}=6{\cdot}28\ \text{Hz}$$

(*a*) $V_0=\omega.a=6{\cdot}28\times0{\cdot}1=0{\cdot}628\ \text{ms}^{-1}$

(*b*) $g_0=\omega^2a=\omega.V_0=6{\cdot}28\times0{\cdot}628=3{\cdot}944\ \text{ms}^{-2}$

(*c*) $F_{max}=mg_0=0{\cdot}2\times3{\cdot}944=0{\cdot}7888$ N

(*d*) $V=\omega\sqrt{a^2-\xi^2}=6{\cdot}28\sqrt{(0{\cdot}1)^2-({\cdot}06)^2}=0{\cdot}5024\ \text{ms}^{-1}$.

Example 4. *A harmonic oscillation is represented by $\xi=0{\cdot}34\cos(3000t+0{\cdot}74)$, where ξ and t are in mm and sec respectively. Calculate (a) the amplitude, (b) angular frequency, (c) frequency, (d) the time period, (e) the initial phase.*

Solution. On comparing the given equation with the standard equation for displacement,

$\xi=a\cos(\omega t+\phi_0)$, we have

(*a*) amplitude, $a=0{\cdot}34$ mm

(*b*) Angular frequency, $\omega=3000$ Hz

(*c*) Frequency, $\nu=\dfrac{\omega}{2\pi}=\dfrac{3000}{2\pi}\ \text{Hz}=\dfrac{1500}{\pi}\ \text{Hz}$

(*d*) Time period, $T=\dfrac{1}{\nu}=\dfrac{\pi}{1500}$ sec.

(*e*) Initial phase $\phi=0{\cdot}74$ radians.

Example 5. *What is the time period of a simple pendulum 2·45 metre long ?*

Solution. Time period,

$$T=2\pi\sqrt{\frac{l}{g}}$$

$$=2\pi\sqrt{\frac{2{\cdot}45}{9{\cdot}8}}=2\pi\times\tfrac{1}{2}=\pi s$$

$$=3{\cdot}14\ s.$$

Example 6. *A simple pendulum of length 1 metre has oscillation-energy equal to 0·3 joule when its amplitude is 0·02 metre. What will be its energy if (a) its length is increased to 1·5 metre, (b) its amplitude is increased to 0·03 metre ?*

Solution. $E=\frac{1}{2}\,mv_0^2$

or $E=\frac{1}{2}\,m\omega^2a^2$

But for a simple pendulum,

$$\omega^2=\frac{g}{l}$$

$$\therefore \qquad E=\frac{1}{2}m\,\frac{g}{l}\,a^2$$

(*a*) $$0{\cdot}3=\frac{1}{2}m\,\frac{g}{1}\,a^2$$

and $$E'=\frac{1}{2}m\,\frac{g}{1{\cdot}5}\,a^2$$

$$\therefore \qquad \frac{E'}{0{\cdot}3}=\frac{1}{1{\cdot}5}$$

$$E=0{\cdot}2\ J$$

(*b*) $$0{\cdot}3=\frac{1}{2}m\,\frac{g}{l}({\cdot}02)^2$$

$$E''=\frac{1}{2}m\,\frac{g}{l}({\cdot}03)^2$$

$$\therefore \qquad \frac{E''}{0{\cdot}3}=\frac{({\cdot}03)^2}{({\cdot}02)^2}$$

$$E''=0{\cdot}675\ J.$$

Example 7. *Certain scale in a spring balance is marked from 0 to 250 g whose length is 10 cm. What is the spring constant ? An object of 100 g is hung from the balance and set in harmonic oscillations. What will be the acceleration of the object when the scale reads 150 g ?*

Solution.

$$m=250\ g=0{\cdot}250\ kg,\ g=9{\cdot}8\ ms^{-2},\ x=10\ cm=0{\cdot}10\ m$$

$$K=\frac{F}{x}$$

$$= \frac{mg}{x}$$

$$= \frac{0{\cdot}25 \times 9{\cdot}8}{0{\cdot}1} = 24{\cdot}5 \text{ Nm}^{-1}$$

Now $\qquad m = 100 \text{ g} = 0{\cdot}100 \text{ kg}$

$\therefore \qquad \omega^2 = \dfrac{K}{m}$

$$= \frac{24{\cdot}5}{0{\cdot}1} = 245$$

Now $\qquad x = \dfrac{(150-100)\times 0{\cdot}1}{250} = {\cdot}02 \text{ m}$

$\therefore$ Acceleration, $\ddot{\xi} = \omega^2 x$

$$= 245 \times {\cdot}02$$

$$= 4{\cdot}90 \text{ ms}^{-2}.$$

Example 8. *A pendulum clock shows correct time. Calculate the error in time per day if the length of the pendulum increases by 0·05%.*

Solution. Let T be the correct time period of pendulum and T′ be its incorrect time. Then

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ and } T' = 2\pi\sqrt{\frac{l'}{g}}$$

$$\therefore \qquad \frac{T'}{T} = \sqrt{\frac{l'}{l}} = \sqrt{\frac{l + \frac{0{\cdot}05}{100}\, l}{l}}$$

$$= (1 + {\cdot}0005)^{1/2}$$

$$= (1 + \tfrac{1}{2} \times {\cdot}0005)$$

$$\therefore \qquad \frac{T'}{T} = (1 + {\cdot}00025)$$

Thus if correct time is 1 sec., the incorrect time is 1·00025 sec. Hence in every second the clock goes slow by ·00025 sec. Therefore in one day *i.e.*, $24 \times 60 \times 60$ sec. the clock will get slow by

$${\cdot}00025 \times 24 \times 60 \times 60 = 21{\cdot}6 \text{ s.}$$

Example 9. *Two springs of spring constant K_1 and K_2 are joined to a body in the following three different ways (Fig. 12.1). Deduce the spring factor for the oscillation of the body in each case.*

Solution. Let the ring (body) be displaced in the downward direction by x, then

(*a*) For Fig. 12.1 (A), the two springs are extended by the same amount x, the restoring force due to them will be given by

$$F_1 = K_1 x$$
$$F_2 = K_2 x$$

and the total restoring force,

$$F = F_1 + F_2$$
$$= K_1 x + K_2 x$$

or
$$F = (K_1 + K_2) x$$

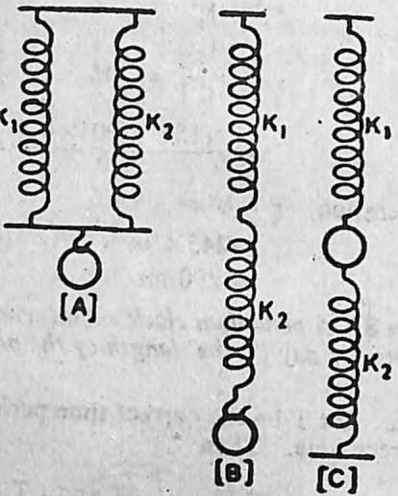

Fig. 12.1.

or $\frac{F}{x} = (K_1 + K_2)$. So the restoring force per unit displacement or spring factor K for the oscillation of the ring will be

$$\boxed{K = (K_1 + K_2)}$$

(*b*) For Fig. 12.1 (B), the two springs are extended by the different amounts say, x_1 and x_2 by the same force F, then

$$F = K_1 x_1 \text{ or } x_1 = F/K_1$$

and
$$F = K_2 x_2 \text{ or } x_2 = F/K_2$$

Therefore resultant displacement x of the ring will be

$$x = x_1 + x_2$$

or
$\frac{F}{K} = \frac{F}{K_1} + \frac{F}{K_2}$, where K is spring factor for the combination.

or
$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\therefore \qquad \boxed{K = \frac{K_1K_2}{K_1+K_2}}$$

(*c*) For Fig. 12.1 (C), the upper spring is extended while the lower spring is compressed. Since the extension of the one spring is equal to the compression of the other, the restoring force due to them will be given by

$$F_1 = K_1x$$
$$F_2 = K_2x$$

$\therefore$ The total restoring force,

$$\begin{aligned} F &= F_1+F_2 \\ &= K_1\,x + K_2\,x \\ &= (K_1+K_2)\,x \end{aligned}$$

or
$$\frac{F}{x} = (K_1+K_2)$$

Now the Spring factor for the combination *i.e.*, oscillation of the ring will be given by

$$K = \frac{F}{x}$$

$$\therefore \qquad \boxed{K = (K_1+K_2)}$$

Example 10. *A vertical U-tube of uniform cross-section contains water upto 80 cm. Find the frequency of oscillation if water on one side is depressed and then released.*

Solution. $L = 2\times 80 = 160$ cm ; $g = 980$ *cm* s^{-2}

$$\begin{aligned} \therefore \qquad \nu &= \frac{1}{2\pi}\sqrt{\frac{2g}{L}} \\ &= \frac{7}{2\times 22}\sqrt{\frac{2\times 980}{160}} \\ &= \frac{7}{44}\times\frac{7}{2} = \frac{49}{88} = 0{\cdot}557 \text{ Hz.} \end{aligned}$$

Example 11. *Derive the expression for the oscillation energy of an LC electrical oscillator with the potential amplitude V_0 and the current amplitude I_0.*

Solution. K and *m* of mechanical system are replaced by $\frac{1}{C}$ and L respectively in a LC electric circuit and '*a*' is replaced by Q_0. Hence $E = \frac{1}{2}Ka^2$ becomes

$$E = \frac{1}{2}\,\frac{1}{C}\,Q_0^2 \qquad \text{...(1)}$$

$$=\frac{1}{2}C\left(\frac{Q_0}{C}\right)^2$$

But
$$\frac{Q_0}{C}=V_0$$

$\therefore$
$$\boxed{E=\tfrac{1}{2}CV_0^2}$$

The angular frequency with which LC circuit oscillates is given by

$$\omega=\frac{1}{\sqrt{LC}} \quad \text{or} \quad \frac{1}{C}={}^{2}\text{ʍ}L$$

and also
$$I_0=\omega Q_0$$

or
$$Q_0=\frac{I_0}{\omega}$$

$\therefore$ Eqn (1) becomes

$$E=\tfrac{1}{2}(\omega^2L)\left(\frac{I_0}{\omega}\right)^2$$

$$\boxed{E=\tfrac{1}{2}LI_0^2}$$

Example 12. *The balance wheel of a watch has a moment of inertia of 2×10^{-8} kg m² and the torsion constant of its hair spring is $9{\cdot}8\times10^{-6}$ Nm rad⁻¹. Calculate its frequency.*

Solution. $I=2\times10^{-8}$ kg m², $C=9{\cdot}8\times10^{-6}$ Nm rad^{-1}. For a balance wheel,

$$\nu=\frac{1}{2\pi}\sqrt{\frac{C}{I}}$$

$\therefore$
$$\nu=\frac{7}{2\times22}\sqrt{\frac{9{\cdot}8\times10^{-6}}{2\times10^{-8}}}$$

$$=\frac{7\times7\sqrt{10}}{44}=3{\cdot}52 \text{ Hz.}$$

Example 13. *A disc of mass 6 kg and radius 10 cm is suspended horizontally by a long wire which is attached to the centre of the disc. The disc is turned through 60° by a force of 1 N acting tangential to the disc. When the force is removed, it executes angular oscillations. Find the time period of oscillation.*

Solution. $M=6$ kg ; $r=10$ cm $=0{\cdot}1$ m ;

$\theta=60^\circ=60\times\frac{\pi}{180}$ rad. $=\frac{\pi}{3}$ rad. and $F=1$ N.

Torque, $\tau = Fr = 1 \times 0{\cdot}1 = 0{\cdot}1$ Nm

$$C = \frac{\tau}{\theta} = \frac{0{\cdot}1 \times 3}{\pi} = \frac{0{\cdot}3 \times 7}{22}$$

Moment of Inertia of disc,

$$I = \tfrac{1}{2}Mr^2 = \tfrac{1}{2} \times 6 \times 0{\cdot}1^2 = 0{\cdot}03 \text{ kg m}^2.$$

Time period of oscillation,

$$T = 2\pi\sqrt{\frac{I}{C}}$$

$$= \frac{2 \times 22}{7}\sqrt{\frac{0{\cdot}03 \times 22}{0{\cdot}3 \times 7}} = 3{\cdot}5 \ s.$$

Example 14. *The bottom of a dip on a road has a radius of curvature 'R'. A rickshaw of mass 'M' left a little away from the bottom oscillates about this dip. Deduce an expression for the period of oscillation.*

Solution. Let the rickshaw be displaced from its initial position A to a new position B by an amount ξ. The component 'Mg cos θ' of weight Mg of the rickshaw is balanced by normal reaction N of the ground and the component 'Mg sin θ' gives the necessary restoring force to bring it to its normal position, hence

Restoring Force=Mg sin θ

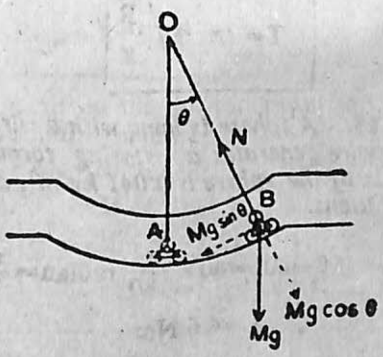

Fig. 12.2.

Now if θ is very small and measured in radian,

$$\sin\theta = \theta = \frac{AB}{OA}$$

$$= \frac{\xi}{R}$$

$$\therefore \qquad F = -Mg\ \frac{\xi}{R}$$

Negative sign shows that ξ and F are opposite in direction.

But $$F=M\ddot{\xi} \text{ (Newton's 2nd law of motion)}$$

$$\therefore \quad M\ddot{\xi}=-Mg\,\frac{\xi}{R}$$

or $$\ddot{\xi}=-\frac{g}{R}\,\xi \qquad ...(1)$$

This equation represents harmonic oscillations of rickshaw. But we also know for a harmonic oscillation,

$$\ddot{\xi}=-\omega^2\xi \qquad ...(2)$$

Hence comparing equations (1) and (2), we have

$$\omega^2=\frac{g}{R}$$

or $$\omega=\sqrt{\frac{g}{R}}$$

But time period of oscillations,

$$T=\frac{2\pi}{\omega}$$

$$\therefore \quad \boxed{T=2\pi\sqrt{\frac{R}{g}}}$$

Example 15. *A sphere is hung with a wire. 60° rotation of the sphere about the wire generates a restoring torque of 4·6 Nm. If the moment of inertia of the sphere is 0·041 kg m², deduce the frequency of angular oscillations.*

Solution. $$\theta=60^\circ=60\times\frac{\pi}{180}\text{ radian}=\frac{\pi}{3}\text{ radian}$$

$$\tau=4{\cdot}6\text{ Nm}$$

$$\therefore \quad C=\frac{\tau}{\theta}=\frac{4{\cdot}6\times3}{\pi}\text{ Nm rad}^{-1}$$

and $$I=0{\cdot}041\text{ kg m}^2$$

$$\therefore \quad \nu=\frac{1}{2\pi}\sqrt{\frac{C}{I}}$$

$$=\frac{7}{2\times22}\sqrt{\frac{4{\cdot}6\times3\times7}{22\times0{\cdot}041}}$$

$$=\frac{7}{44}\sqrt{107{\cdot}09}$$

$$=1{\cdot}65 \text{ Hz.}$$

Example 16. *A person normally weighing 50 kg stands on a platform which oscillates up and down harmonically at a frequency* $2{\cdot}0\ s^{-1}$ *and an amplitude* $0{\cdot}05$ *m. If a machine on the platform gives the person's weight against time, deduce the maximum and minimum readings it will show (Take* $g=10\ ms^{-2}$*).*

Solution. $M=50$ kg ; $g=10\ ms^{-2}$;

$a=0{\cdot}05$ m ; $\nu=2{\cdot}0\ s^{-1}$

Maximum acceleration of platform,

$$g_0=4\pi^2\nu^2 a$$

$$=4\times\left(\frac{22}{7}\right)^2\times 2^2\times{\cdot}05=7{\cdot}9\ ms^{-2}$$

When the platform is at the highest point of oscillation, the machine will show the maximum reading,

$$R_{max}=Mg+Mg_0$$

$$=M\,(g+g_0)$$

$$=50\,(10+7{\cdot}9)$$

$$=895\text{ N}=\frac{895}{10}\text{ kg wt.}$$

or $$\boxed{R_{max}=89{\cdot}5\text{ kg wt.}}$$

When the platform is at the lowest point of oscillation, the machine will show minimum reading,

$$R_{min}=Mg-Mg_0$$

$$=M\,(g-g_0)$$

$$=50\,(10-7{\cdot}9)$$

$$=105\text{ N}=\frac{105}{10}\text{ kg wt.}$$

or $$\boxed{R_{min}=10{\cdot}5\text{ kg wt.}}$$

Example 17. *A cylindrical wooden block of cross-section* $15{\cdot}0$ *cm*^{2} *and mass 200 g is floated over water with an extra weight 80 g attached to its bottom. The cylinder floats vertically. From the state of equilibrium it is slightly depressed and released. Deduce the frequency of oscillation of the block.*

Solution. $m=200+80=280$ g ; $A=15{\cdot}0$ cm^2 ; $\rho=1$ g cm^{-3} (density of water).

If the cylinder of area of cross-section 'A' is depressed by an amount ξ, then

$$K=\frac{F}{\xi}=\frac{\text{weight of extra water displaced}}{\xi}$$

$$=\frac{(A\times\xi)\,\rho\,g}{\xi}$$

where ρ is the density of water.

$$=A\rho g$$

$$=15\times1\times980=14700 \text{ dyne/cm.}$$

$\therefore$ Frequency of oscillation,

$$\nu=\frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

$$=\frac{7}{2\times22}\sqrt{\frac{14700}{280}}$$

$$\boxed{\nu=1\cdot15 \text{ Hz}}$$

Example 18. *Write the displacement's equation representing the following conditions obtained in a simple harmonic motion :*

Amplitude=0·01 m ; Frequency=600 Hz and initial phase π/6.

[D.S.S.E. 1984]

Solution. The general equation of simple harmonic motion is,

$$\xi=a\sin(2\pi\,\nu t+\phi_0)$$

$$=0\cdot01\sin(2\pi\times600t+\pi/6)$$

$$\xi=0\cdot01\sin(3768t+\pi/6)\text{ metre.}$$

Example 19. *Two masses m_1 and m_2 are suspended together on a vertical spring of spring constant 'k'. When the masses are in equilibrium, m_1 is removed without disturbing the system. Find the angular frequency and amplitude of oscillation of m_2.* *[I.I.T. 1981]*

Solution.

Let $AB=a$ $BC=a_1$ and $CD=a_2$

$\therefore$ $a_1k=m_2g$ ...(1)

and $a_2k=m_1g$ ...(2)

On removing m_1, the mass m_2 on the spring will obviously oscillate up and down about the point C in simple harmonic oscillation. Its angular frequency will be

$$\boxed{\omega=\sqrt{\frac{k}{m_2}}}$$

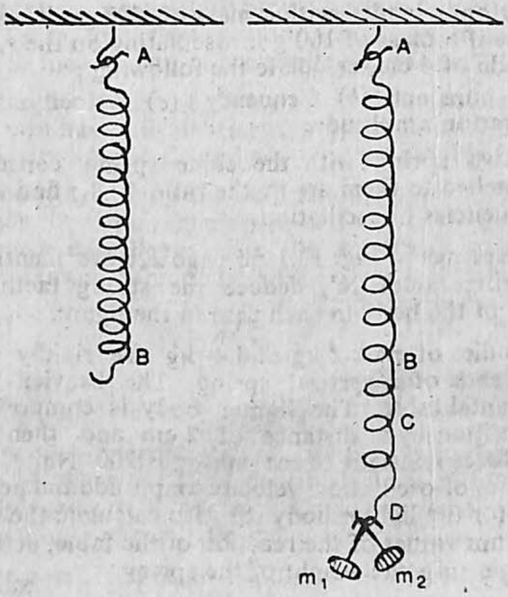

Fig. 12.3.

The amplitude of oscillation of mass m_2 is a_2 which may be given by Eq. (2), as

$$\text{Amplitude} = \frac{m_1 g}{k}.$$

EXERCISE 12

[Take $\pi = 3{\cdot}14$]

1. The bodies of mass 9, 16, 25, 36 kg are oscillating in turn on a spring of force constant 36 $\mathrm{Nm^{-1}}$. Deduce (*a*) angular frequency, (*b*) frequency, and (*c*) time period in each case.
2. When a spring is stretched by 0·1 m, a restoring force of 5 N develops. What is its spring constant ?
3. When a mass of 1 kg is hung from a spring it stretches by 50 cm. What is its force constant ? If the mass is slightly displaced and then released, find the frequency of oscillation.
4. The scale on a spring balance is marked from 0 to 100 g and has a length of 10 cm. What is its spring constant ? An object of 50 g is hung from the balance and set in harmonic oscillation. What will be the acceleration of the object when the scale reads 60 g ?
5. A spring of spring constant K is cut into three equal pieces. What will be the value of spring constant for each piece ?

6. When a spring is loaded with a mass of 200 g, it extends by 5 cm. Now if a mass of 100 g is oscillating on the spring with an amplitude of 4 cm, calculate the following :
   (*a*) spring constant (*b*) frequency (*c*) velocity amplitude (*d*) acceleration amplitude.
7. There are two springs with the same spring constant. The masses attached to them are in the ratio 4 : 1 ; find the ratio of the frequencies of oscillations.
8. If the two springs in Fig. 12.1 on page 228 are identical, each with a spring factor 'K', deduce the spring factor for the oscillation of the body in each case in the figure.
9. The two bodies of mass 2 kg and 4 kg are rigidly connected to the two ends of a vertical spring. The heavier body rests on a horizontal table. The lighter body is compressed from the rest position by a distance of 2 cm and then released. (*a*) If the force constant of the spring is 200 Nm^{-1}, calculate the frequency of oscillation, velocity amplitude and acceleration amplitude for the lighter body (*b*) Also calculate the maximum and minimum values of the reaction of the table, acting on the system, neglecting the weight of the spring.
10. Two springs A and B have their force constant K_1 and K_2 ($K_1 > K_2$). On which spring is more work done (*a*) when their lengths are increased by the same amount, (*b*) when they are stretched by the same force ? [*I.I.T. J.E.E. 1978*]
11. A body of mass 250 g is performing simple harmonic motion with a time period of 0·5 seconds and amplitude 5 cm. Find its (*a*) velocity amplitude, (*b*) acceleration amplitude, (*c*) maximum restoring force.
12. If a particle is executing simple harmonic motion with an amplitude 0·04 m and velocity amplitude 6·28 ms^{-1}, calculate angular frequency and frequency of oscillation.
13. A particle executing simple harmonic motion has an acceleration π^2 ms^{-2} when its displacement is 0·04 m. Find the angular frequency and frequency of oscillation of the particle.
14. A body of mass 0·5 kg oscillates with a frequency of 5 Hz and an amplitude of 0·1 m. Calculate the following :
   (*a*) maximum restoring force.
   (*b*) total energy of motion.
15. A particle undergoing simple harmonic motion has an amplitude of 0·10 m and frequency $\frac{5}{\pi}$ Hz. Find the following :
   (*a*) its maximum velocity, (*b*) its maximum acceleration, and (*c*) its velocity when its displacement is 0·06 m.

16. A particle is executing simple harmonic motion with amplitude of 1·0 m and time period π sec. What is the displacement when the velocity of the particle is 1·2 $\mathrm{ms^{-1}}$?

17. A tuning force of frequency 512 is producing sound. If its prongs have displacement amplitude of 2 mm., calculate its velocity amplitude and acceleration amplitude.

18. A body executing simple harmonic motion, has the velocity V_1 when its displacement is x_1 and velocity V_2 when the displacement is x_2. Prove that the amplitude and time period of oscillation of the body will be given by

$$a=\sqrt{\frac{V_1^2x_2^2-V_2^2x_1^2}{V_1^2-V_2^2}}$$

and $$T=2\pi\sqrt{\frac{x_2^2-x_1^2}{V_1^2-V_2^2}}$$ [*A.I.S.S.E. 1988*]

19. A particle executing simple harmonic motion has a velocity 10 cm $\mathrm{s^{-1}}$ when the displacement is 6 cm and 8 cm $\mathrm{s^{-1}}$ when the displacement is 10 cm. Calculate its time period and the amplitude of oscillation.

20. The amplitude of an oscillating simple pendulum is 0·15 m and its time period is 6·28 sec. Calculate the following :
    (*a*) maximum velocity of the oscillating bob.
    (*b*) maximum acceleration of the oscillating bob.
    (*c*) maximum restoring force if the mass of the bob is 100 g.
    (*d*) length of the pendulum, if $g=9{\cdot}8\ \mathrm{ms^{-2}}$.

21. What will be the length of the second's pendulum at a place where $g=9{\cdot}81\ \mathrm{ms^{-2}}$?

22. If the length of a simple pendulum is increased by 9%, calculate the percentage increase in its time period.

23. A simple pendulum of length 80 cm has oscillation energy equal to 0·2 Joule when its amplitude is 10 cm. What will be its energy :
    (*a*) if its length is doubled ?
    (*b*) if its amplitude is doubled ?

24. A pendulum clock shows correct time. The length of the pendulum increases by 0·03%, calculate the error in time per day.

25. A pendulum clock beats correct time. If the length of the pendulum decreases by 0·04%, calculate the error in time per day.

26. A harmonic oscillation is represented by

$$\xi=2{\cdot}4\cos(2000t+0{\cdot}64)$$

where displacement and time are in mm and *s*. Calculate (*a*) amplitude, (*b*) angular frequency, (*c*) frequency, (*d*) time period, and (*e*) initial phase.

27. A harmonic oscillation is represented by

$$\xi = 0{\cdot}35 \sin (500t + 0{\cdot}42)$$

where the displacement and time are in cm and *s* respectively. Calculate (*a*) amplitude, (*b*) frequency, (*c*) time period, (*d*) initial phase, (*e*) velocity amplitude, and (*f*) acceleration amplitude.

28. A 1·0 g bullet is fired horizontally so that it gets embedded in a suspended sand bag of mass 0·4 kg. The sand bag starts oscillating with an amplitude of 0·02 m and makes 30 oscillations per minute. Deduce the bullet velocity.

29. A heavy sphere of mass 2 kg and radius 5 cm is suspended by a long vertical wire which is attached to the centre of the sphere. The sphere is turned through 90° by a force of 1·5 N acting tangentially to the sphere. When the force ceases to act, the sphere executes angular oscillations. Find the time period and frequency of oscillation.

30. A sphere is hung with a wire, 45° rotation of the sphere about the wire generates a restoring torque of 6 Nm. The moment of inertia of the sphere is 0·072 kg m^2. Calculate the time period and frequency of angular oscillations.

31. A person normally weighing 70 kg stands on a weighing machine in a platform which oscillates harmonically up and down with a frequency of $\frac{4}{\pi}$ Hz and an amplitude of 4 cm. Calculate the maximum and minimum readings of the machine (Take $g = 10$ ms^{-2})..

32. The cross-sectional area of a cylindrical wooden block is 25 cm^2 and its mass is 145 g. An extra mass of 100 g is attached to the bottom of the cylinder and then it is floated vertically over the water. From the state of equilibrium it is slightly depressed and then released. Calculate the frequency and time period of oscillation.

33. A test tube with cross-sectional area 12 cm^2 has a mass of 15 g. After pouring 25 g of mercury in the tube, it is floated vertically over the mercury. From the state of equilibrium it is slightly depressed and then released. Calculate the frequency of oscillations of the tube (density of mercury = 13·6 g cm^{-3}).

34. A body of mass 1 kg is executing simple harmonic motion which is given by

$$x = 6{\cdot}0 \cos\left(100t + \frac{\pi}{4}\right) \text{ cm.}$$

What is the (*a*) amplitude of displacement, (*b*) frequency, (*c*) initial phase, (*d*) velocity-amplitude, (*e*) acceleration-amplitude, (*f*) maximum kinetic energy. [*D.S.S.E. 1982*]

35. A body of mass 1 kg is made to oscillate in turn on two springs, one of force constant 1 N/m and another of 16 N/m. Deduce the time period in each case. [*A.I.S.S.* (*Comp.*) *E. 1985*]

36. The time period of a second's pendulum is 2 seconds. The spherical bob which is empty from inside has a mass of 50 g. This is now replaced by another solid bob of same radius but having different mass of 100 g. What is the new time period of the pendulum. [*N.C.E.R.T. 1972*]

37. A particle execute SHM with a period of 6 sec. and amplitude of 3 cm. What is its maximum speed ? [*A.I.I.M.S. 1982*]

38. Calculate the value of the amplitude and the angular frequency for the following simple harmonic oscillation,

$$y=0{\cdot}2\sin(99\,t+0{\cdot}36).$$

The various units are in S.I. units. [*A.I.S.S.E. 1986*]

39. The pan attached to a spring balance has a mass of 1 kg. A weight of 2 kg when placed on the pan stretches the spring by 10 cm. What is the frequency with which the empty pan will oscillate ? [*D.S.S.E. 1987*]

40. An U-tube with uniform cross-section contains mercury whose density is $13{\cdot}6\times10^3$ kg/m^3. The cross-section of the tube is 1 cm^2 and the total length of mercury column is 1 m. If the mercury column oscillates :

(*a*) What will be its frequency ?

(*b*) How will the frequency change if the cross-section of the tube is doubled ? [*D.S.S.E. 1987 Comp.*]

41. What is the amplitude of the wave represented by

$$y=2{\cdot}0\cos(200\,t-0{\cdot}30\,x).$$

Where distance is in centimetres and time in seconds ? [*A.I.S.S.E. 1987*]

42. Write the value of amplitude and angular frequency in the following simple harmonic wave

$$y=0{\cdot}70\cos(180\,t+0{\cdot}23)$$

where the various quantities are in M.K.S. units. [*D.S.S.E. 1985*]

43. A simple harmonic motion is represented by

$$\xi=0{\cdot}25\times10^{-3}\cos(500\,t-0{\cdot}025\,x)$$

where ξ and x are in metres and t in seconds. What is the value of (*a*) amplitude, (*b*) period ? [*D.S.S.E. 1988 Comp.*]

44. A small body of 100 g is undergoing simple harmonic motion of amplitude 100 cm and period 0·20 s. What is the maximum value of force acting on the body ?

45. A weighed glass tube is floating in liquid with 20 cm of its length immersed. It is pushed a certain distance and then released. Calculate the time period of its oscillation.
[I.I T. J.E.E. 1974]

46. A particle is executing S.H.M. of amplitude 'A'. At what displacement from the mean position is the energy half kinetic and half potential. *[A.I.S.S.E. 1985]*

47. The mass of 10 g is describing S.H.M. along a straight line with a period of 2 s and an amplitude of 10 cm. Compute its K.E. when its displacement is (*a*) 2 cm from the equilibrium position, and (*b*) 5 cm from equilibrium position.
[I.I.T.J.E.E. 1972]

48. Displacement in mm of a harmonic oscillator is represented by equation :

$$\text{ə}\,(\text{mm})=0{\cdot}26\cos(400\,t+\pi/6) \text{ where } t \text{ is in seconds.}$$

What is the (*a*) amplitude, (*b*) angular frequency, (*c*) frequency (*d*) period, and (*e*) initial phase ? *[A.I.S.S E. 1983]*

49. A harmonic oscillation of mass m is represented by

$$\xi\,(m)=a\cos(bt+c)$$

Compute (*a*) amplitude, (*b*) angular frequency, (*c*) frequency, (*d*) velocity and acceleration amplitude, (*e*) particle velocity at time '*t*'.
[A.I.S.S E. 1976]

50. A body A of mass $m_1=1{\cdot}00$ kg and a body B of mass$=4{\cdot}00$ kg are interconnected by a spring. The body A performs free vertical harmonic oscillation with amplitude $a=1{\cdot}6$ cm and angular frequency $\omega=25$ rad/sec. Neglecting the mass of the spring and taking $g=10$ N kg^{-1}. Find the maximum and minimum values of force that this system exerts on bearing surface.
[I.I.T. J.E.E. 1980]

OBJECTIVE TYPE QUESTIONS

51. Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants K_1 and K_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of vibration of M to that of N is

(*a*) $\dfrac{K_1}{K_2}$

(*b*) $\sqrt{\dfrac{K_1}{K_2}}$

(*c*) $\dfrac{K_2}{K_1}$

(*d*) $\sqrt{\dfrac{K_2}{K_1}}$

[I.I.T. J.E.E. 1988]

52. A particle executes simple harmonic motion with a frequency 'f'. The frequency with which its kinetic energy oscillates is
(a) $f/2$
(b) f
(c) $2f$
(d) $4f$.

53. In simple pendulum experiment length of the thread is 125·2 cm and diameter of the bob is 3·42 cm. The effective length will be
(a) 126·9 cm
(b) 126·91 cm
(c) 126·912 cm
(d) None of the above.
[*C.P.M.T. 1988*]

54. The length of the thread is $81{\cdot}7 \times 10^{-2}$ m and radius of the bob is $1{\cdot}74 \times 10^{-2}$ m. The effective length will be
(a) $83{\cdot}44 \times 10^{-2}$ m
(b) $83{\cdot}41 \times 10^{-2}$ m
(c) $83{\cdot}41 \times 10^{2}$ m
(d) None of the above.
[*C.P.M.T. 1988*]

55. Two simple harmonic motions are represented by the equations

$$y_1 = 10 \sin (3\pi t + \pi/4) \text{ and}$$
$$y_2 = 5 \sin (3\pi t + \sqrt{3} \cos 3\pi t)$$

Their amplitudes are in the ratio of :
(a) 2 : 1
(b) 1 : 1
(c) 1 : 2
(d) 4 : 1.

56. The frequency of a simple pendulum in a lift falling freely will be
(a) zero
(b) infinite
(c) finite
(d) undecided.

57. If the length of the pendulum is made 4 times, its time period will become
(a) 4 times
(b) twice
(c) one-fourth
(d) one-half.

58. The time period of a second pendulum is
(a) ½ s
(b) 1 s
(c) 2 s
(d) 4 s.

59. The acceleration amplitude 'g_0' of a particle executing S.H M. is
(a) $\omega^2 a$
(b) ωa
(c) ωa^2
(d) ω^2/a.

60. If the area of cross-section of U-tube is doubled without charging the length of liquid column in it, frequency of oscillation of liquid
(a) increases
(b) decreases
(c) becomes double
(d) uncharged.

61. If the water (density=1 g/c.c.) in a U-tube is replaced by mercury (density=13·6 g/c.c.). If the length of oscillating liquid column in each case is same, the ratio of frequency of oscillation of water and mercury is
    (*a*) 13·6 : 1 (*b*) 1 : 13·6
    (*c*) 1 : 1 (*d*) $\sqrt{13{\cdot}6} : 1$.

62. Resonance between driven body and driver body takes place when they have same
    (*a*) displacement amplitude (*b*) frequency
    (*c*) velocity amplitude (*d*) energy.

63. The following equation of force gives rise to S.H.M.
    (*a*) $F=-k\xi$ (*b*) $F=-k\,\xi^2$
    (*c*) $F=-k/\xi$ (*d*) $F=-k/\xi^2$.

64. If the differential equation representing the free oscillation of a body is given by $\frac{d^2\xi}{dt^2}+\omega^2\xi=0$, then the natural frequency of the body is given by
    (*a*) ω (*b*) $\frac{\pi}{\omega}$
    (*c*) $\frac{2\pi}{\omega}$ (*d*) $\frac{\pi}{2\pi}$.

65. In case of S.H.M.
    (*a*) Potential energy is conserved
    (*b*) Kinetic energy is conserved
    (*c*) Total energy is conserved
    (*d*) All the above are conserved.

66. If amplitude of oscillation is double, its energy becomes
    (*a*) four times (*b*) double
    (*c*) same (*d*) none of these.

67. If the oscillation of a particle is represented by

$$\xi=a\sin\omega t+b\cos\omega t,$$

its displacement amplitude is
    (*a*) $\frac{a}{b}$ (*b*) $\frac{b}{a}$
    (*c*) $a+b$ (*d*) $(a^2+b^2)^{1/2}$.

68. If amplitude of oscillation of a simple pendulum is doubled, its frequency will become
    (*a*) double (*b*) half
    (*c*) same (*d*) none of them.

☐☐

13

Wave Motion

IMPORTANT FORMULAE

1. The wave velocity,

$$\boxed{c=\nu\lambda}\ ;\ \lambda=\text{wavelength}$$

2. The velocity of transverse waves on a string under tension,

$$\boxed{c=\sqrt{\frac{T}{m}}}$$

T = Tension (in Newton or dyne) on the string.

; m = mass per unit length (in kg m^{-1} or g cm^{-1}) of the string.

3. The velocity of longitudinal waves in any medium (whether a solid, a liquid or a gas),

$$\boxed{c=\sqrt{\frac{E}{\rho}}}$$

; E = Elasticity of the medium

ρ = density of the medium

4. The velocity of longitudinal waves in a gaseous medium is given by

$$\boxed{c=\sqrt{\frac{\gamma P}{\rho}}}$$

(Laplace's formula ; $\gamma=\frac{C_p}{C_v}$, the ratio of two sp. heats of that gas.)

or

$$\boxed{c=\sqrt{\frac{\gamma RT}{M}}}$$

R = Gas constant

T = Temperature in K

M = Molecular weight of the gas.

5. Effect of temperature on velocity of sound,

$$\boxed{\frac{c_1}{c_2}=\sqrt{\frac{T_1}{T_2}}}$$

$$=\frac{340}{480}\text{ m.}$$

$$=\frac{17}{24}\times 100\text{ cm}=70{\cdot}83\text{ cm.}$$

Example 2. *The velocity of sound in air at 17°C is 340 ms*$^{-1}$. *Determine the velocity of sound if the temperature increases to 37°C.*

Solution. Here $c_1=340\text{ ms}^{-1}$, $T_1=273+17=290\text{ K}$,

$$T_2=273+37=310\text{ K.}$$

$$\therefore \quad \frac{c_2}{c_1}=\sqrt{\frac{T_2}{T_1}}$$

$$\therefore \quad c_2=\sqrt{\frac{310}{290}}\times 340=351{\cdot}53\text{ ms}^{-1}.$$

Example 3. *Find the temperature at which the velocity of sound in air will be thrice that at 27°C.*

Solution. Here $c_2=3c_1$

and $T_1=273+27=300\text{ K}$.

We know,

$$\frac{c_2}{c_1}=\sqrt{\frac{T_2}{T_1}}$$

$$\therefore \quad T_2=\left(\frac{c_2}{c_1}\right)^2T_1$$

$$=3^2\times 300$$

$$=2700\text{ K}$$

$$=2700-273=2427°\text{C.}$$

Example 4. *At what temperature will the velocity of sound in oxygen be the same as in hydrogen at 25°C ?*

Solution. We know

$$c=\sqrt{\frac{\gamma RT}{M}}$$

$$\therefore \quad \sqrt{\frac{\gamma RT_1}{M_1}}=\sqrt{\frac{\gamma RT_2}{M_2}}$$

$$\therefore \quad T_2=\frac{M_2}{M_1}\times T_1$$

But $\frac{M_2}{M_1}=\frac{16}{1}$, $T_1=273+25=298\text{ K}$

$$\therefore \quad T_2=16\times 298=4768\text{ K}$$

$$=4768-273=4495°\text{C.}$$

Example 5. *What is the ratio of the velocity of sound in hydrogen* $\left(\gamma=\frac{7}{5}\right)$ *to that in the Helium gas* $\left(\gamma=\frac{5}{3}\right)$ *at the same temperature.* **[*I.I.T. 1983*]**

Solution. Since $V=\sqrt{\dfrac{\gamma P}{\rho}}$,

$$V_H=\sqrt{\frac{7P}{5\rho_H}}$$

and
$$V_{He}=\sqrt{\frac{5P}{3\rho_{He}}}$$

$$\therefore \quad \frac{V_H}{V_{He}}=\sqrt{\frac{7}{5}\times\frac{3}{5}\times\frac{\rho_{He}}{\rho_H}}$$

Now
$$\frac{\rho_{He}}{\rho_H}=4$$

$$\therefore \quad \frac{V_H}{V_{He}}=\sqrt{\frac{21}{25}\times 4}=\frac{2\sqrt{21}}{2}=1833:1000\cdot$$

Example 6. *A steel wire of length 50 cm, has a mass of 0·05 gm and is stretched with a tension of* 4×10^5 *dynes. Find the velocity of propagation of transverse wave in the wire.*

Solution. Here

$$m=\frac{0\cdot05}{50}=0\cdot001\text{ gm. cm}^{-1}$$

$$T=4\times10^5\text{ dynes.}$$

$$\therefore \quad c=\sqrt{\frac{T}{M}}$$

$$=\sqrt{\frac{4\times10^5}{0\cdot001}}=2\times10^4\text{ cm s}^{-1}$$

$$=200\text{ ms}^{-1}$$

Example 7. *Calculate the velocity of sound in steel having Bulk modulus of elasticity as* $2\cdot14\times10^{11}$ Nm^{-2} *and the density as* $7\cdot6\times10^3$ $kg\ m^{-3}$.

Solution. Here $E=2\cdot14\times10^{11}\text{ Nm}^{-2}$, $\rho=7\cdot6\times10^3\text{ kg m}^{-3}$

$$\therefore \quad c=\sqrt{\frac{E}{\rho}}$$

$$=\sqrt{\frac{2\cdot14\times10^{11}}{7\cdot6\times10^3}}=5306\cdot4\text{ ms}^{-1}$$

Example 8. *A displacement wave is represented by*

$$\xi=0{\cdot}25\times10^{-3}\ \sin\ (500t-0{\cdot}20\ x)$$

where ξ, t *and* x *are in cm, sec and metre respectively. Deduce (a) the amplitude, (b) the period, (c) the angular frequency, (d) the wavelength, (e) the amplitude of particle velocity, and (f) the amplitude of particle acceleration.*

Solution. Comparing the given equation with the standard equation of wave, $\xi=A\sin\left(\frac{2\pi}{T}\,t-\frac{2\pi}{\lambda}x\right)$, we have

$$A=0{\cdot}25\times10^{-3}\text{ cm}\ ;\ \frac{2\pi}{T}=500\text{ and }\frac{2\pi}{\lambda}=0{\cdot}20.$$

(*a*) the amplitude, $A=0{\cdot}25\times10^{-3}$ cm.

(*b*) the period, $T=\frac{2\pi}{500}=0{\cdot}0126$ sec.

(*c*) the angular frequency, $\omega=2\pi/T=500\text{ s}^{-1}$.

(*d*) the wavelength, $\lambda=\frac{2\pi}{0{\cdot}2}=31{\cdot}4$ m.

(*e*) the amplitude of particle velocity,

$$V_0=\frac{2\pi}{T}A=500\times0{\cdot}25\times10^{-3}=0{\cdot}125\text{ cm s}^{-1}.$$

(*f*) the amplitude of particle acceleration, $g_0=\left(\frac{2\pi}{T}\right)^2A$

$$=(500)^2\times{\cdot}25\times10^{-3}=62{\cdot}50\text{ cm s}^{-2}.$$

Example 9. *Spherical waves are emitted from 3·14 watt source in air. What is the intensity of the waves at a distance 2 metres from the source?*

Solution. Here $R=2$ m, $P=3{\cdot}14$ W

$\therefore$ Surface Area, $S=4\pi R^2=4\times3{\cdot}14\times2^2\text{ m}^2$.

$\therefore$ Intensity, $I=\frac{P}{4\pi R^2}$

$$I=\frac{3{\cdot}14}{16\times3{\cdot}14}=\frac{1}{16}=0{\cdot}0625\text{ W m}^{-2}$$

Example 10. *A drop of water, 2 mm in diameter, falling from a height of 50 cm in a bucket, generates sound which can be heard from the 5 metre distance. Take all the gravitational energy as going into the sound form. The transformation is being spread in time 0·4 second. Deduce the average intensity.*

Solution. $r=1$ mm$=10^{-3}$ m, $h=50$ cm.$=0{\cdot}5$ m., R$=5$ m., $t=0{\cdot}5$ S, Density of water, $\rho=10^3$ kg m^{-3}.

$$\begin{aligned} \mathrm{E} &= mgh \\ &= \mathrm{V}\rho gh = \frac{4}{3}\pi r^3 \rho gh \\ &= \frac{4}{3}\pi(10^{-3})^3 \times 10^3 \times 9{\cdot}8 \times 0{\cdot}5 \text{ J} \\ &= \frac{4}{3}\pi \times 49 \times 10^{-7} \text{ J} \end{aligned}$$

$$\mathrm{S}=4\pi\mathrm{R}^2=4\pi\times5^2 \text{ m}^2=4\pi\times25 \text{ m}^2$$

$$\begin{aligned} \therefore \quad \text{Intensity} &= \frac{\mathrm{E}}{\mathrm{S}\times t} \\ &= \frac{\frac{4}{3}\pi\times49\times10^{-7}}{4\pi\times25\times0{\cdot}4} \\ &= \frac{49\times10^{-7}}{3\times25\times0{\cdot}4} = 1{\cdot}633\times10^{-7} \text{ Wm}^{-2} \end{aligned}$$

Example 11. *The displacement amplitude of the sound waves produced by a tuning fork of frequency 480 Hz. in air is $0{\cdot}7\times10^{-7}$ m at a point. Calculate the intensity of the sound waves at this point. (Velocity of sound in air$=340$ m s^{-1} and density of air$=1{\cdot}3$ kg m^{-3}.)*

Solution. Here A$=0{\cdot}7\times10^{-7}$ cm., $\nu=480$ Hz.,

$\rho=1{\cdot}3$ kg m^{-3} $\qquad c=340$ ms^{-1}.

$$\begin{aligned} \therefore \quad \mathrm{I} &= 2\pi^2 \mathrm{A}^2\nu^2 \rho c \\ &= 2\left(\frac{22}{7}\right)^2 \times (0{\cdot}7\times10^{-7})^2 \times (480)^2 \times 1{\cdot}3 \times 340 \\ &= 9{\cdot}858\times10^{\ 6} \text{ Wm}^{-2}. \end{aligned}$$

Example 12. *Show that the volume strain in any longitudinal wave is given by $\frac{d\xi}{dx}$, where ξ is the displacement at the plane x. Hence, show that for a harmonic wave of amplitude A and wavelength λ in a medium of volume elasticity E, the excess pressure p is given by*

$$p=EA\frac{2\pi}{\lambda}\sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right).$$

Hence deduce the pressure amplitude, if $E=1{\cdot}6\times10^3$ Nm^{-2} $A=2\times10^{-7}$ m and $\lambda=0{\cdot}5$ m.

Solution. Imagine column of thickness δx and cross-section A in air through which longitudinal waves are passing. Let $\delta\xi$ be the change in thickness of the column due to compression or rarefaction of the layers of air contained in this column.

Hence the original volume of the column, $V = A.\delta x$ and change in volume of the column when the longitudinal waves pass through it, $\Delta V = A.\ \delta\xi$

Hence volume strain,

$$\frac{\Delta V}{V} = \frac{A\delta\xi}{A\delta x}$$

$$= \frac{\delta\xi}{\delta x}$$

when the column is of infinitesimal thickness *i.e.*, $\delta x \to 0$, we have

$$\text{volume strain} = \underset{\delta x \to 0}{\mathrm{L}t} \left(\frac{\delta\xi}{\delta x}\right)$$

$$= \frac{d\xi}{dx}$$

Now volume stress,

$$p = E \times \text{volume strain}$$

$$p = E\frac{d\xi}{dx} \qquad ...(i)$$

But

$$\xi = A \cos 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$\therefore \quad \frac{d\xi}{dx} = A\frac{2\pi}{\lambda} \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) \qquad ...(ii)$$

Combining (*i*) and (*ii*), we get,

$$p = EA\frac{2\pi}{\lambda} \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Since the maximum value of sin for any angle is 1, the pressure amplitude,

$$p_0 = EA\,\frac{2\pi}{\lambda}$$

$$\therefore \quad p_0 = \frac{1{\cdot}6 \times 10^3 \times 2 \times 10^{-7} \times 2 \times 22}{7 \times 0\ 5}$$

$$= 4{\cdot}023 \times 10^{-3}\ \mathrm{Nm^{-2}}.$$

Example 13. *Two polaroids are set to give the maximum transmission I_0. Then if (a) the first polaroid is rotated by 30° clockwise, (b) the second polaroid is now rotated by 30° clockwise, and (c) the first is rotated by 60° anticlockwise, calculate, in each case, the ratio of the intensity transmitted to I_0.*

Solution. We know $I = I_0 \cos^2\theta$

(*a*)

$$\theta = 30°$$

$$\therefore \quad \frac{I}{I_0} = (\cos 30°)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

(*b*) $$\theta=30°-30°=0°$$

$$\therefore \quad \frac{I}{I_0}=(\cos 0°)^2=1$$

(*c*) $$\theta=60°$$

$$\therefore \quad \frac{I}{I_0}=(\cos 60°)^2=\left(\frac{1}{2}\right)^2=\frac{1}{4}.$$

Example 14. *In the two diagrams below AB is the dividing surface between two media, and 1—1, 2—2 are the wave fronts at the time t_0 and (t_0+t_1) respectively. Find the wave fronts in the upper medium for the time (t_0+2t_1) and (t_0+3t_1) in each case.*

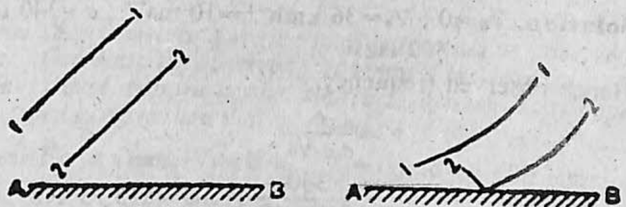

Fig. 13.1.

Solution.

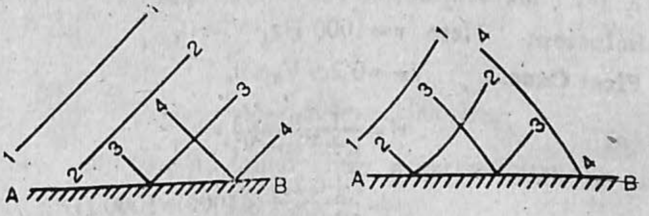

Fig. 13.2.

Example 15. *If c_1 is the root mean square thermal speed of molecules in a gas, and c is the velocity of sound waves in that gas, show that the ratio c/c_1 is independent of the temperature and is same for all mono-atomic gases.*

Solution. Root mean square thermal speed of gas molecules is given by

$$c_1=\sqrt{\frac{3P}{\rho}}$$

velocity of sound waves in a gas is given by

$$c=\sqrt{\frac{\gamma P}{\rho}}$$

$$\therefore \qquad \frac{c}{c_1}=\sqrt{\frac{\gamma}{3}}\,,$$ which is independent of the temperature.

Since value of γ is same for all mono-atomic gases, the value of $\frac{c}{c_1}$ will also be same.

Example 16. *A man standing on the railway platform hears the whistle of an engine which is approaching him with a velocity of 36 km h^{-1}. What frequency does the man hear, if the true frequency of the whistle is 500 Hz ? (Take Velocity of sound=340 ms^{-1}.)*

Solution. $V_0=0$; $V_s=36\ \text{kmh}^{-1}=10\ \text{ms}^{-1}$; $c=340\ \text{ms}^{-1}$; and $\nu=500$ Hz.

Hence observed frequency,

$$\nu'=\frac{c}{c-V_s}\nu$$

$$=\frac{340}{340-10}\times 500=515{\cdot}15\ \text{Hz}.$$

Example 17. *A stationary source emits sound of $\nu=1000$ Hz. If wind blows at the speed 0·2 c, calculate for a stationary observer, (a) the change in frequency, (b) the percentage change in the wavelength. Recalculate these values if instead of wind the observer moves with the speed of 0·2 c towards the source.*

Solution. Here $\nu=1000$ Hz, $V_s=0$.

First Case. $V_m=0{\cdot}2c$, $V_0=0$.

(*a*) $$\therefore \qquad \nu'=\frac{c+V_m-V_0}{c+V_m-V_s}\,\nu$$

$$=\frac{c+0{\cdot}2c}{c+0{\cdot}2c}\times 1000=1000\ \text{Hz}.$$

Hence change in frequency $=0$.

(*b*) $$\lambda=\frac{c}{\nu}=\frac{c}{1000}$$

$$\lambda'=\frac{c+V_m-V_s}{\nu}=\frac{c+0{\cdot}2c}{1000}=\frac{1{\cdot}2c}{1000}$$

$$\therefore \text{ Increase in } \lambda,\ \lambda'-\lambda=\frac{1{\cdot}2c}{1000}-\frac{c}{1000}=\frac{0{\cdot}2c}{1000}$$

$$\%\text{ increase in } \lambda=\frac{0{\cdot}2c/1000}{c/1000}\times 100=20\%$$

Second Case. $V_m=0,\ V_0=-0{\cdot}2c$

(*a*)
$$\nu'=\frac{c+V_m-V_0}{c+V_m-V_s}\nu$$
$$=\frac{c+0{\cdot}2c}{c}\times 1000=1200\text{ Hz}$$

Therefore, % increase in frequency
$$=\frac{1200-1000}{1000}\times 100=20\%$$

(*b*)
$$\lambda=\frac{c}{\nu}=\frac{c}{1000}$$
$$\lambda'=\frac{c+V_m-V_s}{\nu}=\frac{c}{1000}$$

Hence change in wavelength $=0$.

Example 18. *Consider a source moving towards an observer at the speed 0·9c. Calculate the observed frequency if the true frequency is 600 Hz. What would happen if the source is moving at the speed 1·1c i.e., at a speed greater than that of sound*

Solution. First Case. $V_s=0{\cdot}9c$; $V_0=0$ and $\nu=600$ Hz.

$\therefore$
$$\nu=\frac{c-V_o}{c-V_s}\nu$$
$$=\frac{c}{c-0{\cdot}9c}\times 600=6000\text{ Hz.}$$

Second Case. $V_s=1{\cdot}1c$; $V_0=0$ and $\nu=600$ Hz.

(*a*) If the source is approaching the observer,
$$\nu'=\frac{c}{c-V_s}\nu$$
$$=\frac{c}{c-1{\cdot}1c}\times 600=-6000\text{ Hz.}$$

The observer will hear no sound since negative frequency is meaningless.

(*b*) If the source is going away from the observer,
$$\nu'=\frac{c}{c+V_s}\nu$$
$$=\frac{c}{c+1{\cdot}1c}\times 600=285{\cdot}7\text{ Hz.}$$

The observer will hear a sound of less frequency.

Example 19. *A stationary siren has a frequency of 400 Hz. If the velocity of sound is 340 ms*$^{-1}$*, calculate the frequency of the sound of siren as heard by the driver of a car moving at 54 Km h*$^{-1}$ *(a) towards the siren, (b) away the siren.*

Solution. Here $V_s=0$, $\nu=400$ Hz., $c=340\ \text{ms}^{-1}$,
$V_0=54\ \text{Km hr}^{-1}=15\ \text{ms}^{-1}$

(*a*) ∵ The observer is moving towards the source,

$$\therefore \quad V_0=-15\ \text{ms}^{-1}$$

$$\therefore \quad \nu'=\frac{c-V_0}{c}\,\nu$$

$$=\frac{340+15}{340}\times 400=417{\cdot}65\ \text{Hz.}$$

(*b*) ∵ The observer is moving away the source,

$$V_0=+15\ \text{ms}^{-1}$$

$$\therefore \quad \nu'=\frac{340-15}{340}\times 400=382{\cdot}35\ \text{Hz.}$$

Example 20. *The spectral line for a given element in light received from a distant star is shifted towards the longer wavelength by 0·02%. Deduce the velocity of the star in the line of sight. Velocity of light in vacuum is* $3\times 10^8\ ms^{-1}$.

Solution. Here only the star, which is the source, is in motion, hence

$$\nu'=\frac{c}{c=V_s}\,\nu$$

or
$$\frac{1}{\lambda'}=\frac{c}{c-V_s}\times\frac{1}{\lambda} \qquad \left(\because \quad \nu\propto\frac{1}{\lambda}\right)$$

$$\lambda'=\frac{c-V_s}{c}\lambda$$

or
$$\frac{\lambda'}{\lambda}=1-\frac{V_s}{c}$$

or
$$\frac{V_s}{c}=\left(1-\frac{\lambda'}{\lambda}\right)$$

$$\therefore \quad V_s=-c\left(\frac{\lambda'-\lambda}{\lambda}\right)$$

$$=-3\times 10^8\times\frac{{\cdot}02}{100}$$

$$=-6\times 10^4\ \text{ms}^{-1}.$$

It means the star is moving away from the observer with the velocity $6\times 10^4\ \text{ms}^{-1}$.

Example 21. *When a motor car crosses a policeman, his instrument detects a drop of 8% in the pitch of the car. Calculate the speed of the car if the velocity of sound is* $340\ ms^{-1}$.

Solution. The change in frequency as the car crosses the policemen,

$$\Delta\nu=\frac{c}{c-V_s}\nu-\frac{c}{c+V_s}\nu$$

$$\frac{\Delta\nu}{\nu}=\frac{2cV_s}{(c-V_s)(c+V_s)}=\frac{2cV_s}{c^2}$$

$$(\because V_s^2<<c^2 \text{ and so neglecting } V_s^2)$$

$$\frac{\Delta\nu}{\nu}=\frac{2V_s}{c}$$

$$\therefore \quad \frac{8}{100}=\frac{2V_s}{340}$$

$$V_s=13{\cdot}6 \text{ ms}^{-1}$$

$$=48{\cdot}96 \text{ Km h}^{-1}.$$

Example 22. *A radar wave has a frequency of 8×10^9 s^{-1}. The reflected wave from an aeroplane shows a frequency difference $2{\cdot}5\times10^3$ s^{-1} on the higher side. Calculate the velocity of aeroplane.*

Solution. Here $\nu=8\times10^9 \text{ s}^{-1}$; $(\nu'-\nu)=2{\cdot}5\times10^3 \text{ s}^{-1}$:

$$C=3\times10^8 \text{ ms}^{-1}$$

Now
$$\nu'=\frac{c}{c-V_s}\nu$$

$$\frac{\nu'}{\nu}=\left(1-\frac{V_s}{c}\right)^{-1}$$

or
$$=\left(1+\frac{V_s}{c}\right) \qquad \left[\text{Since } \frac{V_s}{c}<<1\right]$$

$$\therefore \quad V_s=\left(\frac{\nu'-\nu}{\nu}\right)c$$

$$=\frac{2{\cdot}5\times10^3\times3\times10^8}{8\times10^9}$$

$$=93{\cdot}75 \text{ ms}^{-1}$$

Hence the velocity of the approaching aeroplane,

$$V_a=\frac{V_s}{2}$$

$$=\frac{93{\cdot}75}{2} \text{ ms}^{-1}=168{\cdot}75 \text{ Km h}^{-1}$$

Example 23. *Calculate the displacement amplitude when the wave intensity is 5 watt m^{-2} and its frequency is 700 Hz. Compare it with the average distance between the gas molecules at N.T.P. (Given that Avogadro number $=6\times10^{26}$ per kg-mole ; at N.T.P. volume occupied by 1 kg-mole of a gas $=22{\cdot}4$ m^3 : density of air $1{\cdot}3$ kg m^{-3} and velocity of sound $=340$ ms^{-1}).*

Solution. Here $I=5 \text{ W m}^{-2}$; $\nu=700$ Hz : $\rho=1{\cdot}3 \text{ Kg m}^{-3}$.

$$c=340 \text{ ms}^{-1} \text{ and } A=?$$

We know
$$I=\pi^2\nu^2A^2\rho c$$

$$\therefore \qquad A=\frac{1}{\pi\nu}\sqrt{\frac{1}{2\rho c}}$$

$$A=\frac{7}{700\times22}\sqrt{\frac{5}{2\times1\cdot3\times340}}$$

$$=3\cdot42\times10^{-5}\text{ m.}$$

Let the average distance between gas molecule be 'x' metre, then one molecule will occupy the volume equal to volume of a cube of x metre side *i.e.*, x^3 metre.^{3}

But 6×10^{26} molecules are occupying volume $=22{\cdot}4$ metre^3

$\therefore$ Volume occupied by one molecule $=\dfrac{22\cdot4}{6\times10^{26}}$ metre^3

$$\therefore \qquad x^3=\frac{22\cdot4}{6\times10^{26}}$$

or
$$x=\left(\frac{224}{6}\times10^{-27}\right)^{1/3}$$

$$=(37\cdot33)^{1/3}\times10^{-9}$$

Now let
$$y=(37\cdot33)^{1/3}$$

$$\therefore \qquad \log y=\tfrac{1}{3}\log(37\cdot33)$$

$$=\tfrac{1}{3}\times1\cdot5720$$

$$=0\cdot5240$$

$$\therefore \qquad y=\text{Antilog }(0\cdot5240)=3\cdot342$$

$$\therefore \qquad x=3\cdot342\times10^{-9}\text{ m}$$

$\therefore \quad \dfrac{A}{x}\simeq10^4$. It means amplitude is 10^4 times the spacing between the molecules.

Example 24. *One end of a steel pipe of length 660 m stuck a blow which produces loud sound. A listener at the other end hears two sounds at an interval of 1·89 s, one from the wave that has travelled along the metal of the pipe and the other from that has travelled through the air. If the density of the steel is 8×10^3 kg/m^3. What is the Bulk modulus of elasticity ?*

[*Velocity of sound in air* $=330$ *m/s*] [*A.I.S.S.E. 1983*]

Solution. Time in air,

$$t_a=\frac{660}{330}=2\text{s}$$

Time in steel, $\quad t_s=2-1\cdot89=0\cdot11$ s

$\therefore$ Velocity in steel,

$$c=\frac{660}{0\cdot11}=6000\text{ m/s}$$

Now
$$c=\sqrt{\frac{E}{\rho}}$$

$$\therefore \quad E = c^2\rho$$
$$= 6000^2 \times 8 \times 10^3$$
$$= 2{\cdot}88 \times 10^{11} \text{ Nm}^{-2}.$$

Example 25. *Out of two interfering sources A and B, source A is ahead of phase by 60° relative to B. If an observation point P is such that PB—PA=2·5λ, calculate the phase difference between the waves from A and B reaching P.*

Solution. The initial phase difference between the two waves,

$$\phi_0 = 60° = 60 \times \frac{\pi}{180} \text{rad} = \frac{\pi}{3} \text{ rad}.$$

Now $\quad p = PB - PA = 2{\cdot}5\lambda$

$\therefore$ Equivalent phase difference, $\phi = \left(\frac{2\pi}{\lambda}\right) p$

$$= \frac{2\pi}{\lambda} \times 2{\cdot}5\lambda$$
$$= 5\pi$$

$\therefore$ Total phase difference,

$$= \phi_0 + \phi$$
$$= \frac{\pi}{3} + 5\pi = 5\tfrac{1}{3}\pi$$

Example 26. *Waves from two sources have amplitude ratio as 4 : 1. Calculate the ratio of intensities at the maxima and minima in the interference.*

Solution.

$$r = \frac{A_1}{A_2} = \frac{4}{1}$$

$$\therefore \quad \frac{I_{max.}}{I_{min.}} = \frac{(r+1)^2}{(r-1)^2}$$
$$= \frac{(4+1)^2}{(4-1)^2} = \frac{25}{9}$$

Example 27. *Sound waves from the two sources superpose on each other with intensity ratio 16 : 9. Find the amplitude ratio and the ratio of intensities at maxima and interference.*

Solution.

$$\frac{I_1}{I_2} = \frac{16}{9}$$

$\therefore$ Amplitude ratio,

$$r = \frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}}$$

$$=\sqrt{\frac{16}{9}}=\frac{4}{3}$$

and $$\frac{I_{max.}}{I_{min.}}=\frac{(r+1)^2}{(r-1)^2}$$

$$=\frac{(\frac{4}{3}+1)^2}{(\frac{4}{3}-1)^2}=\frac{49}{1}.$$

Example 28. *In an experiment with Quinck's tube, one passed from one maximum to the next maximum sound when the U-tube was slided by 40 cm outwards. Calculate the wave length and frequency of the sound waves if the velocity of sound is 340 ms^{-1}.*

Solution. When one passes from one maximum to the next, the path difference will be λ.

Sliding of U-tube by 40 cm will create a path difference

$$=2\times 40=80 \text{ cm.}$$

$\therefore$ $$\lambda=80 \text{ cm.}=0{\cdot}80 \text{ m.}$$

Now $$c=340 \text{ ms}^{-1}$$

$\therefore$ $$\nu=\frac{c}{\lambda}=\frac{340}{0{\cdot}8}=425 \text{ Hz.}$$

Example 29. *Two identical sources are separated by a distance of 40 cm. The observation plane is 1 m away from the line joining the sources and fringe width is found to be 0·25 m. Calculate the frequency of the source. The velocity of sound in air is 340 m^{-1}.*

Solution. $$w=0{\cdot}25 \text{ m}\ ;\ d=40 \text{ cm}=0{\cdot}40 \text{ m}\ ;$$

$$D=1 \text{ m}\ ;\ c=340 \text{ ms}^{-1},$$

$\therefore$ $$\lambda=\frac{wd}{D}$$

$$=\frac{0{\cdot}25\times 0{\cdot}40}{1}=0{\cdot}100 \text{ m}$$

Now $$\nu=\frac{c}{\lambda}$$

$$=\frac{340}{0{\cdot}1}=3400 \text{ Hz.}$$

Example 30. *Two tuning forks A and B when sounded together give 6 bears per second When tuning fork A is filed a little, number of beats per second increases. Find the frequency of tuning fork B when that of A is 256 Hz.*

Solution. $\nu_1=256$ Hz, $\nu=6$ Hz.

$\therefore$ Frequency of tuning fork B,

$$\nu_2=\nu_1\pm\nu$$

$$=256\pm 6$$

$$=262 \text{ or } 250 \text{ Hz}$$

On filing T.F. 'A', its frequency will increase and since at the same time, number of beats increases, it means frequency of T.F. 'A', is more than that of 'B'.

∴ Frequency of T.B. 'B'$=250$ Hz.

Example 31. *Tuning fork A is sounded with tuning fork B whose frequency is 284 Hz. Then 4 beats per second are heard. If tuning fork A is loaded with a little wax, number of beats per second remains the same. Find the frequency of tuning fork A.*

Solution. $\nu_1=284$ Hz, $\nu=4$ Hz

∴ Frequency of T.F. 'A',

$$\begin{aligned}\nu_2&=\nu_1\pm\nu\\&=284\pm4\\&=288 \text{ or } 280 \text{ Hz.}\end{aligned}$$

On loading T.F. 'A', its frequency will decrease and since at the same time number of beats remains the same, it means initially the frequency of T.F. 'A', was higher than that of B by 4 and then it was less than that of B by 4.

∴ Frequency of T.F. 'A'$=288$ Hz.

Example 32. *When two sound waves of wavelengths 80 cm and 81 cm travelling in the same direction in air superpose on each other, 11 beats are produced in 2 seconds. Calculate the velocity of sound in air.*

Solution. No. of beats per second,

$$\nu=\nu_1-\nu_2$$

$$\nu=\frac{c}{\lambda_1}-\frac{c}{\lambda_2}$$

$$\therefore \quad \frac{11}{2}=\frac{c}{80}-\frac{c}{81}$$

$$=c\left(\frac{81-80}{81\times80}\right)$$

$$\therefore \quad c=\frac{11\times81\times80}{2}$$

$$=35640 \text{ cm s}^{-1}=356{\cdot}40 \text{ ms}^{-1}$$

Example 33. *Two tuning forks A and B together give 10 beats in 3 seconds, when A is in resonance with a closed column of air 25 cm long and B is in resonance with an open column of air 50·5 cm long. Calculate the following :*

(*a*) *velocity of sound in air*

(*b*) *frequency of each of the tuning forks A and B.*

Solution. The frequency of T.F. 'A' (in resonance with closed column of air)

$$\nu_1=\frac{c}{4l}=\frac{c}{4\times 25}=\frac{c}{100}$$

The frequency of T.F. 'B' (in resonance with open column of air)

$$\nu_2=\frac{c}{2l}=\frac{c}{2\times 50{\cdot}5}=\frac{c}{101}$$

Now $\nu=\nu_1-\nu_2$

$$\therefore \quad \frac{10}{3}=\frac{c}{100}-\frac{c}{101}$$

$$\therefore \quad c=\frac{100\times 101}{(101-100)}\times\frac{10}{3}$$

$$=33{,}666{\cdot}6 \text{ cm s}^{-1}$$

$$=336{\cdot}67 \text{ ms}^{-1}$$

Now $$\nu_1=\frac{c}{100}=\frac{33666{\cdot}6}{100}=336{\cdot}66 \text{ Hz}$$

and $$\nu_2=\frac{c}{101}=\frac{33666{\cdot}6}{101}=333{\cdot}33 \text{ Hz.}$$

Example 34. *An observer moving towards a wall at 10 ms^{-1} hears a sound directly from a source at some distance behind him as well as after reflection from the wall. Calculate the number of beats per second between these two sounds. True frequency is 170 Hz and velocity of sound in air is 340 ms^{-1}.*

Solution. For the direct sound from the source, the observer is moving away the source,

$$V_0=+10 \text{ ms}^{-1}$$

$\therefore$ Observed frequency in this case,

$$\nu_1=\frac{c-V_0}{c}\nu$$

$$=\frac{340-10}{340}\times 170$$

$$=\frac{33}{34}\times 170 \text{ Hz.}$$

For the reflected sound, the observer is moving towards the source (wall),

$$\therefore \quad V_0=-10 \text{ ms}^{-1}$$

$\therefore$ Observed frequency in this case,

$$\nu_2=\frac{340-(-10)}{340}\times 170$$

$$=\frac{35}{34}\times 170 \text{ Hz.}$$

$\therefore$ No. of beats **per Second,**

$$\nu=\nu_2-\nu_1$$

$$=\frac{35}{34}\times 170-\frac{33}{34}\times 170$$

$$=\frac{2}{34}\times 170=10\ \text{s}^{-1}.$$

Example 35. *The number of beats per second is 6 when a tuning fork is sounded together with a string when its length is either 95 cm or 100 cm. What is the frequency of the fork ?*

Solution. For fundamental note,

$$\nu=\frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\therefore \quad \nu_1=\frac{1}{2\times 95}\sqrt{\frac{T}{m}} \text{ for first case}$$

$$\text{and} \quad \nu_2=\frac{1}{2\times 100}\sqrt{\frac{T}{m}} \text{ for second case}$$

$$\therefore \quad \frac{\nu_1}{\nu_2}=\frac{100}{95}=\frac{20}{19} \qquad \ldots(1)$$

If the frequency of tuning fork is ν, then

$$\left.\begin{aligned}\nu_1&=\nu+6\\ \text{and} \quad \nu_2&=\nu-6\end{aligned}\right\} \qquad (\because \nu_1>\nu_2)$$

$\therefore$ Eqn. (1) becomes

$$\frac{\nu+6}{\nu-6}=\frac{20}{19}$$

$$19(\nu+6)=20\ (\nu-6)$$

$$19\nu+114=20\nu-120$$

$$\therefore \quad 20\nu-19\nu=120+114$$

$$\therefore \quad \nu=234\ \text{Hz}.$$

Example 36. *On a quiet day, two persons A and B each sounding a note of frequency 580 Hz, are standing a few metres apart. Calculate the number of beats heard by each in one second when A moves towards B with a velocity 4 m/s (C=330 m/s)*

[*A.I.S.S.E., 1987 Comptt.*]

Solution. (*i*) Let the moving person 'A' be the source. Then the frequency of the sound of A as received by stationary observer B,

$$\nu_1=\frac{c}{(c-V_s)}=\frac{330}{(340-4)}\times 580=587\ \text{Hz}$$

(*ii*) Let the stationary person **B** be the source. Then the frequency of the sound of B as received by the observer A moving towards the source,

$$\nu_2=\frac{(c+V_0)}{c}=\frac{(330+4)}{330}\times 580=587\text{ Hz}$$

$\therefore$ No. of beats heard per sec. by

$$A=\nu_2-\nu=587-580=7$$

and number of beats heard per sec. by

$$B=\nu_1-\nu=587-580=7.$$

Example 37. *Calculate the fundamental frequency of a sonometer wire of the length 40 cm. The mass per unit length of the wire is* $4{\cdot}9\times10^{-3}$ *kg* m^{-1} *and is under tension of 16 N.*

Solution. For fundamental note,

$$\nu=\frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$=\frac{1}{2\times0{\cdot}40}\sqrt{\frac{16}{4{\cdot}9\times10^{-3}}}$$

$$=71{\cdot}4\text{ Hz.}$$

Example 38. *A string of length 50 cm which weights 0·484 gr is stretched by a load of 2 kg. Calculate the following :*

(*a*) *speed of the transverse waves in the wire.*

(*b*) *the frequency of fundamental note.*

(*c*) *the frequency of the second overtone.*

Solution. $l=50$ cm ;

$$m=\frac{0{\cdot}484}{50}=0{\cdot}00968\text{ g cm}^{-1}$$

and

$$T=Mg=2000\text{ g}\times980\text{ cm s}^{-2}=19{\cdot}6\times10^5\text{ dyne.}$$

(*a*)

$$c=\sqrt{\frac{T}{m}}$$

$$=\sqrt{\frac{19{\cdot}6\times10^5}{0{\cdot}00968}}=142{\cdot}3\times10^2\text{ cm s}^{-1}$$

$$=142{\cdot}3\text{ ms}^{-1}.$$

(*b*) For fundamental note, $p=1$

$$\therefore\quad \nu=\frac{p}{2l}\sqrt{\frac{T}{m}}$$

$$=\frac{1}{2\times50}\times142{\cdot}3\times10^2=142{\cdot}3\text{ Hz.}$$

(*c*) For the second overtone or third harmonic, $p=3$

$$\therefore\quad \nu=\frac{p}{2l}\sqrt{\frac{T}{m}}$$

$$=\frac{3}{2\times50}\times142{\cdot}3\times10^2=426{\cdot}9\text{ Hz.}$$

Alternative. The second overtone for a string under tension is the same as the third harmonic for the string.

Frequency of third harmonic $=3$ times the frequency of fundamental note.

$$=3\times 142{\cdot}3=426{\cdot}9 \text{ Hz.}$$

Example 39. *A sonometer wire under tension vibrates with a fundamental frequency of 480 Hz. What would be the fundamental frequency if the wire is under one-fourth the tension, half as long and twice a thick.*

Solution. For fundamental frequency,

$$\nu=\frac{1}{l\mathrm{D}}\sqrt{\frac{\mathrm{T}}{\pi\rho}}$$

$$\therefore \qquad \nu_1=\frac{1}{l\mathrm{D}}\sqrt{\frac{\mathrm{T}}{\pi\rho}}$$

and

$$\nu_2=\frac{2}{l\times 2\mathrm{D}}\sqrt{\frac{\mathrm{T}/4}{\pi\rho}}$$

$$=\frac{1}{2}\ \frac{1}{l\mathrm{D}}\sqrt{\frac{\mathrm{T}}{\pi\rho}}$$

$$\therefore \qquad \nu_2=\frac{1}{2}\nu_1$$

$$=\frac{1}{2}\times 480=240 \text{ Hz.}$$

Example 40. *A one metre long copper wire, having density 9000 kg m^{-3} is stretched by a load of 5 kg. It is seen that it vibrates in a fundamental mode of 252 Hz. Find the diameter of the wire.*

Solution. $l=1$ m, $\nu=252$ Hz, $\mathrm{T}=5\times 9{\cdot}8=49{\cdot}0$ N and for fundamental **mode** of vibration, $p=1$, $\rho=9000$ kg m^{-3}

$$\therefore \qquad \nu=\frac{1}{l\mathrm{D}}\sqrt{\frac{\mathrm{T}}{\pi\rho}}$$

$$252=\frac{1}{1\times\mathrm{D}}\sqrt{\frac{4\times 7}{22\times 9000}}$$

or

$$252=\frac{1}{\mathrm{D}}0{\cdot}04162$$

$$\therefore \qquad \mathrm{D}=\frac{0{\cdot}04162}{252}\text{ m}$$

$$=1{\cdot}65\times 10^{-4}\text{ m}$$

$$=0{\cdot}165 \text{ mm.}$$

Example 41. *A turning fork of frequency 284 Hz is sounded along with a sonometer wire of length 25 cm and stretched to a tension of 5 kg wt. Mass per unit length of the wire is $2{\cdot}5\times 10^{-3}$ kg m^{-1}. Calculate the number of beats produced per second.*

Solution. $l=25\text{ cm}=0{\cdot}25\text{ m}$, $m=2{\cdot}5\times10^{-3}\text{ kg m}^{-1}$ and $T=Mg=5\times9{\cdot}8\text{ N}$.

For the fundamental note of sonometer wire,

$$\nu=\frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$=\frac{1}{2\times0{\cdot}25}\sqrt{\frac{5\times9{\cdot}8}{2{\cdot}5\times10^{-3}}}$$

$$=280\text{ Hz.}$$

∴ No. of beats per sec. $=284-280=4\text{ s}^{-1}$.

Example 42. *A resonance air column resonates with a tuning fork of frequency 512 Hz at the column lengths 16·8 cm and 50 8 cm. Calculate the following if the room temperature is 27°C.*

(*a*) *velocity of sound at the room temp.*

(*b*) *the end correction.*

(*c*) *the velocity of sound at 0°C.*

Solution. $\nu=512\text{ Hz}$; $l_1=16{\cdot}8\text{ cm}$ and $l_2=50{\cdot}8\text{ cm}$.

(*a*) Velocity of sound,

$$c=2\nu(l_2-l_1)$$

$$=2\times512(50{\cdot}8-16{\cdot}8)$$

$$=34816\text{ cm s}^{-1}=348{\cdot}16\text{ m s}^{-1}.$$

(*b*) The end correction,

$$x=\frac{l_2-3l_1}{2}$$

$$=\frac{50{\cdot}8-3\times16{\cdot}8}{2}=0{\cdot}2\text{ cm}$$

(*c*) $c_1=348{\cdot}16\text{ ms}^{-1}$, $T_1=273+27=300\text{ K}$ and $T_0=273+0=273\text{ K}$.

$$\therefore\quad c_0=\sqrt{\frac{T_0}{T_1}}\times c_1$$

$$=\sqrt{\frac{273}{300}}\times348{\cdot}16$$

$$=332{\cdot}12\text{ m s}^{-1}.$$

Example 43. *In a resonance air column experiment the first resonance occurs at column length of 17·9 cm with a tuning fork of frequency 480 Hz. Calculate the velocity of sound in air, neglecting the end correction.*

Solution. On neglecting the end correction,

$$\frac{\lambda}{4}=(l_1+0)$$

$$\therefore \quad \lambda = 4l_1$$

$$= 4 \times 17{\cdot}9 = 71{\cdot}6 \text{ cm.}$$

Now $c = \nu\lambda$

$$= 480 \times 71{\cdot}6 = 34368 \text{ cm s}^{-1} = 343{\cdot}68 \text{ ms}^{-1}.$$

Example 44. *What will be the frequency of sound emitted by an organ pipe of length 34 cm in the following cases if the velocity of sound is 340 ms^{-1}.*

(*a*) *when the pipe is closed.*

(*b*) *when the pipe is open.*

Solution. $L = 34 \text{ cm} = 0{\cdot}34 \text{ m}, c = 340 \text{ ms}^{-1}$.

(*a*) For closed pipe, frequency of the **fundamental note**

$$\nu = \frac{c}{4L}$$

$$= \frac{340}{4 \times 0{\cdot}34} = 250 \text{ Hz.}$$

(*b*) For open pipe, **frequency of the fundamental note,**

$$\nu = \frac{c}{2L}$$

$$= \frac{340}{2 \times 0{\cdot}34} = 500 \text{ Hz.}$$

Example 45. *An open pipe 38 cm long and a closed pipe 32 cm long, both of the same diameter, each sounding its second overtone, are in unison. If the frequency of sound emitted by them is 1300 Hz. Calculate the following :*

(*a*) *the end correction of these pipes.*

(*b*) *the velocity of sound in air.*

Solution. (*a*) For second overtone in **open pipe,**

$$\nu_0 = 3\,\frac{c}{2(L_0 + 2x)}$$

and for second overtone in closed pipe,

$$\nu_c = 5\,\frac{c}{4(L_c + x)}$$

But $\nu_b = \nu_c$

$$\therefore \quad \frac{3c}{2(L_o + 2x)} = \frac{5c}{4(L_c + x)}$$

$$6(L_c + x) = 5(L_o + 2x)$$

or

$$x = \frac{6L_c - 5L_o}{4}$$

$$= \frac{6 \times 32 - 5 \times 38}{4} = 0{\cdot}5 \text{ cm.}$$

(*b*)
$$1300=\frac{5c}{4(32+0{\cdot}5)}$$

$$\therefore \quad e=\frac{1300\times4\times32{\cdot}5}{5}=33800\text{ cm s}^{-1}$$
$$=338\text{ ms}^{-1}.$$

Example 46. *A tuning fork of frequency 340 Hz produces resonance with a column of air. If the velocity of sound in air is 340 ms^{-1}, calculate the length of air column, when*

(*a*) *it is open at both end.*
(*b*) *it is closed at one end.*

Solution. (*a*) For an air column open at both ends,

$$\nu=\frac{c}{2L}$$

$$\therefore \quad L=\frac{340}{2\times340}=0{\cdot}5\text{ m}=50\text{ cm}.$$

(*b*) For an air column closed at one end.

$$\nu=\frac{c}{4L}$$

$$\therefore \quad L=\frac{340}{4\times340}=0{\cdot}25\text{ m}=25\text{ cm}.$$

Example 47. *A rod 3 m long is clamped in the middle and is set into longitudinal vibrations. If the frequency of the fundamental note produced by rod is 900 Hz, calculate the velocity of sound in rod.*

Solution. For fundamental note in a rod clamped in middle,

$$\lambda=2\,l$$
$$=2\times3=6\text{ m}$$

Now
$$c=\nu\lambda$$
$$=900\times6$$
$$=5400\text{ m s}^{-1}.$$

Example 48. *In Kundt's tube experiment, the mean distance between two successive heaps in air column of tube is 5 cm. The length of the rod is 75 cm. Calculate the velocity of sound in the material of the rod, if the velocity of sound in air is 340 ms^{-1}.*

Solution. $\lambda_{air}=2\times$ mean distance between two successive heaps.

$$=2\times5=10\text{ cm}$$
$$\lambda_{rod}=2\times\text{length of the rod}$$
$$=2\times75=150\text{ cm}.$$

Now
$$c_{rod}=\frac{\lambda_{rod}}{\lambda_{air}}\times c_{air}$$

$$=\frac{150}{10}\times 340$$
$$=1500\ \text{ms}^{-1}.$$

Example 49. *Two tuning forks A and B give 5 beats/sec. A resonates with column of air 15 cm long closed at one end and B with a column 30·5 cm long which is open at both ends. Calculate their frequencies (Neglect end correction).* [*A.I.S.S.E. 1985 (Compt.)*]

Solution. For closed air column,

$$\lambda_1=4l_1=4\times 15=60\ \text{cm}$$

For open air column,

$$\lambda_2=2l_2=2\times 33{\cdot}5=61\ \text{cm}$$

Now $\quad c=\nu_1\lambda_1=\nu_2\lambda_2$

$\therefore \quad \nu_1\times 60=(\nu_1-5)\times 61 \qquad$ (since $\lambda_2<\lambda_1$)

$$\nu_1=305\ \text{Hz}$$

$\therefore \quad \nu_2=305-5=300\ \text{Hz}$

$\because$ Frequency of tuning fork A=305 Hz

and Frequency of tuning fork B=300 Hz.

Example 50. *The vibrations of a string of length 60 cm fixed at both ends are represented by*

$$y=4\sin\left(\frac{\pi x}{15}\right)\cos\left(96\,\pi\, t\right)$$

where x and y are in cm and t in seconds. Calculate (a) The maximum displacement of a point at x=5 cm (b) the position of nodes located along the string (c) the velocity of the particle at x=7·5 cm at and t=0·25 sec. (*I.I.T. 1985*)

Solution. (*a*) At $x=5$ cm

$$y=4\sin\left(\frac{\pi\times 5}{15}\right)\cos\left(96\,\pi\, t\right)$$

$$=4\sin\frac{\pi}{3}\cos\left(96\,\pi\, t\right)$$

$\therefore \quad y_{max}=4\times\frac{\sqrt{3}}{2}\times 1=3{\cdot}464\ \text{cm}.$

(*b*) Nodes must have zero displacement at all times.

$\therefore \quad \sin\left(\frac{\pi x}{15}\right)=0$

$\because \quad \frac{\pi x}{15}=n\pi \qquad$ where n is a integer.

or $\quad x=15\,n$

$$=0,\ 15\ \text{cm},\ 30\ \text{cm},\ 45\ \text{cm},\ldots\ldots$$

(*c*) At $x=7{\cdot}5$ cm.

$$y=4\sin\left(\frac{\pi\times7{\cdot}5}{15}\right)\cos(96\pi t)$$

$$=4\sin\frac{\pi}{2}\cos(96\pi t)$$

$$=4\times1\times\cos(96\pi t)$$

$$\therefore\quad y=4\cos(96\pi t)$$

$$\therefore\ \text{Velocity}\left(\frac{dy}{dt}\right)=-4[\sin(96\pi t)]\times96\pi$$

At $t=\frac{1}{4}$ sec.

$$\text{Velocity}=-4\left[\sin\left(96\pi\times\frac{1}{4}\right)\right]96\pi$$

$$=-4\times0\times96\pi=0.$$

EXERCISE 13

(*Take* $\pi=3{\cdot}14$)

1. The audio range of frequency for human being is from 20 Hz to 20 KHz. If the velocity of sound in air is 340 ms^{-1}, calculate the range of audio wavelengths.
2. The frequency of a factory siren is 1000 Hz. If the velocity of sound at that place is 330 ms^{-1}, calculate the wavelength of the sound produced.
3. If the velocity of waves in medium A is 150 ms^{-1}, find the velocity in medium B. The wavelengths of waves in two media are 12 cm and 15 cm respectively.
4. The volume elasticity and density of water are $2{\cdot}24\times10^9$ Nm^{-2} and 10^3 kg m^{-3} respectively. Find the velocity of sound in water.
5. For aluminium, the Bulk modulus of elasticity is $7{\cdot}5\times10^{10}$ Nm^{-2} and density is $2{\cdot}70\times10^3$ kg m^{-3}. Find the velocity of longitudinal waves in aluminium.
6. For iron, the Bulk modulus of elasticity is $2{\cdot}0\times10^{11}$ Nm^{-2} and density is $7{\cdot}8\times10^3$ kg m^{-3}. Find velocity of longitudinal waves in iron.
7. A piece of wire 40 cm long stretched by load of 4 Kg and has a mass 1·24 g. Find the velocity of transverse waves in the wire.
8. A piece of wire 6 metre long has a mass of 8 g and is stretched with tension of 100 N. What is the velocity of waves in the wire ?

9. Find the temperature at which the velocity of sound in air will be twice of that at 17°C.

10. At what temperature sound travels in oxygen with the same velocity as in nitrogen at 7°C. Atomic weights of the two gases are 16 and 14 respectively.

11. The velocity of sound in air at 17°C is 340 ms^{-1}. What will be the velocity of sound at 127°C and if the pressure becomes thrice ?

12. A harmonic waves is represented by

$$\xi = 0{\cdot}14 \cos(157t - 0{\cdot}98x)$$

where the ξ, t and x are in mm, s and m respectively. Calculate (*a*) the displacement amplitude (*b*) the time-period (*c*) the angular frequency (*d*) the wavelength (*e*) the wave velocity (*f*) the amplitude of particle velocity and (*g*) the amplitude of particle acceleration.

13. A harmonic wave is given by

$$\xi = 12 \sin \pi (0{\cdot}01\, x + 2{\cdot}0t)$$

Where ξ, x and t are in mm, m and s respectively. Calculate all the 7 quantities as in question number 12.

14. A plane wave is represented by

$$\xi = 2{\cdot}5\, e^{-0{\cdot}02x} \cos(1000t - 0{\cdot}75x + \pi/4)$$

where ξ, x and t are $\times 10^{-4}$ cm, m and s respectively. Calculate the following :

(*a*) the phase at $x=0$, $t=0$.
(*b*) the amplitude at $x=40$ m.
(*c*) phase change at a given place in 10^{-3} s.
(*d*) phase difference between the points separated by 20 cm.

15. If ρ is the density of a medium and c is the wave velocity in that medium, prove that the pressure amplitude P_0 of the waves in that medium is related to the wave intensity I by the following formula

$$P_0 = \sqrt{2I\rho c}$$

16. Find out the ratio of pressure amplitudes of sound waves having equal intensity in air and water. The ratio of velocities of sound waves in air and water is 1 : 4. [Density of air is $1{\cdot}3$ kg m^{-3}.]

17. A 50 watt source sends out sound waves of 500 Hz, equally in all directions in air. Calculate the intensity of sound at a point 10 m away from the source. If $c = 340$ m s^{-1} and $\rho = 1{\cdot}3$ kg m^{-3}, calculate the displacement amplitude.

18. A drop of water, 6 mm in diameter falls from a height of 2 m and produces sound which can be heard from 10 m distance.

Whole of the gravitational energy is converted into sound energy in 0·4 s. Calculate the intensity. If $c=340 \text{ m s}^{-1}$, $\rho=1{\cdot}3 \text{ kg m}^{-3}$ and $\nu=1000$ Hz, calculate the displacement amplitude.

19. Two polaroids are adjusted to give the maximum intensity of light. To what fraction of its maximum value is the intensity reduced when one of the polaroids is rotated through (*a*) 90° (*b*) 60° (*c*) 45° and (*d*) 30° ?

20. Two polariod are ajdusted to give the maximum intensity of light. Calculate the ratio of intensity transmitted to the maximum intensity if :
    (*a*) the first polaroid is rotated through 45° anticlockwise.
    (*b*) then second polaroid is rotated through 45° anticlockwise.
    (*c*) the first polaroid is now rotated through 30° clockwise.

21. A locomotive moves towards you with a speed of 108 km/h blowing a whistle of frequency 1000 Hz. At the same time you move towards the locomotive in a car with the speed of 90 km/h. Determine apparent frequency of the whistle. The velocity of sound in air is 335 ms^{-1}.

22. Two aeroplanes having velocities of 216 and 252 km h^{-1} are approaching each other. A sound is emitted by the first. The frequency of this sound as heard by the pilot in the other is 600 Hz. If the velocity of sound is 340 ms^{-1}, calculate the ture frequency.

23. What is the speed of a boat if the frequency of the sound produced by the whistle of the boat changes from 620 Hz to 660 Hz when the boat is approaching a vertical cliff. The velocity of sound is 350 ms^{-1}.

24. A man standing near a railway line hears the whistle of an engine which has the velocity 108 km h^{-1}. What frequency does the man hear, if the true frequency of the whistle is 570 Hz ? (Velocity of sound in air is 350 ms^{-1}.)

25. A siren on a police car has a frequency 660 Hz. What frequency will be heard by a man standing by the side of the road when the police car in moving at a speed of 54 km h^{-1} (*a*) towards the man (*b*) away from the man. The velocity of sound in air is 345 ms^{-1}.

26. Two trains A and B are approaching one another with the relative velocity of 72 km h^{-1}. If the frequency of the sound of the whistle produced by train A is 500 Hz, calculate the frequency of the sound as heard by a passenger in the train B under the following conditions :
    (*a*) All velocity is in the train A alone.
    (*b*) All velocity is in the train B alone.

(*c*) The train A moves at 90 km h^{-1} towards train B, but the train B also moves in the same direction.
(The velocity of sound is 340 m^{-1}).

27. When a motor car crosses a policeman the frequency of its horn is detected to drop by 10%. If the true frequency of the horn is 500 Hz and velocity of sound in air is 340 ms^{-1}, calculate the speed of the car.

28. When a motor car crosses a policeman, his instrument detects a drop of 11% in the pitch of the car. Calculate the speed of the car if the velocity of sound is 330 ms^{-1}.

29. A passenger standing on a railway platform observes that when a train crosses him at a speed of 54 km h^{-1}, the frequency of the whistle appears to drop by 68 Hz. If the velocity of sound is 340 ms^{-1}, calculate the true frequency of the whistle.

30. Consider a source of sound moving towards an observer at the speed (*a*) 0·75 c (*b*) 1·25 c. Deduce the oberved frequency of the sound in each case if the true frequency is 500 Hz.

31. A submarine is approaching a ship with a certain velocity. If the true frequency of sonar is 60,000 and it measures an increase in the frequency as 500 Hz, what is the speed of submarine ? The velocity of sound in water is 1400 ms^{-1}.

32. An aeroplane is approaching a radar with a certain velocity. If the radar wave has true frequency as $7{\cdot}5 \times 10^9$ s^{-1} and it shows a frequency difference of $2{\cdot}5 \times 10^3$ s^{-1} on the higher side, calculate the velocity of aeroplane.

33. The spectral line of $\lambda = 5000$ Å in the light coming from a distant star is observed as 5200 Å. Determine the recession velocity of the star. [*A.I S.S.E. 1982*]

34. The equation of a harmonic wave is given by

$$y(x, t) = 6{\cdot}0 \cos 56 \left(t - \frac{x}{v}\right) \text{ metre}$$

where $v = 280$ ms^{-1} what is the time period and wavelength ? [*D.S.S.E.* (*Compt.*) *1981*]

35. Write the value of amplitude and angular frequency in the following simple harmonic oscillation :

$$y = 0{\cdot}70 \cos (180t + 0{\cdot}23)$$

where the various quantities are in MKS units.
[*D.S.S.E.* (*Compt.*) *1985*]

36. What is the phase difference between two sound waves $y = a \sin (wt - kx)$ and $y = a \cos (wt - kx)$. [*C.P.M.T. 1975*]

37. The displacement of a particle is given by

$$y = 5 \times 10^{-4} \sin (100t - 50x)$$

where x is in metres and t is seconds. Find out the velocity of the wave. [*C.P.M.T. 1982*]

38. The velocity of sound is measured in Hydrogen and Oxygen gases at a given temperature. What is the ratio of two velocities ? [*C.P.M.T. 1976*]

39. The speed of sound in a gas at N.T.P. is 300 m/s. If the pressure is increased four times without change in temperature, what will be velocity of sound ? [*N.C.E.R.T. 1980*]

40. The equation of wave travelling in a spring can be written as $y=3 \cos \pi (100t-x)$. What is its wavelength if x is measured in cm ? [*M.N.R. 1985*]

41. A transverse wave of amplitude 0·5 m and wavelength 1 m and frequency 2 Hz is propagating in a string in negative x-direction. What is the expression for the wave ? [*A.I.I.M.S. 1980*]

42. A source emitting sound of frequency 1600 Hz and an observer are approaching one another with the velocity 80 ms^{-1}. Calculate the observed frequency :

    (*a*) if the source alone is moving.

    (*b*) if the observer alone is moving.

    (the velocity of sound=330 m s^{-1}) [*A.I.S.S.E. 1986*]

43. If two waves due to two sources having intensity ratio 25 : 9 interfere, find the ratio of the intensities at maximum and minimum.

44. When the two tuning forks are sounded together, these produce 6 beats per second. The frequency of one fork is 256 Hz. When the other fork is loaded with a little wax, the beats per second is 4. Calculate the frequency of the other fork.

45. When two tuning forks A and B are sounded together, 5 beats per second are heard. If fork 'A' is filed a little, only 3 beats per second are heard. Calculate the frequency of fork 'A' when that of 'B' is 512.

46. Two tuning forks when sounded together produce 4 beats per second. When one fork of frequency 480 Hz is loaded with a little wax, the beats are found to occur at longer intervals. Calculate the frequency of the other fork.

47. Two tuning forks when sounded together produce 3 beats per second. If one of the fork of frequency 326 Hz is filed a little, the beats are found to occur at shorter intervals. Find the frequency of the other fork.

48. Two tuning forks produce 4 beats per second when sounded together. The frequency of one fork is 384 Hz. Calculate the frequency of the other fork. The beats stop if the second fork is loaded with a little wax.

49. Frequency of a tuning fork is 256 Hz. When it is sounded with the other fork, it produces 6 beats per second. When the first fork is filed a little, the number of beats still remains the same. Calculate the frequency of the other fork.

50. If the waves from two sources have the amplitude ratio 5 : 1, calculate the intensity ratio of the two waves and the ratio of intensities at maximum and minimum in case of their interference.

51. Sound waves from two sources superpose on each other with intensity ratio 25 : 16. Find the amplitude ratio and the ratio of intensities at maximum and minimum in interference.

52. Compare the ratio of fundamental frequencies of open end and closed end pipes of the same lengths.

53. Among two interfering sources, let S_1 be ahead in phase by 90° relative to S_2. If an observation point is such that $PS_2 - PS_1 = 2{\cdot}0\ \lambda$, calculate the phase difference between the waves from S_1 and S_2 reaching P.

54. In an experiment with Quinck's tube, one passed from one maximum to the next maximum sound when the U-tube was slided by 33·5 cm. Calculate the wavelength and frequency of the sound wave if the velocity of sound was 335 ms^{-1}.

55. In a Quinck's tube experiment, one passes from one maximum sound to the next minimum sound when the U-tube is slided by 16 cm. Calculate the wavelength of the waves. Calculate the velocity of sound if the frequency of sound is 530 Hz.

56. Two closed organ pipes of lengths 50 cm and 55 cm respectively produce 15 beats per second when sounded together. Calculate the velocity of sound.

57. When two sound waves of wavelengths 76 cm and 75 cm travelling in the same direction in air, superpose on each other, 6 beats per second are produced. Calculate the velocity of sound in air.

58. Two tuning forks 'A' and 'B' together give 11 beats in 3 seconds when the fork 'A' is in resonance with a closed column of air 25 cm long and 'B' is in resonance with an open column of air 49·5 cm long. Calculate the following :

(*a*) velocity of sound in air.

(*b*) frequency of each of the tuning forks 'A' and 'B'.

59 Two open organ pipes give 8 beats per second when sounded together in air at 27°C. How many beats would be heard when the temperature is raised by 100°C. The length of the two pipes may be supposed to remain unchanged.

60. A cyclist moving away from a hill at 6 ms^{-1} hears a sound directly from a source in front of him at some distance as well

as after reflection from the hill. Calculate the number of beats produced per second between these two sounds. True frequency of sound is 220 Hz and velocity of sound in air is 330 m s^{-1}.

61. An observer moving towards a wall at 3 ms^{-1} hears a sound directly from a source at some distance behind him and as well as after reflection from the wall. If the true frequency of the sound is 110 Hz and the velocity of sound is 330 ms^{-1}, find the beat frequency.

62. The number of beats per second is 7 when a tuning fork is sounded together with a string when its length is either 90 cm. or 100 cm. What is the frequency of the fork ?

63. A tuning fork of frequency 170 Hz is sounded along with a sonometer wire of length 20 cm stretched to a tension of 1·25 kg wt. Calculate the number of beats produced per second. Mass per unit length of the wire is $2{\cdot}5 \times 10^{-3}$ kg m^{-1}.

64. A sitar wire is under tension of 20 N and the length between the two bridges is 70 cm. A 5 metre sample of that wire has a mass of 10 g. Deduce (*a*) speed of the transverse waves on the wire, (*b*) the frequency of fundamental note and (*c*) the frequencies of first two overtones.

65. A steel guitar is under tension of 90 N. If the mass per unit length of the string on the guitar is 1×10^{-3} kg m^{-1}, calculate the velocity of wave on string.

66. A sonometer wire of length 50 cm is under tension of 400 N. If the mass per unit length of the wire is $4{\cdot}9 \times 10^{-3}$ Kg m^{-1}, calculate the frequency of the fundamental note.

67. A sonometer wire is stretched by a load of 2 kg wt. Its 60 cm length is in resonance with a tuning fork. Calculate the frequency of the tuning fork, if 4 metres sample of the wire has a mass 4 g.

68. A wire of length 70 cm is stretched by a load of 1·5 kg. If its frequency of vibration is 50 Hz, calculate the mass per unit length of the wire.

69. A sonometer wire of length 50 cm gives a fundamental note of 256 Hz. When it has a tension of 1 kg wt., how will you make the wire to emit a note of 512 Hz. (*a*) keeping the tension constant, (*b*) keeping the length constant ?

70. A piano wire of length 40 cm produces a note of frequency 480 Hz. Calculate the frequency of the wire when its length is increased to 60 cm keeping all other factors constant.

71. A wire under tension vibrates with a fundamental frequency of 512 Hz. What will be the fundamental frequency if the wire is under four times the tension, twice as long and half as thick ?

72. A 50 cm long steel wire, having density 7·8 g/c.c. is under tension of 40 N. If it vibrates in the fundamental note of frequency of 256 Hz, calculate the diameter of the wire.

73. A 80 cm long wire, having radius 0·18 mm is stretched by a load of 2 kg wt. If it vibrates with fundamental note of 480 Hz, calculate the density of the material of the wire.

74. The length of an organ pipe is 68 cm. If the velocity of sound in air is 340 m/s, calculate the frequency of the fundamental note emitted by the pipe in the following cases :

(*a*) the pipe is open at both the ends,

(*b*) the pipe is closed at one end.

75. In resonance air column experiment, if a tuning fork of frequency 480 Hz is used, two resonances occur at 17·6 cm and 53 cm. Calculate the following if the room temperature is 15°C :

(*a*) velocity of sound in air at room temperature.

(*b*) the end correction.

(*c*) velocity of sound at 0°C.

76. In a resonance air column experiment the first resonance occurs at column length of 20·2 cm with a tuning fork of frequency 426 Hz. Calculate the velocity of sound in air, neglecting the end correction.

77. An open pipe 40·0 cm long and a closed pipe 30·4 cm long are of the same diameter. They are in unison and producing their first overtones. If the frequency of sound emitted by them is 850 Hz, calculate the following :

(*a*) the end correction of these pipes.

(*b*) the velocity of sound in air.

78. A tuning fork of frequency 510 Hz, produces resonance with a column of air. If the velocity of sound in air 340 ms^{-1}, calculate the length of air column, when

(*a*) it is open at both ends.

(*b*) it is closed at one end.

79. A one metre long steel rod clamped in the middle is set into longitudinal vibrations in the fundamental note with the frequency 2·8 KHz. Determine the velocity of longitudinal waves in steel.

80. A rod 2 m long is clamped in the middle and is set into longitudinal vibrations. If the velocity of waves in rod is 5 km s^{-1}, calculate the frequency of fundamental note produced by rod.

81. In Kundt's tube experiment, the mean distance between two successive heaps in the gas column of tube is 6 cm The length

of the steel rod is 70 cm. Calculate the velocity of longitudinal waves in gas if that in steel is 5600 ms^{-1}.

82. In a Kundt's tube experiment, the mean distance between two successive heaps in air column of tube is 5·5 cm. The length of the rod is 80 cm. Calculate velocity of sound in rod material if that in air is 340 ms^{-1}.

83. A wire is under tension of 32 N and length between the two bridges is 1 metre. A 10 metre length of the sample of the wire has a mass of 2 g. Deduce the speed of the transverse waves on the wire and frequency of the fundamental note. *[A.I.S.S.E. 1981]*

84. In a resonance tube using a tuning fork of frequency 325 Hz two successive resonance lengths are observed at 25·4 cm and 77·4 cm respectively. Calculate the velocity of sound in air and end correction. *[D.S.S E. 1985]*

85. A stretched string is observed to vibrate with a frequency of 30 Hz in its fundamental mode when the supports are 60 cm apart. The vibrating string has a mass of 30 g. Calculate (*a*) the speed of propagation of the transverse waves in the string; (*b*) the tension in the string. *[A.I.S.S.E. 1985]*

86. An air column in a pipe which is closed at one end, is in resonance with a tuning fork of frequency 264 Hz. What is the length of air column in cm ? (C=330 m/s) *[I.I.T. 1985]*

87. A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to free end of the rope. A transverse pulse of wavelength 0·06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope ? *[I.I.T 1984]*

88. A sonometer wire under a tension of 64 Newton vibrating in its fundamental mode is in resonance with a vibrating tuning fork. The vibrating portion of the sonometer wire has a length of 10 cm and a mass of 1 g. The vibrating tuning fork is now moved away from the vibrating wire with a constant speed and an observer standing near the sonometer hears one beat per sec. Calculate the speed with which the tuning fork is moved if the speed of sound in air is 300 m/s. *[I.I.T. 1983]*

89. A man is watching two trains one leaving and the other coming in with equal speeds of 4 m/s If they sound their whistles, each of natural frequency of 240 Hz, what will be the number of beats heard by the man (C=350 m/s) ? *[N.C.E.R.T. 1984]*

90. An open organ pipe has a fundamental frequency of 300 Hz. The first overtone of this organ pipe is the same as the first overtone of a closed organ pipe. What is the length of the closed organ pipe ? (C=328 m/s) *[N.C.E.R.T. 1982]*

91. A tuning fork of frequency of 480 Hz produce 10 beats per sec. when sounded with a vibrating sonometer string. What must have been the frequency of string if slight increase in tension produces fewer beats/sec. than before. [*N.C.E.R.T. 1984*]

92. A cylindrical tube open at both end has a fundamental frequency '*f*' in air. The tube is dipped vertically in water so that half of it is in water. What will be fundamental frequency of the air column now ? [*I.I.T. 1981*]

93. A metal wire of diameter 1 mm is held on two knife edges separated by a distance of 50 cm. The tension in the wire is 100 N. The wire vibrating with its fundamental frequency and a vibrating tuning fork produces 5 beats/s.

The tension in the wire is then reduced to 81 N. When the two are excited, beats are heard again at the same rate. Calculate (*a*) the frequency of the fork (*b*) the density of the material of the wire. [*I.I.T. 1980*]

94. The velocity of sound is 300 m/s. What is the distance between two successive nodes of a stationary wave of frequency 1000 Hz. [*C.P.M.T. 1979*]

95. Calculate the frequency of the fundamental note in a sonometer with the length of the vibrating wire as 20 cm. tension=25 N. area of cross-section of wire=10^{-3} cm^2 and density of the material of the wire is 10^4 kg m^{-3}. [*D.S.S.E. 1986*]

96. A tuning fork of frequency 500 Hz produces resonance in an air column successively at length 0·2 m. and 0·65 m. Calculate the end correction and speed and sound in air. [*A.I.S.S.E. 1986 Compt.*]

97. A string 25 cm long and having mass 2·5 g is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per sec are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 ms^{-1}, find the tension in the string. [*I.I.T.J.E.E. 1982*]

98. A rope of mass 0·1 kg/m is held with a tension of 1000 N. As a wave passes along the rope, every point on the rope moves with S.H.M. with a frequency of 10 Hz and an amplitude of 0·1 m. For this wave what is (*a*) the velocity, (*b*) the wavelength, (*c*) the energy density ? [*D.S.S.E. 1987*]

99. At what speed should an observer in a car approach a stationary siren so that its frequency apprears to be 10% higher ? (Velocity of sound=340 m/s). [*D.S.S.E. 1987 Compt.*]

100. A Sitar wire is under tension 30 N and the length between the bridges is 80 cm. A 10 metre sample of wire has mass 2·2 g. Deduce

(*a*) the speed of the transverse waves on the wire

(*b*) the frequency of the fundamental note. [*D.S.S.E. 1988*]

101. The prongs of a tuning fork A, originally in unison with a tuning fork B, are filed. Now the two tuning forks when sounded together produce 4 beats in 2 seconds. What is the frequency of A after filing, if the frequency of B is 250 Hz? [*D.S.S E. 1988 Compt.*]

102. The siren of a police car emits a note of frequency 500 Hz. This car is moving at a speed of 30 ms^{-1}. A passenger car moving at a speed of 15 ms^{-1} is approaching it from opposite direction. What is the frequency of the siren note which is heard by the driver of the passenger car :

(*a*) before crossing the police car, and

(*b*) after crossing the police car. ($c=330\ ms^{-1}$). [*D.S S.E. 1988 Compt.*]

103. The density of air at S.T.P. is 1·293 kg m^{-3}. The ratio of the two principal specific heats is 1·40. Calculate the velocity of sound at 20°C. (The standard pressure$=1{\cdot}013\times10^5$ Nm^{-2}) [*D.S.S.E. 1989*]

104. Two identical Sonometer wires resonate at fundamental frequency of 300 Hz. when subjected to a tension of 6 kg wt. The tension of one of the wires is increased by 200 g. wt. Calculate the beat frequency if both the wires are sounded simultaneously. [*D.S.S.E. 1989*]

105. A tuning fork of frequency 120 Hz is sounded alongwith a sonometer wire of length 35 cm. The wire is stretched by a tension of 2·5 kg wt. Calculate the number of beats per second (mass per unit length of the wire 0·05 g/cm). [*A.I.S.S.E. 1989*]

106. A tuning fork produces two successive resonances in a resonance tube (closed pipe) experiment at lengths 22·0 cm and 68·2 cm respectively. Calculate the end correction and the frequency of the fork. (Velocity of sound is 340 ms^{-1}.) [*D.S.S.C.E. 1989*]

107. The fundamental note produced by a wire 50 cm. long has a frequency which is four times the frequency of the fundamental note of another wire 150 cm. long. Assuming that both the wires are subjected to the same tension, calculate the ratio of the masses per unit length of the two wires. [*D.S.S.C.E 1989*]

OBJECTIVE TYPE QUESTIONS

108. The ratio of intensities of two waves is 1 : 25. The ratio of their amplitude will be
(*a*) 1 : 25 (*b*) 5 : 1
(*c*) 26 : 24 (*d*) 1 : 5 [*C.P.M.T. 1988*]

109. The reflected sound is heard after 0·6 S from a hill 100 m away. The velocity of sound is
(*a*) 166·6 m/s (*b*) 60 m/s
(*c*) 333·3 m/s (*d*) 332 m/s. [*C.P.M.T. 1988*]

110. The resultant of two amplitudes a_1 and a_2 will be maximum when phase difference angle is
(*a*) 180° (*b*) 0°
(*c*) 90° (*d*) 45° [*C.P M.T. 1988*]

111. The tuning fork 'B' is sounded with tuning fork A of frequency 200 cps and 3 beats are heard. If B is loaded with wax. 5 beats are heard. The frequency of 'B' is
(*a*) 203 Hz. (*b*) 197 Hz.
(*c*) 200 cps. (*d*) 206 cps. [*C.P.M.T. 1988*]

112. In which of the following medium sound has maximum velocity ?
(*a*) air (*b*) hydrogen
(*c*) oxygen (*d*) carbon dioxide.
[*C.P.M.T. 1988*]

113. Waves transmit from one place to the other
(*a*) energy (*b*) wavelength
(*c*) amplitude (*d*) matter. [*C.P.M.T. 1988*]

114. The equation of progressive waves is

$$y=0{\cdot}3 \sin (31{\cdot}4\, t-1{\cdot}57\, x)$$

and $y=0{\cdot}1 \sin (31{\cdot}4\, t-1{\cdot}57\, x+1{\cdot}57)$

where x, t, y are in metre, second and metre respectively. The frequency of the wave is
(*a*) 100 cps. (*b*) 200 cps.
(*c*) 50 cps. (*d*) 75 cps. [*C.P.M.T. 1988*]

115. A wave represented by equation $y=a \cos (kx-wt)$ is superposed with another wave to form a stationary wave such that the point $x=0$ is a node. The equation for the other wave is
(*a*) $a \sin (kx+wt)$ (*b*) $-a \cos (kx-wt)$
(*c*) $-a \cos (kx+wt)$ (*d*) $-a \sin (kx-wt)$
[*I.I.T. J.E.E. 1988*]

116. The displacement of particles in a string stretched in the x direction is represented by y. Among the following expressions for y, those describing wave motion are :
(*a*) $\cos kx \sin wt$ (*b*) $k^2x^2-w^2t^2$
(*c*) $\cos^2 (kx+wt)$ (*d*) $\cos (k^2x^2-w^2t^2)$.

117. If the temperature of the atmosphere is 30°C, the velocity of sound in air will be
(*a*) 332 ms^{-1} (*b*) 340 ms^{-1}
(*c*) 350 ms^{-1} (*d*) 362 ms^{-1}.

118. For every 1°C rise in temperature, the velocity of sound in air increases nearly by
(*a*) 0·3 ms^{-1} (*b*) 0·6 ms^{-1}
(*c*) 0·9 ms^{-1} (*d*) 1·2 ms^{-1}.

119. In which of the following media the velocity of sound is maximum
(*a*) hydrogen (*b*) air
(*c*) water (*d*) steel.

120. The ratio of velocity of sound in oxygen to that in hydrogen is
(*a*) 1 : 16 (*b*) 16 : 1
(*c*) 4 : 1 (*d*) 1 : 4.

121. A railway engine which is moving towards a stationary observer on the platform with a speed of 30 km h^{-1}, blows a whistle of frequency 480 Hz. The apparent frequency as heard by the observer will be
(*a*) greater than 480 (*b*) less than 480
(*c*) equal to 480 (*d*) 0.

122. When the tension in a string is made four times, the velocity of transverse waves over the string becomes
(*a*) four times (*b*) two times
(*c*) one-fourth (*d*) one-half.

123. When the diameter of vibrating wire under tension in sonometer is doubled, the frequency of vibration of wire becomes
(*a*) twice (*b*) one-half
(*c*) four times (*d*) one-fourth.

124. How will the intensity of sound waves change if the amplitude of the waves is doubled
(*a*) twice (*b*) one-half
(*c*) four times (*d*) one-fourth.

125. The wave velocity is given by
(*a*) $c=\nu/\lambda$ (*b*) $c=\lambda/\nu$
(*c*) $c=\nu\lambda$ (*d*) $c=\dfrac{1}{\nu\lambda}$.

126. In a stationary wave the distance between two successive antinodes is

(a) $\lambda/4$　(b) $\lambda/2$

(c) λ　(d) 2λ.

127. The distance between the two successive heaps of powder in Kundt's tube is

(a) 2λ　(b) λ

(c) $\lambda/2$　(d) $\lambda/4$.

128. The fringe width '*w*' of interference fringes is equal to

(a) $\lambda d/D$　(b) $\lambda D/d$

(c) $D/\lambda d$　(d) $d/\lambda D$.

129. Two tuning forks produces the same beats/sec. as before even after loading one of the tuning forks with a little wax. Is it possible ?

(a) yes　(b) no

(c) not decided　(d) question is incomplete.

130. Beats are produced due to

(a) constructive interference

(b) destructive interference

(c) constructive and destructive interference

(d) diffraction.

131. What is the phase difference between the two points separated by $\lambda/2$.

(a) $\pi/2$　(b) π

(c) 2π　(d) $\pi/4$.

132. When the humidity of the atmosphere increases, the velocity of sound in air

(a) increases　(b) decreases

(c) remains same　(d) not decided.

☐☐

Answers

EXERCISE 1

1. (*i*) *c* (*ii*) *e* (*iii*) *f* (*iv*) *b* (*v*) *d* (*vi*) *a* **2.** (*i*) *b* (*ii*) *a* (*iii*) *d* (*iv*) *e*
(*v*) *c*. **3.** (*i*) *f* (*ii*) *f* (*iii*) *e* (*iv*) *b* (*v*) *c* (*vi*) *a* (*vii*) *d* **4.** (*i*) *e* (*ii*) *c* (*iii*) *a*
(*iv*) *b* (*v*) *d* **5.** *b* **6.** *a* **7.** *c* **8.** *d*.

EXERCISE 2

1. LT^{-1}, LT^{-2}, MLT^{-2} **2.** L^2, L^3, ML^{-3} **3.** ML^2T^{-2}, ML^2T^{-2}
4. ML^2T^{-1}, $ML^{-1}T^{-1}$ **5.** ML^2 **6.** ML^2T^{-3} **7.** ML^2T^{-2}, T^{-1}
8. MT^{-2}, $MT^{-2}I^{-1}$ **9.** $MT^{-2}I^{-1}$ **11.** $M^{-1}L^3T^{-2}$, $ML^2T^{-2}I^{-1}$
12. IT, $MLT^{-3}I^{-1}$, $ML^2T^{-2}I^{-2}$ **13.** $T=2\pi\sqrt{l/g}$ **15.** $a=-5/6$,
$b=\frac{1}{2}$, $c=1/3$ **16.** $p=4\sigma/r$ **17.** $h=\dfrac{2\,T\cos\theta}{h\rho g}$ **19.** $m=k\rho v^6g^3$
26. $v=\dfrac{k}{l}\sqrt{Y/\rho}$ **27.** $c=\sqrt{\dfrac{p}{\rho}}$ **29.** (*a*) $\rho v^4\nu^{-2}$ (*b*) $\rho v^4\nu^{-3}$ (*c*) $\rho v^3\nu^{-1}$
30. (*a*) $va^{-1}F$ (*b*) $v^3a^{-2}F$ (*c*) $v^{-4}a^2F$ **31.** $E=2\pi^2mv^2a^2$
32. $v=k\sqrt{\lambda g}$ **33.** $ML^3T^{-4}I^{-2}$ **34.** $p=h\rho g$ **35.** $v=k\sqrt{gR}$
36. $T=2\pi\sqrt{\dfrac{m}{k}}$ **37.** $MLT^{-3}\,k^{-1}$ **38.** (*a*) $F=mv^2/r$ (*b*) $E=\frac{1}{2}mv^2$
39. 50 divisions on circular scale **40.** 0·01 cm **41.** 0·5 mm,
0·005 cm **42.** (*a*) 6·8%, (*b*) 5·0% **43.** (*a*) 5·66%, (*b*) 5·44%
44. (*a*) 0·53%, (*b*) 1·34% **45.** (*a*) 0·348%, (*b*) 0·172% **46.** 3·84
$\times 10^8$ *m* **47.** $1{\cdot}4\times10^3$ kg m^{-3}.

Objective Type Questions

48. *c* **49.** *d* **50.** *c* **51.** (*a*) 3, (*b*) 6, (*c*) 1, (*d*) 5, (*e*) 2, (*f*) 4
52. *d* **53.** *b* **54.** *c* **55.** *c* **56.** *b* **57.** (*a*) 3, (*b*) 4, (*c*) 5,
(*d*) 1, (*e*) 2 **58.** *a* **59.** *a* **60.** *a* **61.** *d* **62.** *b* **63.** *c*
64. (*a*) $1{\cdot}25\times10^{-4}$ (*b*) 5029 (*c*) 40 (*d*) 13·6, 13600 **65.** (*a*) 4
(*b*) 4 (*c*) 3 (*d*) 3 (*e*) 4 (*f*) 4 (*g*) 5 (*h*) 2 **66.** (*a*) 10^{15} (*b*) 1·057
$\times10^{-16}$ (*c*) 10^{-8} (*d*) 12·5 (*e*) 1000 **67.** *d* **68.** *c* **69.** *d*.

EXERCISE 3

1. $-1{\cdot}75$ ms^{-2}, 87·5 m **2.** 5 kg m/s **3.** 3 kg ms^{-1} **4.** 2×10^4
N **5.** (*a*) 62·5 kg, (*b*) (*i*) 50 kg, (*ii*) 25 kg **6.** $1/3\times10^8$ ms^{-1} **7.** 9·6 m,
Loss in K.E. appears in the form of sound and heat energy. **8.** (*a*) 300
ms^{-1} (*b*) 900 ms^{-1} (*c*) 900 ms^{-1} **9.** (*a*) 2 ms^{-2}, (*b*) 110 m
10. 12·5 ms^{-1} **11.** 17 6 ms^{-1}, 15 6 m **12.** 15 ms^{-1} **13.** 500 ms^{-1}
14 The boat should be rowed up stream making an angle 120°
with water stream **15.** 35 m, $S\propto t^2$ **16.** 2·2 ms^{-2}, 4·4 m **17.** (*a*) at
the bottom of the mast (*b* 16 m away from the mast **18.** (*a*) 35 s
(*b*) 900 m (*c*) 29·4 ms^{-1} **19.** 2431 m **20.** 2s, 9 8 ms^{-1} **21.** (*a*)
28·28 s (*b*) No, it hits 4·713 km away from the target **22.** No,
1·984 km **23.** (*a*) 2 mg (*b*) 2 mg (*c*) 3 mg **24.** 0·1 ms^{-1} **25.** 14·1
ms^{-1} **26.** 4·42 s **27.** 19 ms^{-1} **28.** (*a*) 178·4 m (*b*) 59·13 ms^{-1}

(*c*) 10·03 s **29.** (*a*) 180 m (*b*) 60 ms^{-1} (*c*) 9 s **30.** 12 s, 98 ms^{-1} **31.** 0·2 kg **32.** 0·6 kg **33.** 0·3535 **34.** 800 km/h **35.** 176·4 m, 12 s **36.** 10 s, 10 s **37.** (*a*) 6 km/h, 6 km/h (*b*) 1·5 km/h, 4·5 kmh^{-1} (*c*) 0, 4 kmh^{-1} **38.** Yes, $R_{max}=12$ km **39.** 500 N, if applied at 2 kg mass, 200 N if applied at 5 kg mass **40.** 28 ms^{-2}, 63 N **41.** 0·4 kg ms^{-1} **42.** 273·6 N, 715·7 N **43.** 173·6 N, 258·8 N **44.** (*a*) 6·25 ms^{-1}, 8·86 ms^{-1}, 45° (*b*) 7·8 ms^{-1}, 2·0 ms^{-1} 38°48′ **45.** 18°25′, 63·08 N **46.** 2·55 ms^{-1} **47.** 3·27 ms^{-2}, 13·33 kg wt **48.** 1 : 2 **49.** (*a*) 2 ms^{-2} (*b*) 48 N **50.** All the four graphs are impossible (*a*) A particle can't have two different position at the same time (*b*) A particle can't have velocity in opposite direction at the same time (*c*) Speed is never negative (*d*) Total path length of a particle can never decrease with time. **51.** It is wrong, X-*t* plot does not show the trajectory of a particle, context : a body dropped from a tower **52.** Greatest in 3, least in 2 ; $V>0$ in 1 and 2 and $V<0$ in 3.

53.

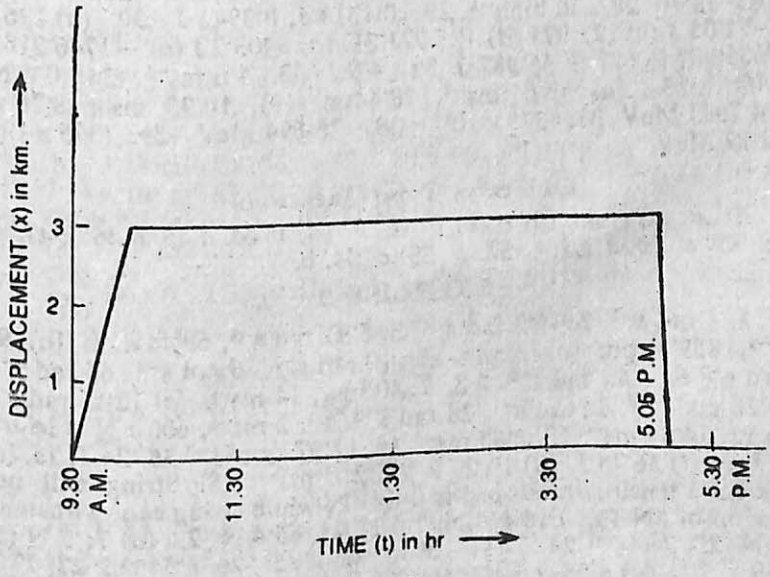

Fig. 51

54. 22°36′ due west of south, 13 min **55.** 8 kmh^{-1}, 30° to verticle towards east **56.** 138° 36′ with the water stream **57.** 109·86 m **58.** 5·1 m **59.** 0·5774 **60.** 70·86 m **61.** (*a*) 0 (*b*) ·018 J **62.** 12·25 s, 5·25 s **63.** 1 s, 10 m from the ground **64.** 2·86° **65.** 10 s, 99·6 ms^{-1} **66.** 0 **67.** (*a*) The force on the ball is normal to the wall in each case (*b*) 13 : 10 **68.** 37·5 ms^{-2} **69.** 50 N, 245 N **70.** 31·25 m **71.** Parabola, $x=\frac{1}{2}\frac{qE}{mv_0^2}y^2$ **72.** Q hits the ground in the middle of AB vertically, $5\sqrt{2}$ s **73.** $\frac{75}{32}$ J, $\frac{75}{32}$ J, $\frac{50}{32}$ J, **74.** 45° with horizontal, 31·3 ms^{-1}.

Objective Type Questions

75. *c* **76.** *a* **77.** *b* **78.** *a* **79.** *b* **80.** *b* **81.** *a* **82.** *b* **83.** *d* **84.** *a* **85.** *c* **86.** *a* **87.** *d* **88.** *d* **89.** *b* **90.** *c* **91.** *b* **92.** *d* **93.** *b* **94.** *a* **95.** *d* **96.** *c* **97.** *c* **98.** *c* **99.** *d* **100.** *c* **101.** *d* **102.** *c* **103.** *c* **104.** *b* **105.** *a* **106.** *a* **107.** *c* **108.** *a* **109.** *a* **110.** *c* **111.** *b* **112.** *d* **113.** *b*.

EXERCISE 4

1. 8×10^8 J **2.** 2×10^4 J **3.** 25 m/s **4.** 0, 9800 J **5.** 328·3 J **6.** 205·8 kW **7.** 40·8 m **8.** $2{\cdot}4 \times 10^6$ J **9.** 2×10^6 J **10.** 9·8 J, 19·6 J **11.** 0·98 J, 0·8488 J **12.** 9×10^{13} J **13.** $14{\cdot}94 \times 10^{-11}$ J **14.** $1{\cdot}11 \times 10^{-11}$kg **15.** 20 kW **16.** 49 W **17.** 2960 N, (*a*) 10 kW (*b*) 29·6 kW **18.** 120 kW **19.** $11{\cdot}76 \times 10^5$ J, 544·4 W **20.** $2{\cdot}94 \times 10^5$ J, $1{\cdot}96 \times 10^4$ W **21.** (*a*) 1·47 $\times 10^5$ J (*b*) $7{\cdot}5 \times 10^5$ J (*c*) 14·95 kW **22.** $9{\cdot}8 \times 10^6$ J, 3·4 kW **23.** $4{\cdot}5 \times$ 10^{-19} J **24.** 600 Nm^{-1}, $4{\cdot}563 \times 10^6$ J **25.** 5·2 ms^{-1} **26.** 0·4 cm. **27.** Rs. 38·40 **28.** 36 kmh^{-1} **29.** 0·1314 J, 0·3942 J **30.** (*a*) 196 J (*b*) −72·03 J (*c*) 123·97J (*d*) 123·97J **31.** (*a*) 9403·2 J (*b*) −1746·213 J (*c*) 7656·987 J (*d*) 7·56·987 J **32.** 4·9 J **33.** 4·6 ms^{-1} **34.** 0·6 N, 288 J/kg **35.** (*a*) 13·05 ms^{-1}, 10·44 m (*b*) 10·95 ms^{-1} 8·76 m **36.** 4·7835 MeV **37.** 375×10^{13} **38.** 25·694 MeV **39.** 375×10^{15} **40.** 2·22 MeV.

Objective Type Questions

41. *c* **42.** *c* **43.** (*a*) F (*b*) F (*c*) F (*d*) F **44.** *d* **45.** *a* **46.** *c* **47.** *a* **48.** *d* **49.** *c* **50.** *d* **51.** *c* **52.** *b* **53.** *c* **54.** *b*.

EXERCISE 5

1. 8 rad s^{-2} **2.** 15·7 rad s^{-1} **3.** 6·91 rad s^{-1}, 69·44 s **4.** (*a*) 90 rads^{-1}, 859·4 rpm (*b*) 3 min. **5.** 10 rad s^{-1}, 6 rad s^{-2}, 45 rad s^{-1}, 22 rad s^{-2} **6.** -4π rad s^{-2}, 5 S **7.** 40 rev., 14 rev **8.** (*a*) 12·56 rad s^{-1} (*b*) 6·28 ms^{-1} **9.** 65 rad s^{-1}, 28 rad s^{-2} **10.** 2 ms^{-2}, 6000 N **11.** 0·5 rev/s **12.** 14·85 ms^{-1} **13.** 8·85 ms^{-1} **14.** 12·57 $\mathrm{kgm}^2\,\mathrm{s}^{-1}$ **15.** 24 J, **16.** (*a*) 24·5 J (*b*) (*i*) 36·75 J (*ii*) 10·29 J **17.** 2·7 ms^{-2} **18.** String will not break since tension developed is only 4N while string can withstand a tension of 5N **19.** 1815 N **20.** 7·6 N **21.** 65·4 N **22.** (*a*) 71·9 N (*b*) 79·7 N **23.** 7 ms^{-1} **24.** $3{\cdot}41 \times 10^3$ m **25.** 2·85° **26.** 4·54 cm **27.** 7° 7′ **28.** 49 J **29.** (*a*) 6·25 rad s^{-2} (*b*) 100 rad s^{-1} **30.** (*a*) 12 rad s^{-2} (*b*) 150 rad **31.** 0·5 N, 25 J **32.** 2·94 kg m^2 **33.** 0·628 Nm, 3·14N **34.** 0·786 Nm, 9·88 watt **35.** $13/20\ MR^2$ **36.** (*a*) $\frac{1}{2} Mr^2$ (*b*) $\frac{3}{2} Mr^2$ **37.** $Ml^2/3$ **38.** 160 J, 6·4 $\mathrm{kgm}^2\,\mathrm{s}^{-1}$ **39.** 10 ms^{-1} **40.** 30 km **41.** 11°46′ **42.** outer rail, 27·5° **43.** $\mu mg \geqslant m\omega^2 T$ or $\mu g \geqslant \omega^2 r$, $\mu g = 1{\cdot}176$, $\omega^4 r = 0{\cdot}242$ for coin at 5 cm & $\omega^2 r = 0{\cdot}484$ for coin at 10 cm. So both the coins will revolve with record. **44.** (*a*) 14 rad s^{-1} (*b*) 90 J, 490 J, Increase in energy is supplied by the child when he brings his hands inwards. **45.** 5·8 rad s^{-1} **46.** 12·5 ms^{-2} **47.** 11·34 S 19·84 S the sphere will revolve earlier. **48.** (*a*) 10·63 S (*b*) (*i*) 0, 156·3 J (*ii*) 625 J, 156·3 J (*c*) 312·4 J, against friction. **49.** (*a*) 5·1 S (*b*) (*i*) 0, 500 J (*ii*) 2250 J.

250 J (*c*) 1500 J, against fraction **50.** −0·02 kgm² s^{-2} **51.** 100 : 49 **52.** 3·535 ms^{-2}, 45° with the direction of velocity.

Objective Type Questions

53. *b* **54.** *a* **55.** *c* **56.** *a* **57.** *d* **58.** *b* **59.** *c* **60.** *d* **61.** *a* **62.** *a* **63.** *b* **64.** *c* **65.** *a* **66.** *b*.

EXERCISE 6

1. 6·67×10^{-9} N **2.** (*a*) 1·635×10^{19} N (*b*) 24·2 kms^{-1} **3.** 6·39 ×10^{6}m **4.** (*a*) 1·7 ms^{-2} (*b*) 2·4 ms^{-1} **5.** 2·935 ms^{-2} **6.** $T=\dfrac{2\pi R}{R\sqrt{\dfrac{4}{3}\pi e G}}=\sqrt{\dfrac{3\pi}{Ge}}$=5·12 kms^{-1} **7.** 8·45 ms^{-2}, 7·69 km/s, 1 hr 35 min. **8.** 10·4 kms^{-1} **9.** 7·35×10^{22} kg. **10.** 8 kms^{-1} **11.** 11·2 kms^{-1} **12.** 7·99 kms^{-1} **13.** (*a*) 7·699 kms^{-1} (*b*) 91 min 50 s **14.** 11·52 kg **15.** 17 rev./day **16.** 688·7 days **17.** 29·4 ms^{-2} **18.** 2·4 kms^{-1} **19.** 1·605 ms^{-2} 2·35 kms^{-1} **20.** 6·06×10^{24} kg **21.** 4·048×10^{8} m **22.** 10·56 kms^{-1} **23.** 164·3 days **24.** $M=\dfrac{4\pi^2}{G}\,\dfrac{a^3}{T^3}$=2·0×$10^{30}$ kg **25.** 865 km **26.** 129 days **27.** 11·857 : 1 **28.** 6 J kg^{-1}, 30 J **29.** 16·7 kms^{-1} **30.** 11·2 kms^{-1} **31.** 18·72 kms^{-1} **32.** 1·7 ms^{-2} **33.** 684·47 days, 223·15 days **34.** 7·8 ×10^{11} m, 14·3×10^{11} m **35.** 6·52 Nm^2/kg^2 **36.** 9·79 ms^{-2} **37.** 7·537 kms^{-1}, 1 hr. 37·3 min **38.** 36 N **39.** 15 kg **40.** 5·9×10^{9} km **41.** 13061 km **42.** 7·34×10^{6} m **43.** 10·34 kms^{-1} **44.** 6 kg **45.** (*a*) 5·8×10^{9} J (*b*) −11·6×10^{9} J (*c*) −5·8×10^{9} J **46.** (*a*) 13·132×10^{9} J (*b*) −26·264 ×10^{9} J (*c*) −13·132×10^{9} J **47.** 1245 km **48.** 3·48×10^{8} m **49.** 0, $-4\sqrt{2}\,\dfrac{Gm}{l}$ **50.** 0, 1·334×10^{-8} J kg^{-1} **51.** 5·558 ms^{-2} directed towards 40 kg, 2×10^{-8} J kg^{-1} **52.** 0·00476 Å **53.** 38000 km.

Objective Type Questions

54. *b* **55.** *a* **56.** *d* **57.** *c* **58.** *b* **59.** *c* **60.** *c* **61.** *a* **62.** *a* **63.** *b* **64.** *d* **65.** *c* **66.** *a*.

EXERCISE 7

1. 2 m **2.** 16 **3.** 72 **4.** 100 ml/s **5.** 36 : 1. **6.** N_2 **7.** −204·75°C **8.** 4·4×10^{5} J **11.** 3·944g **13.** law of definite proportions. **19.** (*a*) 12·55 g (*b*) 2 g (*c*) 40·3225 g (*d*) 11·449 g **20.** 6·0×10^{26} **21.** 3·125 g atom, 1·79 g atom, 3·704 g atom **22.** 8·412×10^{25} atoms **23.** 200·5659 g **24.** 3·228×10^{22} **25.** 0·5 mole, 3×10^{-23} **26.** 1·3 g atom **27.** 6×10^{23} **28.** 1·125×10^{-4} **29** MCl_3 **30.** 0·105 **31.** 2·034×10^{-8} ms^{-1} **32.** 71290 **33.** 1·35×10^{-23} J/k **34.** (*a*) 7×10^{-15} m, 2·4×10^{17} kg/m^3 (*b*) 3·024 ×10^{-15} m, 2·34×10^{17} kg/m^3 **35.** 6·1×10^{-14} m **36.** 5·7 MeV **37.** 10^{-12} **38.** 3·355×10^{-15} m **39.** (*a*) 5·696×10^{-15} m (*b*) 4·236×10^{-15} m (*c*) 2·746 ×10^{-15} m **40.** 45·7748 J **41.** 4·4 eV **42.** 4·0296 eV **43.** 3/4000 **44.** −2·775 eV **45.** 0·56 eV.

Objective Type Questions

46. *a* **47.** *c* **48.** *b* **49.** *d* **50.** *c* **51.** *a* **52.** *d* **53.** *c* **54.** *d* **55.** *b* **56.** *a* **57.** *c* **58.** *a* **59.** *a* **60.** *c* **61.** *b* **62.** *d* **63.** *b*.

EXERCISE 8

1. 10^{-3} **2.** (*a*) 1·5 $\times 10^{-5}$ (*b*) 1·66 $\times 10^{7}$ Nm^{-2} (*c*) 11·1 $\times 10^{11}$ Nm^{-2} **3.** 8·445 $\times 10^{10}$ Nm^{-2} **4.** 0·035 mm **5.** 7 $\times 10^{10}$ Nm^{-2} **6** 6 $\times 10^{8}$ Nm^{-2} **7.** 19250 N **8.** 10·204 km **9.** 499·07 cc **10.** 2 $\times 10^{9}$ Nm^{-2}, 0·506 $\times 10^{-4}$ per atmosphere **11.** 1·25 $\times 10^{5}$ Nm^{-2} **12.** 4·4% decrease **13.** 1425·6 N **14.** 2·328 cm **15.** 0·5 $\times 10^{-4}$ J **16.** 3·77 $\times 10^{6}$ N **17** 2 mm **18.** 4 $\times 10^{8}$ Nm^{-2}, 2 $\times 10^{-3}$, 10^{-6} m^2 **19.** 8 $\times 10^{10}$ Nm^{-2} **20.** 11 Hz **21.** $ML^{-1}T^{-2}$ **22.** Second wire **23.** 10^{11} Nm^{-2} **24.** 43·67 N **25.** 44 N **26.** 0·22 mm **27.** Same **28.** 9·98 mm **29.** 2·355 J **30.** 9·043 cm^3.

Objective Type Questions

31. *c* **32.** *d* **33.** *a* **34.** *b* **35.** *c* **36.** *d* **37.** *a* **38.** *a*.

EXERCISE 9

1. 1·00062 $\times 10^{5}$ N/m^2 **2.** 1216 cm **3.** 12·92 m **4.** 5000 kg wt **5.** 36 **6.** 8064 kg wt, 0·25 cm **7.** 400 kg wt **8.** 1 kg wt **9.** 442·4 g **10.** 1·048 g/cc **11.** 0·64, 2·5 **12.** 18 cc, 3·33 **13.** 0·86 g/m^3 **14.** 0·91 **15.** 2·77 g cm^{-3} **16.** 2·5 cm^3 **17.** 7·319 kg Gold 2·681 kg Silver **18.** Float **19.** 9·48 g, 3·02 g **20.** $\frac{2}{11}$ **21.** 12·5 g **22.** 4/51 **23.** $2^{8/3}\pi r^2\sigma$ **24.** 895·3 $\times 10^{-7}$ J **25.** 2·2 $\times 10^{-5}$ J **26.** 0·011 J **27.** 609·84 R^2 **28.** $2\pi D^2\sigma$ **29.** 9·856 $\times 10^{-5}$ J **30.** 0·0125 J **31.** 6·896 $\times 10^{-6}$ J **32.** 2·56 $\times 10^{3}$ Nm^{-2} **33.** 31·6512 $\times 10^{-6}$ J **34.** $4\pi\sigma R^2$ **35.** 1·884 $\times 10^{-3}$ J **36.** 0·1 ms^{-1} **37.** 8·4 mm **38.** 3·4 mm **39.** 3·56 mm **40.** 0·356 mm **41.** 714·3 kg m^{-3} **42.** 2·8 ms^{-1} **43.** 6·51 **44.** 76·936 cm **45.** $\sqrt{2gh}$ **46.** 4 m **47.** 9·8 ms^{-1} **48** 2732·8 Nm^{-2} **49.** 7·2 cm s^{-2} **50.** 3·26 mm s^{-1} **51.** 9·97 $\times 10^{-5}$ m **52.** 79·2 ms^{-1} **53.** (*a*) 3·13 ms^{-1} (*b*) 11·54 ms^{-1} **55.** 0·40 ms^{-1} **56.** 152·44 Pas **57.** 0·319 mm **58.** 0·795 ms^{-1} **59.** 5·3 cm of Hg **60.** (*a*) 1·57 $\times 10^{-13}$ m^3 (*b*) 8·08 $\times 10^{-11}$ J **61.** 3·2 ms^{-1} **62.** 1·743 cm^2 **63.** 0·14 $\times 10^{8}$ $m^{-3}s^{-1}$ **64.** 1 ms^{-1} **65.** 1·095 ms^{-1} **66.** 1·5 Pas **67.** 10·69 ms^{-1} **68.** 40 cm s^{-1}, 9·2125 $\times 10^{4}$ Nm^{-2}.

Objective Type Questions

69. *b* **70.** *d* **71.** *a* **72.** *d* **73.** *d* **74.** *c* **75.** *b* **76.** *a* **77.** *b* **78.** *d* **79.** *b* **80.** *d* **81.** *b* **82.** *d* **83.** *a* **84.** *b* **85.** *c* **86.** *a* **87.** *d* **88** *d* **89** *a* **90.** *d* **91.** *b*.

EXERCISE 10

1. 1·025 km s^{-1}, 1·596 km s^{-1} **2.** 175 : 197 **3.** $\frac{V_1+V_2+V_3}{3}$, $\sqrt{\frac{V_1^2+V_2^2+V_3^2}{3}}$ **4.** (*a*) 5 km s^{-1} (*b*) 5·477 km s^{-1} **5.** 0·583 kms^{-1} **6.** 819°C **7.** (*a*) 0·462 km s^{-1} (*b*) 0·492 km s^{-1} **8.** 2·98 cal. $mole^{-1}$ k^{-1}, 4·96 cal. $mole^{-1}$ k^{-1} **9.** 4·94 cal. mol^{-1} k^{-1}, 6·916 cal. $mole^{-1}$ k^{-1} **10.** 0·46 km s^{-1} **11.** 527°C **12.** 3·34 $\times 10^{-27}$ kg, 1·369 $\times 10^{-23}$ J $mole^{-1}$ k^{-1} **13.** 0·583 km s^{-1} **14.** 0·376 km s^{-1} **15.** −204·75°C **16.** 4922 cm^3 **17.** 1·4 **18.** 2·2 km s^{-1} **19.** 2·075 J mol^{-1} k^{-1} **20.** 2145 J.

Objectives Type Questions

21. *c* **22.** *d* **23.** *b* **24.** *b* **25.** *d* **26.** *a* **27.** *c* **28.** *b* **29.** *a* **30.** *b* **31.** *a* **32.** *b* **33.** *c* **34.** *b* **35.** *d* **36.** *a* **37.** *b* **38.** *b* **39.** *c* **40.** *a*.

EXERCISE 11

1. 30·8 cm **2.** 3 63 cm **3.** 1109·74°C **4.** 10·016 cm **5.** 1002·16 cc **6.** 2924°C **7.** 1425·6 N **8.** 30 cm, 40 cm **9.** 24 cm., 36 cm. **10.** 312 cal **11.** 20·024 cm **12.** 0·5 cal/sec/cm/°C **13.** 50 min **14.** 101·8°C **15.** 408·208 cc. **16.** 12×10^{-6} °C^{-1} **17.** (*a*) 539 gm, (*b*) ·667 sec **18.** 3000 cal s^{-1}, 94°C **19.** 1·20 **20.** 69·284 cm^3 **21.** 0·7 k **22.** 0·98 °C **23.** 2·92 °C **24.** 4·2 J cal^{-1} **25.** 0·116°C **26.** 0·49°C **27.** 34·28 km **28.** 44·64 cal **29.** 257·14 m **30.** 200 m **31.** 1036·9 m/s **32.** 50°C **33.** 525 watt **34.** −10320 J **35.** 410 ms^{-1} **36.** 75% **37.** (*a*) 25%, (*b*) $6{\cdot}3\times10^5$ J, (*c*) 450 kcal **38.** −4012·5 J **39.** 42 kJ **40.** 60·8 kJ **41.** 682·5 J **42.** 4·052 J **43.** $3{\cdot}383\times10^5$ J **44.** 10030 J **45.** 6245·82 kJ **46.** 40% **47.** 33·3% **48.** 40% **49.** 320 k, 33·3% **50.** (*a*) 14, (*b*) 30 J **51.** (*a*) 50 (*b*) 1·2 k cal (*c*) 840 J **52.** 40 cal **53.** (*a*) 50160 J (*b*) 72 k cal (*c*) 5 **54.** 3·128 HP **55.** 164·1 watt **56.** (*a*) 5 min, 36 s (*b*) 257·143 k cal **57.** (*a*) 13·5 (*b*) $3{\cdot}11\times10^4$ J, (*c*) 129·6 watt **58.** 15 **59.** 85 cal **60.** Fig. 52.

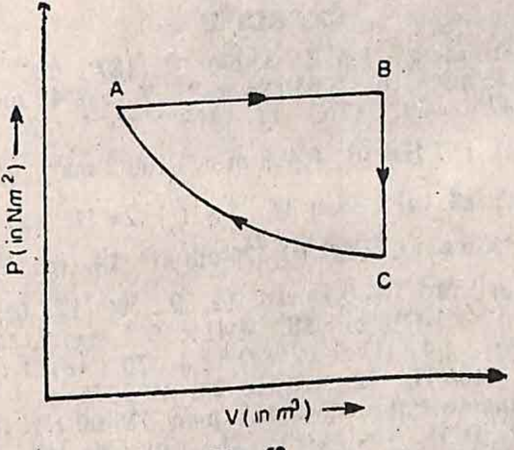

Fig. 52

Objective Type Questions

61. *b* **62.** *d* **63.** *c* **64.** *a* **65.** *d* **66.** *b* **67.** *c* **68.** *d* **69.** *a* **70.** *c* **71.** *d* **72.** *c* **73.** *d* **74.** *b* **75.** *a* **76.** *a* **77.** *b* **78.** *d* **79.** *a* **80.** *d*.

EXERCISE 12

1. (*a*) 2 Hz, 1·5 Hz, 1·2 Hz, 1 Hz (*b*) 1/π Hz, 3/4π Hz, 3/5π Hz, 1/2π Hz (*c*) π S, 4π/3 S, 5π/3 S, 2π S **2.** 50 Nm^{-1} **3.** 19·6 Nm^{-1}, 0·704 Hz **4.** 9·8 Nm^{-1}, 1·96 ms^{-2} **5.** 3 K **6.** (*a*) 39·2 Nm^{-1}, (*b*) 3·15 Hz (*c*) 0·792 ms^{-1}, (*d*) 15·68 ms^{-2} **7.** 1 2 **8.** - k, k/2, 2k **9.** (*a*) 5/π Hz, 0·20 ms^{-2}, 2 ms^{-1} (*b*) 62·8 N 54·8 N **10.** (*a*) Spring 'A' (*b*) Spring 'B' **11.** (*a*) 0·628 ms^{-1} (*b*) 7·88 ms^{-2} (*c*) 1·972 N **12.** 157 Hz, 25 Hz **13.** 15·7 Hz, 2·5 Hz

14. (*a*) 49·298 N (*b*) 2·465 J **15.** (*a*) 1 ms^{-1} (*b*) 10 ms^{-2} (*c*) 0·8 ms^{-1} **16.** 0·8 m **17.** 6·431 ms^{-1}, 2·067 $\times 10^4$ ms^{-2} **19.** 8·37 s, 14·62 cm **20.** (*a*) 0·15 ms^{-1} (*b*) 0·15 ms^{-2} (*c*) 0·015 N (*d*) 9·8 m **21.** 99·497 cm **22.** 4·4% **23.** (*a*) 0·1 J (*b*) 0·8 J **24.** 12·96 s slow **25.** 17·28 s fast **26.** (*a*) 2·4 mm (*b*) 2000 Hz (*c*) 1000/π Hz (*d*) π/1000 s (*e*) 0·64 rad **27.** (*a*) 3·5 mm (*b*) 250/π Hz (*c*) π/250 s (*d*) 0·42 rad (*e*) 1·75 ms^{-1} (*f*) 875 ms^{-2} **28.** 25·18 ms^{-1} **29.** 1·285 s, 0·778 Hz **30.** 0·61 s, 1·638 Hz **31.** 87·92 kg, 52·08 kg **32.** 1·59 Hz 0·628 s **33.** 10·069 Hz **34.** (*a*) 6 cm (*b*) 50/π Hz (*c*) π/4 rad (*d*) 6 ms^{-1} (*e*) 600 ms^{-2} (*f*) 18 J **35.** 2π s, π/2 s **36.** 2 s **37.** π cm/s **38.** 0·2 m, 99 rad s^{-1} **39.** 2·227 Hz **40.** (*a*) 0·705 Hz (*b*) Since length of mercury column will reduce to half, ν=0·498 Hz **41.** 2·0 cm **42.** 0·7 m, 180 Hz **43.** (*a*) 0·25 mm (*b*) 0·01256 s **44.** 98·596 N **45.** 44/49 s **46.** A/$\sqrt{2}$ **47.** (*a*) 48 $\pi^2 \times 10^{-6}$ J (*b*) 37·5 $\pi^2 \times 10^{-5}$ J **48.** (*i*) 0·26 mm (*ii*) 4000 s^{-1} (*iii*) 2000/π Hz (*iv*) 1·57 ms^{-1} (*v*) π/6 rad **49.** (*i*) a (*ii*) b (*iii*) $b/2\pi$ (*iv*) ab, ab^2 (*v*) $-ab \sin (bt+c)$ **50.** 60 N & 40 N.

Objective Type Questions

51. *d* **52.** *c* **53.** *b* **54.** *a* **55.** *b* **56.** *a* **57.** *b* **58.** *c* **59.** *a* **60.** *d* **61.** *c* **62.** *b* **63.** *a* **64.** *d* **65.** *c* **66.** *a* **67.** *d* **68.** *c*.

EXERCISE 13

1. 17 m to 1·7 cm **2.** 33 cm **3.** 187·5 ms^{-1} **4.** 1496·663 ms^{-1} **5.** 5270 ms^{-1} **6.** 5063·696 ms^{-1} **7.** 112·45 ms^{-1} **8.** 273·86 ms^{-1} **9.** 887°C **10.** 47°C **11.** 399·3 ms^{-1} **12.** (*a*) 0·14 mm (*b*) $\frac{2\pi}{157}$ s (*c*) 157 Hz (*d*) 6·408 m (*e*) 160·2 ms^{-1} (*f*) 2·198 ms^{-1} (*g*) 3·451 ms^{-2} **13.** (*a*) 1·2 cm (*b*) 1 s (*c*) 2π Hz (*d*) 200 m (*e*) 200 ms^{-1} (*f*) 7·536 cm s^{-1} (*g*) 47·326 cm s^{-2} **14.** (*a*) $\frac{\pi}{4}$ rad (*b*) 1·123 $\times 10^{-4}$ cm (*c*) 1 rad (*d*) 0·15 rad **16.** 9 : 500 **17.** (*a*) 3·988 $\times 10^{-2}$ wm^{-2} (*b*) 4·2 $\times 10^{-6}$ m **18.** 4·41 $\times 10^{-6}$ wm^{-2}, 2·25 $\times 10^{-8}$ m **19.** (*a*) 0 (*b*) 1 : 4 (*c*) 1 : 2 (*d*) 3 : 4 **20.** (*a*) 1 : 2 (*b*) 1 : 1 (*c*) 3 : 4 **21.** 1180 Hz **22.** 410 Hz **23.** 10·9375 ms^{-1} **24.** 623·4375 Hz, when engine approaches the man 525·00 Hz, when engine is going away from the man. **25.** (*a*) 690 Hz (*b*) 632·5 Hz **26.** (*a*) 531·25 Hz (*b*) 529·41 Hz (*c*) 531·76 Hz **27.** 61·2 kmh^{-1} **28.** 65·34 kmh^{-1} **29.** 769·2 Hz **30.** (*a*) 2000 Hz (*b*) no sound **31.** 20·91 kmh^{-1} **32.** 180 kmh^{-1} **33.** 1·2 $\times 10^7$ ms^{-1} **34.** 0·112 s, 31·4 m **35.** 0·70 m, 180 Hz **36.** π/2 **37.** 2 ms^{-1} **38.** 4 : 1 **39.** 300 ms^{-1} **40.** 2 cm **41.** $y=0{\cdot}5 \cos (4\pi t+2\pi x)$ metre **42.** (*a*) 2112 Hz (*b*) 1988 Hz **43.** 16 : 1 **44.** 262 Hz **45.** 507 Hz **46.** 476 Hz **47.** 323 Hz **48.** 388 Hz **49.** 262 Hz **50.** 25 : 1, 9 : 4 **51.** 5 : 4, 81 : 1 **52.** 2 : 1 **53.** 9π/2 **54.** 67 cm, 500 Hz **55.** 64 cm, 339·2 ms^{-1} **56.** 330 ms^{-1} **57.** 342 ms^{-1} **58.** 363 ms^{-1}, 363 Hz, 366·67 Hz **59.** 9·237 **60.** 8 **61.** 2 **62.** 133 Hz **63.** 5 **64.** (*a*) 100 ms^{-1} (*b*) 71·43 Hz (*c*) 142·86 Hz : 214·28 Hz **65.** 300 ms^{-1} **66.** 285·7 Hz **67.** 116·67 Hz **68.** 3 $\times 10^{-3}$ kg m^{-1} **69.** (*a*) By making the length half *i.e.*, 25 cm (*b*) by making the tension 4 times *i.e.*, 4 kg wt **70.** 320 Hz **71.** 1024 Hz **72.** $3{\cdot}157 \times 10^{-4}$ m **73.** 3268 kg m^{-3} **74.** (*a*) 250 Hz (*b*) 125 Hz **75.** (*a*) 339·84 ms^{-1} (*b*) 0·1 cm (*c*) 330·87 ms^{-1} **76.** 344·208 ms^{-1} **77.** (*a*) 0·8 cm (*b*) 353 6 ms^{-1} **78.** (*a*) 33·33 cm (*b*) 16·67 cm **79.** 5600 ms^{-1} **80.** 1250 Hz **81.** 480 ms^{-1} **82.** 5440 ms^{-1} **83.** 400 ms^{-1}, 200 Hz **84.** 338 ms^{-1}, 0·6 cm **85.** (*a*) 36 ms^{-1} (*b*) 64·8 N **86.** 31·25 cm, 93·75 cm and so on **87.** 0·12 m **88.** 100/133 ms^{-1} **89.** 6 beats s^{-1} **90.** 41 cm **91.** 470 Hz **92.** '*f*' **93.** 95 Hz, 12·74 g cm^{-3} **94.** 15 cm **95.** 125 Hz. **96.** 450 ms^{-1}, 2·5 cm **97.** 27·4 N **98.** (*a*) 100 ms^{-1} (*b*) 10 m (*c*) 6·28 ρ J m^{-3} (Since Value of 'ρ' is not given in question) **99.** 122·4 kmh^{-1}. **100.** (*a*) 369·3 ms^{-1} (*b*) 231 Hz **101.** 252 Hz. **102.** (*a*) 575 Hz (*b* 437·5 Hz **103.** 343·3 ms^{-1} **104.** 4·959 Hz **105.** 20 **106.** 1·1 cm, 318 Hz **107.** 9 : 16.

Objective Type Questions

108. *d* **109.** *c* **110.** *c* **111.** *b* **112.** *c* **113.** *a* **114.** *c* **115.** *c* **116.** *a* **117.** *c* **118.** *b* **119.** *d* **120.** *d* **121.** *a* **122.** *b*. **123.** *b* **124.** *c* **125.** *c* **126.** *b* **127.** *c* **128.** *a* **129.** *a* **130.** *c* **131.** *b* **132.** *a*.

□ □

Appendix

TABLES

1. PHYSICAL CONSTANTS

Symbol	*Name*	*Magnitude*
c	Speed of light	3×10^{8} ms^{-1}
h	Planck's constant	$6{\cdot}63 \times 10^{-34}$ J s
e	Electron charge	$1{\cdot}602 \times 10^{-19}$ coulomb
m_e	Electron mass	$9{\cdot}107 \times 10^{-31}$ Kg
m_n	Neutron mass	$1{\cdot}67479 \times 10^{-27}$ Kg $=1{\cdot}008665$ amu
m_p	Proton mass	$1{\cdot}67239 \times 10^{-27}$ Kg $=1{\cdot}007276$ amu
m_H	Hydrogen atom mass	$1{\cdot}67330 \times 10^{-27}$ Kg $=1{\cdot}007825$ amu
N	Avogadro's number	$6{\cdot}023 \times 10^{26}$ Kg mol^{-1}
R	Gas constant	$8{\cdot}314$ J mol^{-1} K^{-1}
μ_0	Permeability of free space	$4\pi \times 10^{-7}$ Wb A^{1} m^{-1}
ϵ_0	Permitivity of free space	$8{\cdot}854 \times 10^{-12}$ C^{2}N^{-1}m^{-2}
—	Hydrogen atom radius	$5{\cdot}29 \times 10^{-11}$ m
r_0	Hydrogen nucleus radius	$1{\cdot}2 \times 10^{-15}$ m
$\frac{1}{4\pi\epsilon_0}=K$	Coulomb's constant	9×10^{9} Nm^{2} C^{-2}

2. PREFIXES FOR UNIT

Prefix	*Power of ten*	*Abbreviation*
Atto	10^{-18}	a
Femto	10^{-15}	f
Pico	10^{-12}	p
Nano	10^{-9}	n
Micro	10^{-6}	μ
Milli	10^{-3}	m
Centi	10^{-2}	c
Kilo	10^{3}	K
Mega	10^{6}	M
Giga	10^{9}	G
Tera	10^{12}	T

3. CONSTANTS

$\pi = 3{\cdot}1416$, $\log_{10}\pi = 0{\cdot}4971$

1 radian $= 57{\cdot}296$ degree

$e = 2{\cdot}7183$, $\log_{10} e = 0{\cdot}4343$

$\log_e N = 2{\cdot}3026$ $\text{long}_{10}\, N$:

$\log_{10} N = 0{\cdot}4343 \log_e N$

$\sqrt{\pi} = 1{\cdot}7724$;

$\pi^2 = 9{\cdot}8696$

4. THE GREEK ALPHABETS

Name	*Symbol*	*Name*	*Symbol*
Alpha	α	Xi	ξ
Beta	β	Pi	π
Gamma	γ	Rho	ρ
Delta	δ or Δ	Sigma	σ, Σ
Epsilon	ε	Tau	τ
Zeta	ζ	Upsilon	ν
Eeta	η	Phi	ϕ
Theta	θ	Psi	ψ
Iota	*i*	Omega	ω
Kappa	K	Omcrion	o
Lambda	λ	Chi	χ
Mu	μ		
Nu	ν		

5. YOUNG MODULUS

	$\times 10^{10}$ Nm^{-2}
Aluminium	7
Brass	10
Copper	10
Glass	7
Nylon	0·2
Rubber	0·0001
Steel	20
Zinc	11

6. SOME PROPERTIES OF SOLIDS

Substance	*Melting Point °C*	*Specific Heat cal./gm./°C*	*Coefficient of Linear expansion* $\times 10^{-6}$
Aluminium	657	0·22	26
Bismuth	271	0·0303	12
Copper	1083	0·093	17
Gold	1063	0·0316	14
Iron (cast)	1100	0·119	10·2
Steel	1400	0·11	11·0
Lead	327	0·0305	29·2
Silver	961	0·056	19·2
Brass	900	0·092	18·9
Cork	—	0·49	—
Glass	1000	0·16	—
Platinum	1773	0·032	9

7. VISCOSITY OF LIQUIDS (in poise)

Substance	*Viscosity*	*Substance*	*Viscosity*
Alcohol (ethyl)	0·0119	Olive oil	0·98
Alcohol (methyl)	0·00591	Turpentine oil	0·0149
Benzene	0·00649	Water at 0°C	0·01793
Castor oil	9·86	Water at 20°C	0·01006
Chloroform	0·00564	Water at 25°C	0·00893
Ether	0·00234	Water at 30°C	0·00800
Glycerine	8·5	Water at 40°C	0·00657
Mercury	0·0156	Water at 60°C	0·00469
		Water at 100°C	0·00284

8. ACCELERATION DUE TO GRAVITY (metre/sec.^{2})

Place	*Acc. due to gravity* (g)	*Place*	*Acc. due to gravity* (g)
Agra	9·7906	Indore	9·7860
Ajmer	9·7890	Jaipur	9·7852
Aligarh	9·7808	Kanpur	9·7901
Allahabad	9·7895	Lucknow	9·7900
Amritsar	9·792	Ludhiana	9·792
Bangalore	9·780	Madras	9.7828
Bombay	9·7865	Meerut	9·7915
Calcutta	9·7878	Nagpur	9·7854
Dehradun	9·7907	Patna	9·784
Delhi	9·7915	Varanasi	9·7899
Gorakhpur	9·7905	Pole	9·8322
Gwalior	9·7897	Equator	9·7803
Hyderabad (Deccan)	9·789		

9. SURFACE TENSION OF WATER (dynes/cm.)

Temperature °*C*	*Surface Tension*	*Temperature* °*C*	*Surface Tension*	*Temperature* °*C*	*Surface Tension*
0	75·0	30	70·6	70	63·8
10	73·5	40	68·9	80	62·0
15	72·8	50	67·3	90	60·2
20	72·1	60	65·6	100	58·2

10. DENSITIES OF COMMON SUBSTANCES (g./c.c.)

Substance	*Density*	*Substance*	*Density*	*Substance*	*Density*
Solids		*Liquids*		*Gases*	
Aluminium	2·7	Alcohol	0·80	Air	·00129
Brass	8·6	Benzene	0·88	Carbon dioxide	·00198
Copper	8·89	Ether	0·74	Helium	·000179
Glass (Crown)	2·6	Glycerine	1·26	Hydrogen	·000090
Glass (Flint)	4·0	Lubricating oil	0·91	Steam (100°C)	·00061
Gold	19·3	Mercury	13·60		
Iron (cast)	7·5	Methylated spirit	0·83		
Iron (wrought)	7·9	Turpentine	0·87		
Lead	11·34				
Platinum	21·45				
Silver	10·5				
Steel	7·8				
Zinc	7·1				

11. DENSITIES OF WATER AND MERCURY (g./c.c.)

Temperature °C	*Water*	*Mercury*
0	0·9999	13·5955
4	1·0000	13·5856
10	0·997	13·5708
20	0·9982	13·5462
30	0·9957	13·5217
40	0·9922	13·4973
50	0·9881	13·4729
60	0·9832	13·4486
70	0·9778	13·4243
80	0·9718	13·4001
90	0·9653	13·3759
100	0·9584	13·3518
150	0·9170	13·2330
200	0·8630	13·1150
400	0·7000	12·8810

12. SURFACE TENSION AND ANGLE OF CONTACT

Substance	*Surface Tension*	*Angle of contact with glass*
Water	73 dynes/cm.	8°—9°
Mercury	465 dynes/cm.	137°
Turpentine	27·3 dynes/cm.	17°
Soap solution	20—40 dynes/cm.	……

13. VELOCITY OF SOUND (metres/sec.)

Substance	*Velocity*	*Substance*	*Velocity*	*Substance*	*Velocity*
Solids (20°C)		*Liquids* (20°C)		*Gases* (0°C)	
Aluminium	5100	Alcohol	1275	Air	332
Brass	3400	Mercury	1407	Carbon dioxide	259
Copper	3560	Turpentine	1326	Hydrogen	1262
Glass	5000	Water	1447	Nitrogen	338
Iron	5130			Oxygen	316
Steel	4990			Sulphur dioxide	209
				Water vapour	401

□□

APPENDIX

Logarithm Tables

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170						5 9 13	17 21 26	30 34 38
						0212	0253	0294	0334	0374	4 8 12	16 20 24	28 32 36
11	0414	0453	0492	0531	0569						4 8 12	16 20 23	27 31 35
						0607	0645	0682	0719	0755	4 7 11	15 18 22	26 29 33
12	0792	0828	0864	0899	0934						3 7 11	14 18 21	25 28 32
						0969	1004	1038	1072	1106	3 7 10	14 17 20	24 27 31
13	1139	1173	1206	1239	1271						3 6 10	13 16 19	23 26 29
						1303	1335	1367	1399	1430	3 7 10	13 16 19	22 25 29
14	1461	1492	1523	1553	1584						3 6 9	12 15 19	22 25 28
						1614	1644	1673	1703	1732	3 6 9	12 14 17	20 23 26
15	1761	1790	1818	1847	1875						3 6 9	11 14 17	20 23 26
						1903	1931	1959	1987	2014	3 6 8	11 14 17	19 22 25
16	2041	2068	2095	2122	2148						3 6 8	11 14 16	19 22 24
						2175	2201	2227	2253	2279	3 5 8	10 13 16	18 21 23
17	2304	2330	2355	2380	2405						3 5 8	10 13 15	18 20 23
						2430	2455	2480	2504	2529	3 5 8	10 12 15	17 20 22
18	2553	2577	2601	2625	2648						2 5 7	9 12 14	17 19 21
						2672	2695	2718	2742	2765	2 4 7	9 11 14	16 18 21
19	2788	2810	2833	2856	2878						2 4 7	9 11 13	16 18 20
						2900	2923	2945	2967	2989	2 4 6	8 11 13	15 17 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	123	456	789
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1	1 1 1	2 2 2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 2	2 2 2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1	1 2 2	2 3 3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1	1 2 2	2 3 3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1	1 2 2	2 3 3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1	1 2 2	3 3 3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1	2 2 2	3 3 3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1	2 2 2	3 3 3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0 1 1	2 2 3	3 4 4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1 1 2	2 3 3	4 4 5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2285	1 1 2	2 3 3	4 4 5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 3 4	5 6 6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 3 4	5 6 6

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	123	456	789
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1 1 2	3 4 4	5 6 7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1 2 2	3 4 5	5 6 7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1 2 2	3 4 5	5 6 7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1 2 2	3 4 5	6 6 7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1 2 2	3 4 5	6 6 7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1 2 2	3 4 5	6 7 7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1 2 3	3 4 5	6 7 8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1 2 3	3 4 5	6 7 8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1 2 3	4 4 5	6 7 8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1 2 3	4 5 5	6 7 8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1 2 3	4 5 6	6 7 8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1 2 3	4 5 6	7 8 9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1 2 3	4 5 6	7 8 9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1 2 3	4 5 6	7 8 9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1 2 3	4 5 6	7 8 9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1 2 3	4 5 6	7 8 9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1 2 3	4 5 6	7 9 10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1 2 3	4 5 7	8 9 10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1 2 3	4 6 7	8 9 10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1 2 3	5 6 7	8 9 10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1 2 4	5 6 7	8 9 11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1 2 4	5 6 7	8 10 11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1 2 4	5 6 7	9 10 11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1 3 4	5 6 8	9 10 11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1 3 4	5 6 8	9 10 12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1 3 4	5 7 8	9 10 12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1 3 4	5 7 8	9 11 12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1 3 4	5 7 8	10 11 12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1 3 4	6 7 8	10 11 13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1 3 4	6 7 9	10 11 13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1 3 4	6 7 9	10 12 13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2 3 5	6 8 9	11 12 14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2 3 5	6 8 9	11 12 14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2 3 5	6 8 9	11 13 14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2 3 5	6 8 10	11 13 15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2 3 5	7 8 10	12 13 15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2 3 5	7 8 10	12 13 15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2 3 5	7 9 10	12 14 16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2 4 5	7 9 11	12 14 16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2 4 5	7 9 11	13 14 16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2 4 6	7 9 11	13 15 17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2 4 6	8 9 11	13 15 17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2 4 6	8 10 12	14 15 17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2 4 6	8 10 12	14 16 18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2 4 6	8 10 12	14 16 18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2 4 6	8 10 12	15 17 19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2 4 6	8 11 13	15 17 19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2 4 7	9 11 13	15 17 20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2 4 7	9 11 13	16 18 20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2 5 7	9 11 14	16 18 20

NATURAL SINES

Degrees	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean Differences				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	·2419	2436	2452	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	·3256	3273	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	·3746	3762	3778	3795	3811	2827	3843	3859	3875	3891	3	5	8	11	14
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	·5446	5461	6476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	·5878	5892	5906	5920	5934	5848	5962	5976	5990	6004	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES

Degrees	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean Differences				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	1	2	3	4	5
45	·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	·7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	·9205	9212	9219	9225	9232	9239	9345	9252	9259	9265	1	2	3	4	6
68	·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	·9945	9947	9949	9951	9952	8854	9956	9957	9959	9960	0	1	1	1	2
85	·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	·9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0
90	1·000														

NATURAL COSINES

[Numbers in difference columns to be subtracted, not added.]

Degrees	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean Differences				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	1	2	3	4	5
0	1·000	1·000	1·000	1·000	1·000	1·000	9999	9999	9999	9999	0	0	0	0	0
1	·9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	·9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	·9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	·9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	·9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	·9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	·9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	·9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	·9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	·9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	·9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	·9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	·9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	·9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	·9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	·9613	·9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	·9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
18	·9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	·9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	·9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	·9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	·9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	·9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
24	·9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	·9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	·8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	·8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	·8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	·8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	·8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	·8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	·8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	·8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	·8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	·8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	.8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
37	·7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	·7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	·7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	·7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	·7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	·7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	·7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	·7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

NATURAL COSINES

[Numbers in difference columns to be subtracted, not added.]

Degrees	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean Differences				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	1	2	3	4	5
45	·7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	·6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	·6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	·6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	·6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50	·6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	·6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	·6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	·6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	·5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	·5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	·5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	·5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	·5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	·5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	·5000	4985	4970	4955	4939	4924	4900	4894	4879	4863	3	5	8	10	13
61	·4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	·4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	·4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
	·4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	·4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	·4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
67	·3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	14
68	·3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	·3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	·3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	·3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	·3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	·2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	·2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	·2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	·2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	·2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	·1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
80	·1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	·1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	·1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
83	·1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	·1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	·0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	15
86	·0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	·0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	·0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	·0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15
90	·0000														

NATURAL TANGENTS

Degrees	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

Degrees	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean Differences				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	1	2	3	4	5
45	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5830	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	2.0057	2.0145	2.0233	2.0323	2.0413	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2·9042	9208	9375	9544	9714	9887	3.0061	3.0237	3.0415	3.0595	29	58	87	116	145
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4·7046	7453	7867	8288	8716	9152	9594	5·0045	5·0504	5·0970					
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5·6713	7297	7894	8502	9124	9758	6·0405	6·1066	6·1742	6·2432					
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	7·0264					
82	7·1154	2066	3002	3962	4947	5958	6996	8062,	9158	8·0285					
83	8·1443	2636	3863	5126	6427	7769	9152	9·0579	9·2052	9·3572					
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16.83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23.86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40.92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·5	143.2	191·0	286·5	573·0					
90	∞														

Mean differences cease to be sufficiently accurate.

LOGARITHMS OF SINES

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences				
											1	2	3	4	5
0	$-\infty$	$\bar{3}.2419$	$\bar{3}.5429$	7190	8439	9408	$\bar{2}.0200$	$\bar{2}.0870$	$\bar{2}.1450$	$\bar{2}.1961$					
1	$\bar{2}.2419$	2832	3210	3558	3880	4179	4459	4723	4971	5206					
2	$\bar{2}.5428$	5640	5842	6035	6220	6397	6567	6731	6889	7041					
3	$\bar{2}.7188$	7330	7468	7602	7731	7857	7979	8098	8213	8326					
4	$\bar{2}.8436$	8543	8647	8749	8849	8946	9042	9135	9226	9315	16	32	48	64	80
5	$\bar{2}.9403$	9489	9573	9655	9736	9816	9894	9970	$\bar{1}.0046$	$\bar{1}.0120$	13	26	39	52	65
6	$\bar{1}.0192$	0264	0334	0403	0472	0539	0605	0670	0734	0797	11	22	33	44	55
7	$\bar{1}.0859$	0900	0981	1040	1099	1157	1214	1271	1326	1381	10	19	29	38	48
8	$\bar{1}.1436$	1489	1542	1594	1646	1697	1747	1797	1847	1895	8	17	25	34	42
9	$\bar{1}.1943$	1991	2038	2085	2131	2176	2221	2266	2310	2353	8	15	23	30	38
10	$\bar{1}.2397$	2439	2482	2524	2565	2606	2647	2687	2727	2767	7	14	20	27	34
11	$\bar{1}.2806$	2845	2883	2921	2959	2997	3034	3070	3107	3143	6	12	19	25	31
12	$\bar{1}.3179$	3214	3250	3284	3319	3353	3387	3421	3455	3488	6	11	17	23	28
13	$\bar{1}.3521$	3554	3585	3618	3650	3682	3713	3745	3775	3806	5	11	16	21	26
14	$\bar{1}.3837$	3867	3897	3927	3957	3986	4015	4044	4073	4102	5	10	15	20	24
15	$\bar{1}.4130$	4158	4186	4214	4242	4269	4296	4323	4350	4377	5	9	14	18	23
16	$\bar{1}.4403$	4430	4456	4482	4508	4533	4559	4584	4609	4634	4	9	13	17	21
17	$\bar{1}.4659$	4684	4709	4733	4757	4781	4805	4829	4853	4876	4	8	12	16	20
18	$\bar{1}.4900$	4923	4946	4969	4992	5015	5037	5060	5082	5104	4	8	11	15	19
19	$\bar{1}.5126$	5148	5170	5192	5213	5235	5256	5278	5299	5320	4	7	11	14	18
20	$\bar{1}.5341$	5361	5382	5402	5423	5443	5463	5484	5504	5523	3	7	10	14	17
21	$\bar{1}.5543$	5563	5583	5602	5621	5641	5660	5679	5698	5717	3	6	10	13	16
22	$\bar{1}.5736$	5754	5773	5792	5810	5828	5847	5865	5883	5901	3	6	9	12	15
23	$\bar{1}.5919$	5937	5954	5972	5990	6007	6024	6042	6059	6076	3	6	9	12	15
24	$\bar{1}.6093$	6110	6127	6144	6161	6177	6194	6210	6227	6243	3	6	8	11	14
25	$\bar{1}.6259$	6276	6292	6308	6324	6340	6356	6371	6387	6403	3	5	8	11	13
26	$\bar{1}.6418$	6434	6449	6465	6480	6495	6510	6526	6541	6556	3	5	8	10	13
27	$\bar{1}.6570$	6585	6600	6615	6629	6644	6659	6673	6687	6702	2	5	7	10	12
28	$\bar{1}.6716$	6730	6744	6759	6773	6787	6801	6814	6828	6842	2	5	7	9	12
29	$\bar{1}.6856$	6869	6883	6896	6910	6923	6937	6950	6863	6977	2	4	7	9	11
30	$\bar{1}.6990$	7003	7016	7029	7042	7055	7068	7080	7093	7106	2	4	6	9	11
31	$\bar{1}.7118$	7131	7144	7156	7168	7181	7193	7205	7218	7230	2	4	6	8	10
32	$\bar{1}.7242$	7254	7266	7278	7290	7302	7314	7326	7338	7349	2	4	6	8	10
33	$\bar{1}.7361$	7373	7384	7396	7407	7419	7430	7442	7453	7464	2	4	6	8	10
34	$\bar{1}.7476$	7487	7498	7509	7520	7531	7542	7553	7564	7575	2	4	6	7	9
35	$\bar{1}.7586$	7597	7607	7618	7629	7640	7650	7661	7671	7682	2	4	5	7	9
36	$\bar{1}.7692$	7703	7713	7723	7734	7744	7754	7764	7774	7785	2	3	5	7	9
37	$\bar{1}.7795$	7805	7815	7825	7835	7844	7854	7864	7874	7884	2	3	5	7	8
38	$\bar{1}.7893$	7903	7913	7922	7932	7941	7951	7960	7970	7979	2	3	5	6	8
39	$\bar{1}.7989$	7998	8007	8017	8026	8035	8044	8053	8063	8072	2	3	5	6	8
40	$\bar{1}.8081$	8090	8099	8108	8117	8125	8134	8143	8152	8161	1	3	4	6	7
41	$\bar{1}.8169$	8178	8187	8195	8204	8213	8221	8230	8238	8247	1	3	4	6	7
42	$\bar{1}.8255$	8264	8272	8280	8289	8297	8305	8313	8322	8330	1	3	4	6	7
43	$\bar{1}.8338$	8346	8354	8362	8370	8378	8386	8394	8402	8410	1	3	4	5	7
44	$\bar{1}.8418$	8426	8433	8441	8449	8457	8464	8472	8480	8487	1	3	4	5	6

LOGARITHMS OF SINES

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences 1	 2	 3	 4	 5
45	$\bar{1}$·8495	8502	8510	8517	8525	8532	8540	8547	8555	8562	1	2	4	5	6
46	$\bar{1}$·8569	8577	8584	8591	8598	8606	8613	8620	8627	8634	1	2	4	5	6
47	$\bar{1}$·8641	8648	8655	8662	8669	8676	8683	8690	8697	8704	1	2	3	5	6
48	$\bar{1}$·8711	8718	8724	8731	8738	8745	8751	8758	8765	8771	1	2	3	4	6
49	$\bar{1}$·8778	8784	8791	8797	8804	8810	8817	8823	8830	8836	1	2	3	4	5
50	$\bar{1}$·8843	8849	8855	8862	8868	8874	8880	8887	8893	8899	1	2	3	4	5
51	$\bar{1}$·8905	8911	8917	8923	8929	8935	8941	8947	8953	8959	1	2	3	4	5
52	$\bar{1}$·8965	8971	8977	8983	8989	8995	9000	9006	9012	9018	1	2	3	4	5
53	$\bar{1}$·9023	9029	9035	9041	9046	9052	9057	9063	9069	9074	1	2	3	4	5
54	$\bar{1}$·9080	9085	9091	9096	9101	9107	9112	9118	9123	9128	1	2	3	4	5
55	$\bar{1}$·9134	9139	9144	9149	9155	9160	9165	9170	9175	9181	1	2	3	3	4
56	$\bar{1}$·9186	9191	9196	9201	9206	9211	9216	9221	9226	9231	1	2	3	3	4
57	$\bar{1}$·9236	9241	9246	9251	9255	9260	9265	9270	9275	9279	1	2	2	3	4
58	$\bar{1}$·9284	9289	9294	9298	9303	9308	9312	9317	9322	9326	1	2	2	3	4
59	$\bar{1}$·9331	9335	9340	9344	9349	9353	9358	9362	9367	9371	1	1	2	3	4
60	$\bar{1}$·9375	9380	9384	9388	9393	9397	9401	9406	9410	9414	1	1	2	3	4
61	$\bar{1}$·9418	9422	9427	9431	9435	9439	9443	9447	9451	9455	1	1	2	3	3
62	$\bar{1}$·9459	9463	9467	9471	9475	9479	9483	9487	9491	9495	1	1	2	3	3
63	$\bar{1}$·9499	9503	9506	9510	9514	9518	9522	9525	9529	9533	1	1	2	3	3
64	$\bar{1}$·9537	9540	9544	9548	9551	9555	9558	9562	9566	9569	1	1	2	2	3
65	$\bar{1}$·9573	9576	9580	9583	9587	9590	9594	9597	9601	9604	1	1	2	2	3
66	$\bar{1}$·9607	9610	9614	9617	9621	9624	9627	9631	9634	9637	1	1	2	2	3
67	$\bar{1}$·9640	9643	9647	9650	9653	9656	9659	9662	9666	9669	1	1	2	2	3
68	$\bar{1}$·9672	9674	9678	9681	9684	9687	9690	9693	9696	9699	0	1	1	2	2
69	$\bar{1}$·9702	9704	9707	9710	9713	9716	9719	9722	9724	9727	0	1	1	2	2
70	$\bar{1}$·9730	9733	9735	9738	9741	9743	9746	9749	9751	9754	0	1	1	2	2
71	$\bar{1}$·9757	9759	9762	9764	9767	9770	9772	9775	9777	9780	0	1	1	2	2
72	$\bar{1}$·9782	9785	9787	9789	9792	9794	9797	9799	9801	9804	0	1	1	2	2
73	$\bar{1}$·9806	9808	9811	9813	9815	9817	9820	9822	9824	9826	0	1	1	2	2
74	$\bar{1}$·9828	9831	9833	9835	9837	9839	9841	9843	9845	9847	0	1	1	1	2
75	$\bar{1}$·9849	9851	9853	9855	9857	9859	9861	9863	9865	9867	0	1	1	1	2
76	$\bar{1}$·9869	9871	9873	9875	9876	9878	9880	9882	9884	9885	0	1	1	1	2
77	$\bar{1}$·9887	9889	9891	9892	9894	9896	9897	9899	9901	9902	0	1	1	1	1
78	$\bar{1}$·9904	9906	9907	9909	9910	9912	9913	9915	9916	9918	0	1	1	1	1
79	$\bar{1}$·9919	9921	9922	9924	9925	9927	9928	9929	9931	9932	0	0	1	1	1
80	$\bar{1}$·9934	9935	9936	9937	9939	9940	9941	9943	9944	9945	0	0	1	1	1
81	$\bar{1}$·9946	9947	9949	9950	9951	9952	9953	9954	9955	9956	0	0	1	1	1
82	$\bar{1}$·9958	9959	9960	9961	9962	9963	9964	9965	9966	9967	0	0	1	1	1
83	$\bar{1}$·9968	9968	9969	9970	9971	9972	9973	9974	9975	9975	0	0	0	1	1
84	$\bar{1}$·9976	9977	9978	9978	9979	9980	9981	9981	9982	9983	0	0	0	0	1
85	$\bar{1}$·9983	9984	9985	9985	9986	9987	9987	9988	9988	9989	0	0	0	0	0
86	$\bar{1}$·9989	9990	9990	9991	9991	9992	9992	9993	9993	9994	0	0	0	0	0
87	$\bar{1}$·9994	9994	9995	9995	9996	9996	9996	9996	9997	9997	0	0	0	0	0
88	$\bar{1}$·9997	9998	9998	9998	9998	9999	9999	9999	9999	9999	0	0	0	0	0
89	$\bar{1}$·9999	9999	0.0000	0000	0000	0000	0000	0000	0000	0000					
90	0·0000														

LOGARITHMS OF COSINES

[Numbers in difference columns to be subtracted, not added.]

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences	
											1 2 3	4 5
0	0.0000	0000	0000	0000	0000	0000	0000	0000	0000	$\bar{1}.9999$	0 0 0	0 0
1	$\bar{1}.9999$	9999	9999	9999	9999	9999	9998	9998	9998	9998	0 0 0	0 0
2	$\bar{1}.9997$	9997	9997	9996	9996	9996	9996	9995	9995	9994	0 0 0	0 0
3	$\bar{1}.9994$	9994	9993	9993	9992	9992	9991	9991	9990	9990	0 0 0	0 0
4	$\bar{1}.9989$	9989	9988	9988	9987	9987	9986	9985	9985	9984	0 0 0	0 0
5	$\bar{1}.9983$	9983	9982	9981	9981	9980	9979	9978	9978	9977	0 0 0	0 1
6	$\bar{1}.9976$	9975	9975	9974	9973	9972	9971	9970	9969	9968	0 0 0	1 1
7	$\bar{1}.9968$	9967	9966	9965	9964	9963	9962	9961	9960	9959	0 0 1	1 1
8	$\bar{1}.9958$	9956	9955	9954	9953	9952	9951	9950	9949	9947	0 0 1	1 1
9	$\bar{1}.9946$	9945	9944	9943	9941	9940	9939	9937	9936	9935	0 0 1	1 1
10	$\bar{1}.9934$	9932	9931	9929	9928	9927	9925	9924	9922	9921	0 0 1	1 1
11	$\bar{1}.9919$	9918	9916	9915	9913	9912	9910	9909	9907	9906	0 1 1	1 1
12	$\bar{1}.9904$	9902	9901	9899	9897	9896	9894	9892	9891	9889	0 1 1	1 1
13	$\bar{1}.9887$	9885	9884	9882	9880	9878	9876	9875	9873	9871	0 1 1	1 2
14	$\bar{1}.9869$	9867	9865	9863	9861	9859	9857	9855	9853	9851	0 1 1	1 2
15	$\bar{1}.9849$	9847	9845	9843	9841	9839	9837	9835	9833	9831	0 1 1	1 2
16	$\bar{1}.9828$	9826	9824	9822	9820	9817	9815	9813	9811	9808	0 1 1	2 2
17	$\bar{1}.9806$	9804	9801	9799	9797	9794	9792	9789	9787	9785	0 1 1	2 2
18	$\bar{1}.9782$	9780	9777	9775	9772	9770	9767	9764	9762	9759	0 1 1	2 2
19	$\bar{1}.9757$	9754	9751	9749	9746	9743	9741	9738	9735	9733	0 1 1	2 2
20	$\bar{1}.9730$	9727	9724	9722	9719	9716	9713	9710	9707	9704	0 1 1	2 2
21	$\bar{1}.9702$	9699	9696	9693	9690	9687	9684	9681	9678	9675	0 1 1	2 2
22	$\bar{1}.9672$	9669	9666	9662	9659	9656	9653	9650	9647	9643	1 1 2	2 3
23	$\bar{1}.9640$	9637	9634	9631	9627	9624	9621	9617	9614	9611	1 1 2	2 3
24	$\bar{1}.9607$	9604	9601	9597	9594	9590	9587	9583	9580	9576	1 1 2	2 3
25	$\bar{1}.9573$	9569	9566	9562	9558	9555	9551	9548	9544	9540	1 1 2	2 3
26	$\bar{1}.9537$	9533	9529	9525	9522	9518	9514	9510	9506	9503	1 1 2	3 3
27	$\bar{1}.9499$	9495	9491	9487	9483	9479	9475	9471	9467	9463	1 1 2	3 3
28	$\bar{1}.9459$	9455	9451	9447	9443	9439	9435	9431	9427	9422	1 1 2	3 3
29	$\bar{1}.9418$	9414	9410	9406	9401	9397	9393	9388	9384	9380	1 1 2	3 4
30	$\bar{1}.9375$	9371	9367	9362	9358	9353	9349	9344	9340	9335	1 1 2	3 4
31	$\bar{1}.9331$	9326	9322	9317	9312	9308	9303	9298	9294	9289	1 2 2	3 4
32	$\bar{1}.9284$	9279	9275	9270	9265	9260	9255	9251	9246	9241	1 2 2	3 4
33	$\bar{1}.9236$	9231	9226	9221	9216	9211	9206	9201	9196	9191	1 2 3	3 4
34	$\bar{1}.9186$	9181	9175	9170	9165	9160	9155	9149	9144	9139	1 2 3	3 4
35	$\bar{1}.9134$	9128	9123	9118	9112	9107	9101	9096	9091	9085	1 2 3	4 5
36	$\bar{1}.9080$	9074	9069	9063	9057	9052	9046	9041	9035	9029	1 2 3	4 5
37	$\bar{1}.9023$	9018	9012	9006	9000	8995	8989	8983	8977	8971	1 2 3	4 5
38	$\bar{1}.8965$	8959	8953	8947	8941	8935	8929	8923	8917	8911	1 2 3	4 5
39	$\bar{1}.8905$	8899	8893	8887	8880	8874	8868	8862	8855	8840	1 2 3	4 5
40	$\bar{1}.8843$	8836	8830	8823	8817	8810	8804	8797	8791	8784	1 2 3	4 5
41	$\bar{1}.8778$	8771	8765	8758	8751	8745	8738	8731	8724	8718	1 2 3	5 6
42	$\bar{1}.8711$	8704	8697	8690	8683	8676	8669	8662	8655	8648	1 2 3	5 6
43	$\bar{1}.8641$	8634	8627	8620	8613	8606	8598	8591	8584	8577	1 2 4	5 6
44	$\bar{1}.8569$	8562	8555	8547	8540	8532	8525	8517	8510	8502	1 2 4	5 6

LOGARITHMS OF COSINES

[Numbers in difference columns to be subtracted, not added].

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences				
											1	2	3	4	5
45	$\bar{1}.8495$	8487	8480	8472	8464	8457	8449	8441	8433	8426	1	3	4	5	6
46	$\bar{1}.8418$	8410	8402	8394	8386	8378	8370	8362	8354	8346	1	3	4	5	7
47	$\bar{1}.8338$	8330	8322	8313	8305	8297	8289	8280	8272	8264	1	3	4	6	7
48	$\bar{1}.8255$	8247	8238	8230	8221	8213	8204	8195	8187	8178	1	3	4	6	7
49	$\bar{1}.8169$	8161	8152	8143	8134	8125	8117	8108	8099	8090	1	3	4	6	7
50	$\bar{1}.8081$	8072	8063	8053	8044	8035	8026	8017	8007	7998	2	3	5	6	8
51	$\bar{1}.7989$	7979	7970	7960	7951	7941	7932	7922	7913	7903	2	3	5	6	8
52	$\bar{1}.7893$	7884	7874	7864	7854	7844	7835	7825	7815	7805	2	3	5	7	8
53	$\bar{1}.7795$	7785	7774	7764	7754	7744	7734	7723	7713	7703	2	3	5	7	9
54	$\bar{1}.7692$	7682	7671	7661	7650	7640	7629	7618	7607	7597	2	4	5	7	9
55	$\bar{1}.7586$	7575	7564	7553	7542	7531	7520	7509	7498	7487	2	4	6	7	9
56	$\bar{1}.7476$	7464	7453	7442	7430	7419	7407	7396	7384	7373	2	4	6	8	10
57	$\bar{1}.7361$	7349	7338	7326	7314	7302	7290	7278	7266	7254	2	4	6	8	10
58	$\bar{1}.7242$	7230	7218	7205	7193	7181	7168	7156	7144	7131	2	4	6	8	10
59	$\bar{1}.7118$	7106	7093	7080	7068	7055	7042	7029	7016	7003	2	4	6	9	11
60	$\bar{1}.6990$	6977	6963	6950	6937	6923	6910	6896	6883	6869	2	4	7	9	11
61	$\bar{1}.6856$	6842	6828	6814	6801	6787	6773	6759	6744	6730	2	5	7	9	12
62	$\bar{1}.6716$	6702	6687	6673	6659	6644	6629	6615	6600	6585	2	5	7	10	12
63	$\bar{1}.6570$	6556	6541	6526	6510	6495	6480	6465	6449	6434	3	5	8	10	13
64	$\bar{1}.6418$	6403	6387	6371	6356	6340	6324	6308	6292	6276	3	5	8	11	13
65	$\bar{1}.6259$	6243	6227	6210	6194	6177	6161	6144	6127	6110	3	6	8	11	14
66	$\bar{1}.6093$	6076	6059	6042	6024	6007	5990	5972	5954	5937	3	6	9	12	15
67	$\bar{1}.5919$	5901	5883	5865	5847	5828	5810	5792	5773	5754	3	6	9	12	15
68	$\bar{1}.5736$	5717	5698	5679	5660	5641	5621	5602	5583	5563	3	6	10	13	16
69	$\bar{1}.5543$	5523	5504	5484	5463	5443	5423	5402	5382	5361	3	7	10	14	17
70	$\bar{1}.5341$	5320	5299	5278	5256	5235	5213	5192	5170	5148	4	7	11	14	18
71	$\bar{1}.5126$	5104	5082	5060	5037	5015	4992	4969	4946	4923	4	8	11	15	19
72	$\bar{1}.4900$	4876	4853	4829	4805	4781	4757	4733	4709	4684	4	8	12	16	20
73	$\bar{1}.4659$	4634	4609	4584	4559	4533	4508	4482	4456	4430	4	9	13	17	21
74	$\bar{1}.4403$	4377	4350	4323	4296	4269	4242	4214	4186	4158	5	9	14	18	23
75	$\bar{1}.4130$	4102	4073	4044	4015	3986	3957	3927	3897	3867	5	10	15	20	24
76	$\bar{1}.3837$	3806	3775	3745	3713	3682	3650	3618	3586	3554	5	11	16	21	26
77	$\bar{1}.3521$	3488	3455	3421	3387	3353	3319	3284	3250	3214	6	11	17	23	28
78	$\bar{1}.3179$	3143	3107	3070	3034	2997	2959	2921	2883	2845	6	12	19	25	31
79	$\bar{1}.2806$	2767	2727	2687	2647	2606	2565	2524	2482	2439	7	14	20	27	34
80	$\bar{1}.2397$	2353	2310	2266	2221	2176	2131	2085	2038	1991	8	15	23	30	38
81	$\bar{1}.1943$	1895	1847	1797	1747	1697	1646	1594	1542	1489	8	17	25	34	42
82	$\bar{1}.1436$	1381	1326	1271	1214	1157	1099	1040	0981	0920	10	19	29	38	48
83	$\bar{1}.0859$	0797	0734	0670	0605	0539	0472	0403	0334	0264	11	22	33	44	55
84	$\bar{1}.0192$	0120	0046	$\bar{2}.9970$	$\bar{2}.9894$	$\bar{2}.9816$	$\bar{2}.9736$	$\bar{2}.9655$	$\bar{2}.9573$	$\bar{2}.9489$	13	26	39	52	65
85	$\bar{2}.9403$	9315	9226	9135	9042	8946	8849	8749	8647	8543	16	32	48	64	80
86	$\bar{2}.8436$	8326	8213	8098	7979	7857	7731	7602	7468	7330					
87	$\bar{2}.7188$	7041	6889	6731	6567	6397	6220	6035	5842	5640					
88	$\bar{2}.5428$	5206	4971	4723	4459	4179	3880	3558	3210	2832					
89	$\bar{2}.2419$	1961	1450	0870	0200	$\bar{3}.9408$	$\bar{3}.8439$	$\bar{3}.7190$	$\bar{3}.5429$	$\bar{3}.2419$					
90	∞														

LOGARITHMS OF TANGENTS

Degrees	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	1	2	3	4	5
0	$-\infty$	$\bar{3}.2419$	$\bar{3}.5429$	$\bar{3}.7190$	$\bar{3}.8439$	$\bar{3}.9409$	$\bar{2}.0200$	$\bar{2}.0870$	$\bar{2}.1450$	$\bar{2}.1962$					
1	$\bar{2}.2419$	2833	3211	3559	3881	4181	4461	4725	4973	5208					
2	$\bar{2}.5431$	5643	5845	6038	6223	6401	6571	6736	6894	7046					
3	$\bar{2}.7194$	7337	7475	7609	7739	7865	7988	8107	8223	8336					
4	$\bar{2}.8446$	8554	8659	8762	8862	8960	9056	9150	9241	9331	16	32	48	64	81
5	$\bar{2}.9420$	9506	9591	9674	9756	9836	9915	9992	$\bar{1}.0068$	$\bar{1}.0143$	13	26	40	53	66
6	$\bar{1}.0216$	0289	0360	0430	0499	0567	0633	0699	0764	0828	11	22	34	45	56
7	$\bar{1}.0891$	0954	1015	1076	1135	1194	1252	1310	1367	1423	10	20	29	39	49
8	$\bar{1}.1478$	1533	1587	1640	1693	1745	1797	1848	1898	1948	9	17	26	35	43
9	$\bar{1}.1997$	2046	2094	2142	2189	2236	2282	2328	2374	2419	8	16	23	31	39
10	$\bar{1}.2463$	2507	2551	2594	2637	2680	2722	2764	2805	2846	7	14	21	28	35
11	$\bar{1}.2887$	2927	2967	3006	3046	3085	3123	3162	3200	3237	6	13	19	26	32
12	$\bar{1}.3275$	3312	3349	3385	3422	3458	3493	3529	3564	3599	6	12	18	24	30
13	$\bar{1}.3634$	3668	3702	3736	3770	3804	3837	3870	3903	3935	6	11	17	22	28
14	$\bar{1}.3968$	4000	4032	4064	4095	4127	4158	4189	4220	4250	5	10	16	21	26
15	$\bar{1}.4281$	4311	4341	4371	4400	4430	4459	4488	4517	4646	5	10	15	20	25
16	$\bar{1}.4575$	4603	4632	4660	4688	4716	4744	4771	4799	4826	5	9	14	19	23
17	$\bar{1}.4853$	4880	4907	4934	4961	4987	5014	5040	5066	5092	4	9	13	18	22
18	$\bar{1}.5118$	5143	5169	5195	5220	5245	5270	5295	5320	5345	4	8	13	17	21
19	$\bar{1}.5370$	5394	5419	5443	5467	5491	5516	5539	5563	5587	4	8	12	16	20
20	$\bar{1}.5611$	5634	5658	5681	5704	5727	5750	5773	5796	5819	4	8	12	15	19
21	$\bar{1}.5842$	5864	5887	5909	5932	5954	5976	5998	6020	6042	4	7	11	15	19
22	$\bar{1}.6064$	6086	6108	6129	6151	6172	6194	6215	6236	6257	4	7	11	14	18
23	$\bar{1}.6279$	6300	6321	6341	6362	6383	6404	6424	6445	6465	3	7	10	14	17
24	$\bar{1}.6486$	6506	6527	6547	6567	6587	6607	6627	6647	6667	3	7	10	13	17
25	$\bar{1}.6687$	6706	6726	6746	6765	6785	6804	6824	6843	6863	3	7	10	13	16
26	$\bar{1}.6882$	6901	6920	6939	6958	6977	6996	7015	7034	7053	3	6	9	13	16
27	$\bar{1}.7072$	7090	7109	7128	7146	7165	7183	7202	7220	7238	3	6	9	12	15
28	$\bar{1}.7257$	7275	7293	7311	7330	7348	7366	7384	7402	7420	3	6	9	12	15
29	$\bar{1}.7438$	7455	7473	7491	7509	7526	7544	7562	7579	7597	3	6	9	12	15
30	$\bar{1}.7614$	7632	7649	7667	7684	7701	7719	7736	7753	7771	3	6	9	12	14
31	$\bar{1}.7788$	7805	7822	7839	7856	7873	7890	7907	7924	7941	3	6	9	11	14
32	$\bar{1}.7958$	7975	7992	8008	8025	8042	8059	8075	8092	8109	3	6	8	11	14
33	$\bar{1}.8125$	8142	8158	8175	8191	8208	8224	8241	8257	8274	3	5	8	11	14
34	$\bar{1}.8290$	8306	8323	8339	8355	8371	8388	8404	8420	8436	3	5	8	11	14
35	$\bar{1}.8452$	8468	8484	8501	8517	8533	8549	8565	8581	8597	3	5	8	11	13
36	$\bar{1}.8613$	8629	8644	8660	8676	8692	8708	8724	8740	8755	3	5	8	11	13
37	$\bar{1}.8771$	8787	8803	8818	8834	8850	8865	8881	8897	8912	3	5	8	10	13
38	$\bar{1}.8928$	8944	8959	8975	8990	9006	9022	9037	9053	9068	3	5	8	10	13
39	$\bar{1}.9084$	9099	9115	9130	9146	9161	9176	9192	9207	9223	3	5	8	10	13
40	$\bar{1}.9238$	9254	9269	9284	9300	9315	9330	9346	9361	9376	3	5	8	10	13
41	$\bar{1}.9392$	9407	9422	9438	9453	9468	9483	9499	9514	9529	3	5	8	10	13
42	$\bar{1}.9544$	9560	9575	9590	9605	9621	9636	9651	9666	9681	3	5	8	10	13
43	$\bar{1}.9697$	9712	9727	9742	9757	9772	9788	9803	9818	9833	3	5	8	10	13
44	$\bar{1}.9848$	9864	9879	9894	9909	9924	9939	9955	9970	9985	3	5	8	10	13

LOGARITHMS OF TANGENTS

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences				
											1	2	3	4	5
45	·0000	0015	0030	0045	0061	0076	0091	0106	0121	0136	3	5	8	10	13
46	·0152	0167	0182	0197	0212	0228	0243	0258	0273	0288	3	5	8	10	13
47	·0303	0319	0334	0349	0364	0379	0395	0410	0425	0440	3	5	8	10	13
48	·0456	0471	0486	0501	0517	0532	0547	0562	0578	0593	3	5	8	10	13
49	·0608	0624	0639	0654	0670	0685	0700	0716	0731	0746	3	5	8	10	13
50	·0762	0777	0793	0808	0824	0839	0854	0870	0885	0901	3	5	8	10	13
51	·0916	0932	0947	0963	0978	0994	1010	1025	1041	1056	3	5	8	10	13
52	·1072	1088	1103	1119	1135	1150	1166	1182	1197	1213	3	5	8	10	13
53	·1229	1245	1260	1276	1292	1308	1324	1340	1356	1371	3	5	8	11	13
54	·1387	1403	1419	1435	1451	1467	1483	1499	1516	1532	3	5	8	11	13
55	·1548	1564	1580	1596	1612	1629	1645	1661	1677	1694	3	5	8	11	14
56	·1710	1726	1743	1759	1776	1792	1809	1825	1842	1858	3	5	8	11	14
57	·1875	1891	1908	1925	1941	1958	1975	1992	2008	2025	3	6	8	11	14
58	·2042	2059	2076	2093	2110	2127	2144	2161	2178	2195	3	6	9	11	14
59	·2212	2229	2247	2264	2281	2299	2316	2333	2351	2368	3	6	9	12	14
60	·2386	2403	2421	2438	2456	2474	2491	2509	2527	2545	3	6	9	12	15
61	·2562	2580	2598	2616	2634	2652	2670	2689	2707	2725	3	6	9	12	15
62	·2743	2762	2780	2798	2817	2835	2854	2872	2891	2910	3	6	9	12	15
63	·2928	2947	2966	2985	3004	3023	3042	3061	3080	3099	3	6	9	13	16
64	·3118	3137	3157	3176	3196	3215	3235	3254	3274	3294	3	6	10	13	16
65	·3313	3333	3353	3373	3393	3413	3433	3453	3473	3494	3	7	10	13	17
66	·3514	3535	3555	3576	3596	3617	3638	3659	3679	3700	3	7	10	14	17
67	·3721	3743	3764	3785	3806	3828	3849	3871	3892	3914	4	7	11	14	18
68	·3936	3958	3980	4002	4024	4046	4068	4091	4113	4136	4	7	11	15	19
69	·4158	4181	4204	4227	4250	4273	4296	4319	4342	4366	4	8	12	15	19
70	·4389	4413	4437	4461	4484	4509	4533	4557	4581	4606	4	8	12	16	20
71	·4630	4655	4680	4705	4730	4755	4780	4805	4831	4857	4	8	13	17	21
72	·4882	4908	4934	4960	4986	5013	5039	5066	5093	5120	4	9	13	18	22
73	·5147	5174	5201	5229	5256	5284	5312	5340	5368	5397	5	9	14	19	23
74	·5425	5454	5483	5512	5541	5570	5600	5629	5659	5689	5	10	15	20	25
75	·5719	5750	5780	5811	5842	5873	5905	5936	5968	6000	5	10	16	21	26
76	·6032	6065	6097	6130	6163	6196	6230	6264	6298	6332	6	11	17	22	28
77	·6366	6401	6436	6471	6507	6542	6578	6615	6651	6688	6	12	18	24	30
78	·6725	6763	6800	6838	6877	6915	6954	6904	7033	7073	6	13	19	26	32
79	·7113	7154	7195	7236	7278	7320	7363	7406	7449	7493	7	14	21	28	35
80	·7537	7581	7626	7672	7718	7764	7811	7858	7906	7954	8	16	23	31	39
81	·8003	8052	8102	8152	8203	8255	8307	8360	8413	8467	9	17	26	35	43
82	·8522	8577	8633	8690	8748	8806	8865	8924	8985	9046	10	20	29	39	49
83	·9109	9172	9236	9301	9367	9433	9501	9570	9640	9711	11	22	34	45	56
84	·9784	9857	9932	1·0008	1·0085	1·0164	1·0244	1·0326	1·0409	1·0494	13	26	40	53	66
85	1·0580	0669	0759	0850	0944	1040	1138	1238	1341	1446	16	32	48	64	81
86	1·1554	1664	1777	1893	2012	2135	2261	2391	2525	2663					
87	1·2806	2954	3106	3264	3429	3599	3777	3962	4155	4357					
88	1·4569	4792	5027	5275	5539	5819	6119	6441	6789	7167					
89	1·7581	8038	8550	9130	9800	2·0591	2·1561	2·2810	2·4571	2·7581					